



YAŞAR UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

MASTER THESIS

**DESIGNING ASYMMETRIC SHELL SYSTEMS BY
AUTOCLAVED AERATED CONCRETE BLOCKS:**

A PARTICLE BASED COMPUTATIONAL MODEL

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

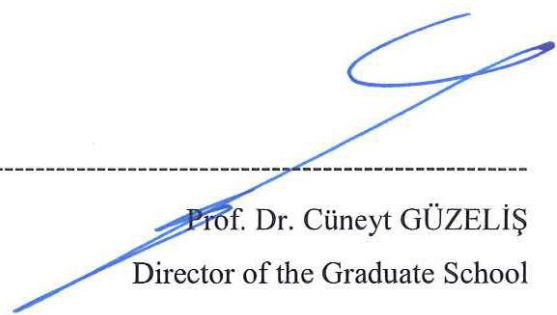
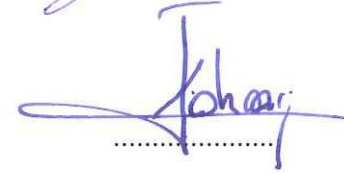

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ABSTRACT

DESIGNING ASYMMETRIC SHELL SYSTEMS BY AUTOCLAVED AERATED CONCRETE BLOCKS: A PARTICLE BASED COMPUTATIONAL MODEL

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Msc in Architecture

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Masonry vault structures have been used in many significant buildings in architecture for many centuries and have been applied by many civilizations as an important knowledge of construction in architecture. Today, vault and shell structures are still being used in various structural types and with various materials. With advances in computer-aided design technologies and modelling techniques, new form-finding methods have enabled us to design more complex structures in various forms. This thesis is on generating a computational model of symmetric and asymmetrically shaped shell systems by using “Autoclaved Aerated Concrete” (AAC) blocks. Thus, this thesis aims to find the appropriateness of AAC for material oriented design of shell systems and to study its behaviour in shell type constructions. In light of this research, it is aimed to develop a generic model of particle based asymmetrically shaped shell, which is more difficult to construct than symmetrical shell, via the geometrical predeterminations on shell making, hanging chain criteria and structural behaviour of AAC blocks.

The significance of the generic model is on the flexibility in parameters change such as material thickness, plan geometry, height and length of spans which bring an overall capability to Architects and designers who are not familiar with structural and statics aspects. This feature carry architects and designers to the idea of digital sketching in the very first steps of decision making while bringing benefits of computational design and integrated form finding methods.

Key Words: Computational Design, Vault and Shell Systems, Masonry Constructions, Autoclaved Aerated Concrete, Form Finding Methods, Digital Sketching



ÖZ

GAZBETON BLOKLARI İLE ASİMETRİK KABUK SİSTEMLERİN TASARLANMASI: PARÇACIK TABANLI JENERİK BİR MODEL

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Yığma yapılar, mimarlık tarihi boyunca yapı stoğunun önemli bir kısmını oluşturmuş ve bir çok önemli yapının bu bilgi ile ayağa kaldırılmasıyla uygarlıkların yapı bilgisi envanterine girmiştir. Günümüzde, yığma yapılar yerlerini daha hafif ve taşıyıcılıkta daha etkili malzemelerle oluşturulmuş olan kabuk sistemlere bırakmasına rağmen halen kullanılmaktadırlar. Bilgisayar destekli tasarım teknolojileri ve modelleme tekniklerindeki ilerlemeler ile, yeni form bulma yöntemleri çeşitli biçimlerde daha karmaşık yapılar tasarlamaya olanak vermektedir. Bu çalışma, ‘Gazbeton’ blokları kullanarak simetrik ve asimetrik biçimli kabuk sistemlerinin yığma taşıyıcılık prensiplere dayalı hesaplamalı modelinin üretilmesi üzerinedir. Bu sebeple, tezin kapsamı, kabuk sistemlerinin materyal odaklı tasarımı için gazbetonun uygunluğunu araştırmak ve kabuk tipi konstrüksiyonlardaki davranışlarını incelemek olarak belirlenmiştir. Bu araştırmanın ışığında, simetrik tonozlardan daha zor inşa edilen asimetrik tonoz ve kabuk sistemlerin, kabuk oluşturmada kullanılan geometrik öntanımları, zincir eğriliği kriterleri ve gazbeton bloklarının yapısal davranışları üzerinden, parçacık tabanlı amaca özgü bir genel modelinin geliştirilmesi amaçlanmıştır.

Bu jenerik modelin önemi, malzeme kalınlığı, plan geometrisi, kemer yüksekliği ve açıklıkları gibi parametrelerin esnek olmasıdır. Bu model yığma yapıların yapısal ve statik özellikleri konusunda mimarlar ve tasarımcılar için bir öngörü oluşturma ve erken tasarım evresinde taşıyıcılığa bağlı karar verebilme olanağı kazandırmaktadır. Bu kazanım ile, mimarlar ve tasarımcılar, bilgisayar ortamında, sayısal biçimlendirme yöntemlerinden yararlanarak, karar verme sürecinin ilk adımlarında bir bakıma dijital eskiz oluştururlar.

Anahtar Kelimeler: Hesaplamalı Tasarım, Tonoz ve Kabuk Sistemler, Yığma Yapılar, Otoklavlı Gazbeton, Form Bulma Yöntemleri, Dijital eskiz



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Esra CEVİZCİ

İzmir, 2017

TEXT OF OATH

I declare and honestly confirm that my study, titled “DESIGNING ASYMMETRIC SHELL SYSTEMS BY AUTOCLAVED AERATED CONCRETE BLOCKS: A PARTICLE BASED COMPUTATIONAL MODEL” and presented as a Master’s Thesis, has been written without applying to any assistance inconsistent with scientific ethics and traditions. I declare, to the best of my knowledge and belief, that all content and ideas drawn directly or indirectly from external sources are indicated in the text and listed in the list of references.

Esra CEVİZCİ

Signature,



February 17, 2017



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SYMBOLS AND ABBREVIATIONS

ABBREVIATIONS:

AAC Autoclaved Aerated Concrete

2D Two Dimensional

3D Three Dimensional

FEM Finite Element Method

FE Finite Element

FEA Finite Element Analysis

FDM Force Density Method

TNA Thrust Network Analysis

PHD Doctor of Philosophy

DRM Dynamic Relaxation Method

DR Dynamic Relaxation

PS Particle Spring Systems

ICD Institute for Computational Design of the University of Stuttgart

ITKE Institute of Building Structures and Structural Design of the University of
Stuttgart

NAAC Non- Autoclaved Aerated Concrete

SYMBOLS:

G	Compression Only Thrust Network
Γ	Form Diagram of TNA
Γ^*	Force Diagram of TNA
d_{uw}	Weight per unit of Volume / Density
n	Number of Blocks
L	Span Length
d	Displacement



CHAPTER ONE

INTRODUCTION

1.1. Statement of the Problem

Shell structures have always taken a significant place in architecture. The term of shell structure is described as; *“in building construction, a thin, curved plate structure shaped to transmit applied forces by compressive, tensile, and shear stresses that act in the plane of the surface”* in Encyclopaedia Britannica (retrieved on 2016). Also, Düzgün and Polatoğlu (2016) make remark about the shell structure as *“... architectural envelope is a very important part of a building, that, as a basic construction element, it defines the building by determining its identity, represents the dynamic tension between interior and exterior, and has semantic, technological, and aesthetic value”*.

Beyond being just a covering for a building, shell systems are known as the constructions that perform as the structural systems envelope and define an interior space in it, and consist of single or composite materials. Considering a shell, the dead load is mostly being the structures self-weight and the system carries its own. Furthermore, with the advantages of being a structural system of its own, shell constructions are comparatively economical in terms of material and safe in terms of structural performance. For the reason that it enables us to cover large spaces with long spans, and to close with structural safety, shell structures are preferred excessively throughout the history and today, they are still in use in the agenda of Contemporary Architecture. Shell structures can be designed with free forms other than symmetrical and noble geometries such as; domes, vaults, hyperbolic paraboloid and cylindrical. In this case, for different usage purposes and in different geometrical properties in terms of aesthetics and applicability, shell and vault designs can be preferred.

The masonry shell systems, which are mostly suitable for wide openings, are implemented by using traditional building materials which have high compressive

strength, such as stone or brick. Ochsendorf and Block (2014) signify that Masonry shells have been used as structural elements for centuries around the world in the forms of arches, domes, and vaults. Moreover, Ochsendorf and Block emphasize that *“... masonry materials such as brick and stone are strong in compression and weak in tension. The challenge is to find geometries that can work entirely in compression under gravity loading. These geometries are not limited only to masonry, and will often provide efficient geometries for structures built of any material.*

When we refer to the history of architecture, construction methods have always been reformulated with the benefit of technological advances in material use or applications. Freeform and asymmetrical shell systems are the structures that evolved from noble geometries such as domes and vaults. In contemporary architectural design, more complex structural requirements with complex shapes and asymmetrical geometries are in serious demand and this brings exploration and validation of a structural system within the geometry. Nevertheless, a serious computational workload is needed in order to overcome the validation process.

Parametric modelling is a powerful way to design this intricate geometries and forms. Computation has a significant place in the process of designing these structures and Rhinoceros 3D with the Grasshopper plug-in is used as computational design software. Physical modelling brings a non-rigid relationship concerning design components and can make simulation of model behaviours. Thus, the challenge based on masonry shell geometry that mentioned by Ochsendorf and Block (2014) can be taken up by using computational modelling methods.

In the conducted literature review, it is seen that materials such as Autoclaved Aerated Concrete (AAC), which is high-efficient in application and construction as it has compressive stress resistance, lightweight, pores and a high efficiency heat impermeability, are used less for asymmetrical shell systems in abroad practices. AAC is not used in shell systems except as a divider wall, in Turkey.

AAC was first produced nearly 100 years ago and has been improved in time. It is an alternative to the masonry building materials such as, stones and bricks, in terms of insulation and structural for architectural constructions. Moreover, it is known as a high quality and innovative material that had been extensively used for the

realization of residential, commercial and industrial buildings, in recent years (Ferretti et al., 2014).

According to the research survey, there is so few studies have been made for ACC in the field of designing or constructing shell structures. For this reason, the study focuses on designing an asymmetric shell by AAC blocks in the matter of making design decisions about covered area, height and number of arches and their sizes. Thereby, the research involves calculations and understanding the performance and the characteristics of AAC as it is the basic material for a freeform shell system. The outcome of this study is believed to be beneficial for form-finding knowledge of present-day technologies and finding an efficient material.

In similar studies, models are generated through the static calculations by engineers. A distinctive characteristic of the thesis is remaining in the forefront of the design and architect-focused circumstances while forming the generic model. Prior to the static calculations, open edges, kind of arches to pass through these edges and covered areas are determined. For this reason, the generic model is produced not only based on engineering aspects but also on the design and an architectural point of view which can be called as an integrated design approach or knowledge based design approach.

1.2. Research Goal and Question

The main goal of this thesis, as previously mentioned above, is to develop a computational design model of a masonry shell having the properties of structural stability and form by using AAC blocks. In this way, the purpose of the project can be determined as; searching the design of symmetrically and asymmetrically shaped shell systems by using AAC blocks as the material with computational design methods, and in light of this research, developing a generic model of structure based asymmetrically shaped shell, which is more difficult to construct than symmetrical shell, with AAC blocks.

Due to its properties, such as lightness, easy and in-situ processing and preferability in wall applications in buildings, AAC is considered as an alternative material for masonry vault and shell designs. Within the scope of the project, firstly, the features of AAC in shell design is examined. Thus, the feasibility of designing shell constructions by AAC material has been validated. Furthermore, methods for finding

a valid form for material oriented design of masonry structures have also been studied on the thesis.

With reference to the main goal, which is pointed out above, the research questions of this thesis are also as follows:

- What are the main methods for understanding and calculating the structural principles of shells?
- What are the material properties of ACC in designing a shell structures with load bearing principles?
- How a computational generic model could be applied as a support tool for designers and architects in order to design a material and structural based model?

1.3. Research Focus and Framework

The thesis focuses on developing of masonry vault and shell structures with AAC material. In this context, literature reviews on AAC material properties and loadbearing shell system and studies on how AAC materials behaves on this system are done. Thus, a framework is determined for asymmetrical shell systems by this material.

As a beginning, basic form finding methods of funicular shaped masonries have been studied. In the light of this research, AAC blocks have been tested in two dimensional according to 'Hooke's Hanging Chain Law. Thus, the span-length relationship of catenaries constructed with AAC has been examined and parameters of designing compression only AAC shells are found. Moreover, in order to enhance knowledge on structural performance of AAC material, examinations are done on simple shell geometries. In this way, the most appropriate design solutions are generated on structural performance of the shells. Considering as not a vital element that determines the shape of the geometry, the area supporters are eliminated in this research. Furthermore, the shape of the blocks and laying pattern are also excluded from the scope of the thesis.

Computational design is an important approach for the research in order to cope with difficulties and entanglement of vault and shell designs. The asymmetrical shell model, which has been developed during this research, was considered to be designed material oriented and therefore, '*Particle Based Design*' has been chosen.

The main concentrations of computational generic design for this study is optimizing the shape of the structure and find a shape statically in equilibrium by using determined form finding methods. Objectives such as minimizing the mass or minimizing the cost are left out of the framework of the study. From given topology of the particle spring network with loads on the particles the stiffness, rest length and stiffness of the springs are defined, and static equilibrium of the structure has been found by shape optimization.

Static Analyses step of the research is done by using '*Finite Element Method (FEM)*'. In FEM analysis program, normal displacement and principle stresses of the shell are calculated, thus static strength of the model is viewed. Mechanical properties of the chosen AAC material obtained from the literature have been used to perform the analysis.

The steps of the design process are all interchangeable with the previous stage. It is possible to change the design criteria of a resulting shape and build a newly generated model as regeneration. But, the results found in two dimensional tests and material examinations can be used universally on masonry shells designed with AAC material, thus, this study has fulfilled to focus on developing general knowledge on material behaviour on shell structures.

1.4. Method of the Research

According to the research survey, it is seen that there are not any studies or information on building masonry shells by using AAC blocks as material. For this reason, the thesis focuses on developing physics based computational design model for freeform vault and shells with AAC blocks. Within this regard "*shells and computational design*", "*autoclave aerated concrete (AAC) and properties*", "*developing particle based computational design model*" and "*results of the model*" are the chapters of the method to follow. These four chapters are reciprocal and interchangeable. The method of the research has been expressed as in the followings;

Shells and Computational Design: This chapter of the research seeks to find the basic design requirements of the vault and shell systems and an integrated design approach. With the help of this study, the generative principles and the design criteria of the shell systems are found out. The knowledge obtained from this study is used to prepare the generative form-finding steps of the model.

Autoclaved Aerated Concrete (AAC) and Properties: In this chapter, chemical, physical, mechanical and functional properties of the material have been studied. The characteristics of the material have been surveyed and comparatively studied through the strength tests under pressure stresses and the Mechanical Characteristics Tables had been prepared in previous studies. From these known values, AAC kinds, from different unit weights and different characteristics are researched. Thus, the material properties required for the modelling and the structural analysis are obtained.

Developing Particle Based Computational Design Model: In the third chapter, the aim is to design a Particle Based Computational Freeform Shell Model. For this section, firstly, hanging chain ratio of AAC has been determined for a material-oriented design. Moreover, structural performance of the shell designed with AAC material has been tested by four different structural examinations. Regarding the hanging chain ratio of AAC material and structural performance examinations, the structural characteristics of the shell structures desired to be designed with AAC material have widely become known. '*Rhinoceros*' modelling software and '*Grasshopper*', which is a graphical algorithmic plug-in running under Rhinoceros, have been used for modelling the structure. With the found hanging chain ratio, it is possible to create generic funicular arch models in this program. '*Kangaroo*' is a physic engine plug-in running in Grasshopper and has been used for generating the model. With the help of this plug-in, whose working principle is expressed as particle based structural form-finding method, the form of the model has set to be relaxed. The total weight of the model has been deduced from the volume, thereby; the applied force from every particle has been calculated and used in the engine. By completing all these steps, final equilibrium shape of the model has been determined. Finally, by analysing the static strength of the obtained model, the results are taken over visually and numerically. '*Millipede*' finite element analysis tool has been run in Grasshopper and structural analysis of the designed model is obtained.

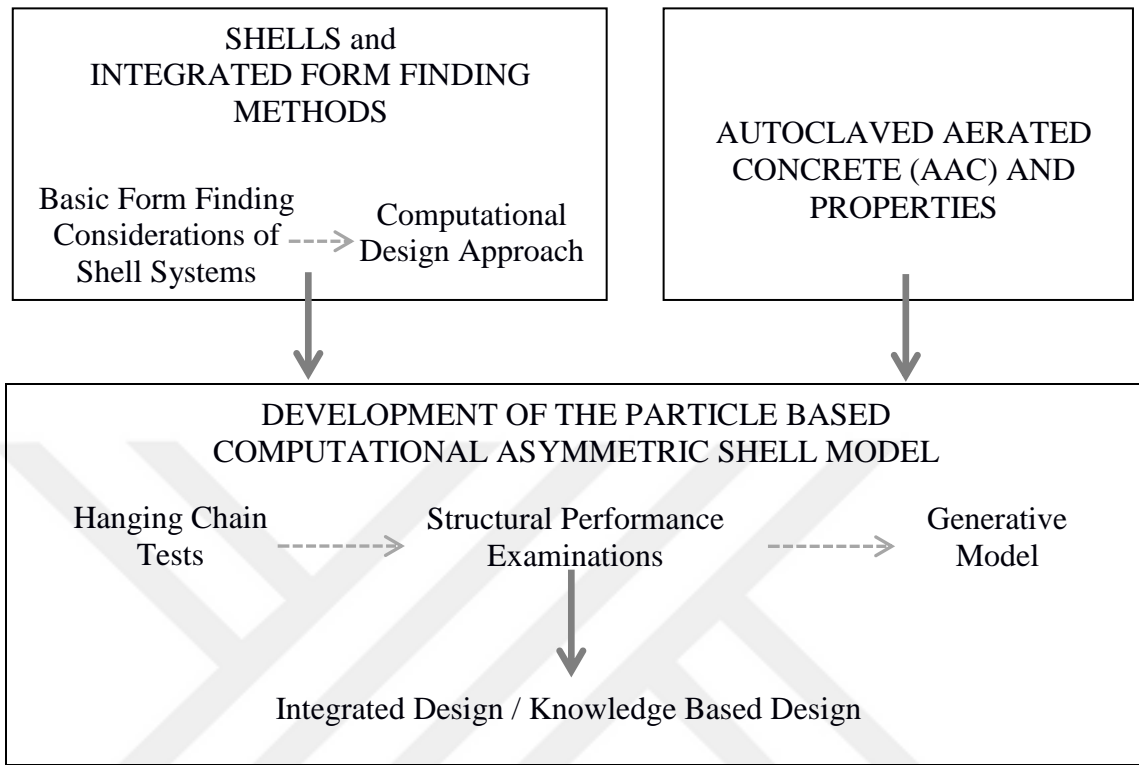


Figure 1. Method of the Research as Schematic Illustration

CHAPTER TWO

SHELLS AND INTEGRATED FORM FINDING METHODS

2.1. Basic Form Finding Methods of Shell Systems

Arches, vaults, and domes have been playing an essential role in architecture for a very long time. The oldest known of true arches were built by the Etruscans, civilization of ancient Italy, in the fourth century BC. These forms are more remarkable and eye-catching, than the other structural systems. Ochsendorf and Block (2014) attribute this case to “... *call for sustainable interest in the mechanics and design of shell structures*”. Williams (2014) describes a shell as “... *call for sustainable interest in the mechanics and design of shell structures*”. Williams (2014) describes shell as “... *a structure defined by a curved surface. It is thin in direction perpendicular to the surface, but there is no absolute rule as to how thin it has to be. It might be curved in two directions, like a dome or a cooling tower, or may be cylindrical and curve only in one direction*”.

There are enormous numbers of different equilibrium shapes for a shell or vault to be designed, and all of them have both advantages and disadvantages. In this regard, the substantial thing is to find the forms that are instinctively structural. At the present, many computer aided form-finding methods serve for this purpose. However, in old times, these advanced techniques were not in existence, and more basic form-finding methods had been used. Moreover, these techniques are ingenerated the fundamentals of today's computer aided form-finding methods.

The aim of basic form finding methods were building self-supporting vaults and shells, which are stable through their shape with formed arches. These arches, under own self-weight, without any bending and without any other loading, form a funicular shape that accurately defines the form of the funicular shell. Excluding advanced techniques to generate the funicular shapes physical testing's and drawings were used. Basic form finding and physical testing methods are examined in three

titles in this section; these are ‘*Hooke’s Hanging Chain*’, ‘*Graphic Statics*’ and ‘*Physical Modelling*’.

2.1.1. Hooke’s Hanging Chain Law

In 1675, Robert Hooke, an engineer and scientist, invented a structural form finding manner and summarized his invention with the quote; “*As hangs the flexible line, so but inverted will stand in the rigid arch*”. According to Hooke’s Law; a hanging chain that forms a catenary shape in tension under its self-weight has been defined as it could be inverted to an arch, which stands in compression. The pair, hanging chain and the arch is required to work in stability.

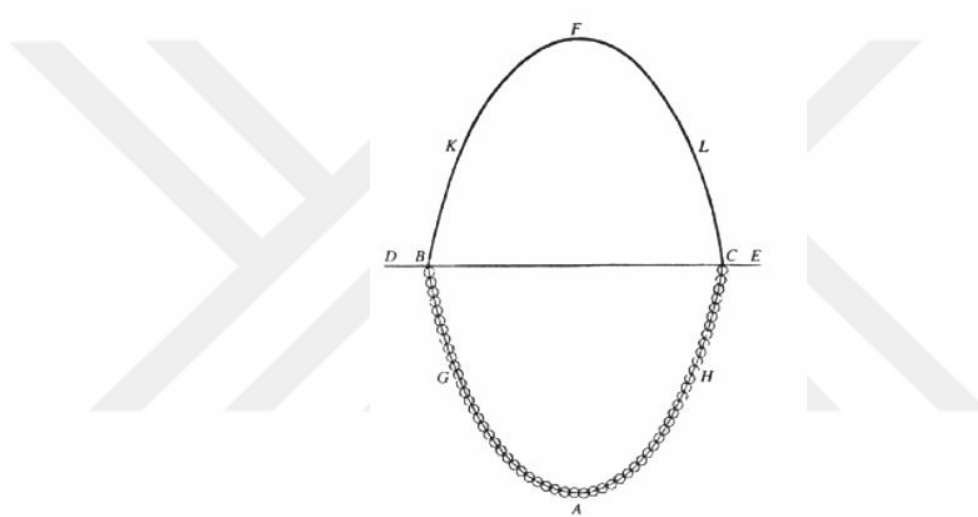


Figure 2. Poleni’s Drawing of Hooke’s Analogy between an Arch and a Hanging Chain (1748)

Dahnien and Ochsendorf (2012) generalized the working principle of the hanging chain as; “*the shape that a string or chain takes under a set of loads, if inverted, is an ideal shape for an arched structure to support the same set of loads*”. Block et al., (2006) defined the form of the chain and the overturned arch as *funicular shape* and these funicular forms of arch bring out a line of thrust, and this can be used in vaulted structures designing and analysing.

This manner can also be used for the designs of vault and shell structures. According to the Heyman (1995); “*materials such as masonry and concrete could carry large compressive loads, but are very weak under tension*”. Hence, the external loads are carried as compressive forces by masonry structures and very thin domes and vaults

could be created (Schenk, 2009).

Thus, it had been used in the design process of many significant buildings by architects. Antoni Gaudí is one of the pioneer architects, who used hanging models as a design method in his buildings. The Casa Mila and Crypt of Colònia Güell, two of his stunning buildings, are examples that are designed by using hanging models in the design process. Also, Frei Otto and his team is another example for these architects. Hanging models were studied to find the shape of the Mannheim grid shell (Burkhardt and Bächer 1978), and according to Chilton (2000), Heinz Isler used hanging cloth models, as he designed his concrete shells.

2.1.2. Graphic Statics

In 1866, Culmann formalized the graphical analysis as an effective method for stability analyses in structural engineering for the first time (Block et al., 2006). The method can be used instead of a hanging model for two dimensional problems. It allows finding the form of possible funicular shapes for given loads, but at the same time also the magnitude of force in them (Block et al., 2014).

The relationship between form and force diagrams of the geometry is defined reciprocal and so, graphic statics is bi-directional in nature. The diagrams of the reciprocal relation between form and forces are represented by Van Mele et al. (2012) as; *“...linked through simple geometric constraints: a form diagram, representing the geometry of the structure, reaction forces and applied loads, and a force diagram, representing both global and local equilibrium of forces acting on and in the structure”*. Regarding the aid of the diagrams, behavior of a structural system can be understood in a graphical sense.

For the vault to be stable under the uniform gravity load, each segment of the vault must be in static equilibrium. This can be achieved when the gravity force for each segment and the two inclined compressive forces from the segments on either side, balance each other. This then develops a funicular shape that defines the ideal shape of a pure compression vault (Asmaljee, 2013).

Graphic statics was used positively in engineering field. However, it has not become very popular and other new methods had taken the place of graphic statics. In 2001 Boothby mentioned that, *“graphical methods gave good but conservative results, though the process and analysis could become very tedious”*. It is known that, in

today, the method is known by a few engineers or architects. Anyhow, by implementing graphic statics to modern computer science, graphic statics drawings can be created with parametric tools and can be continuous to be used digitally.

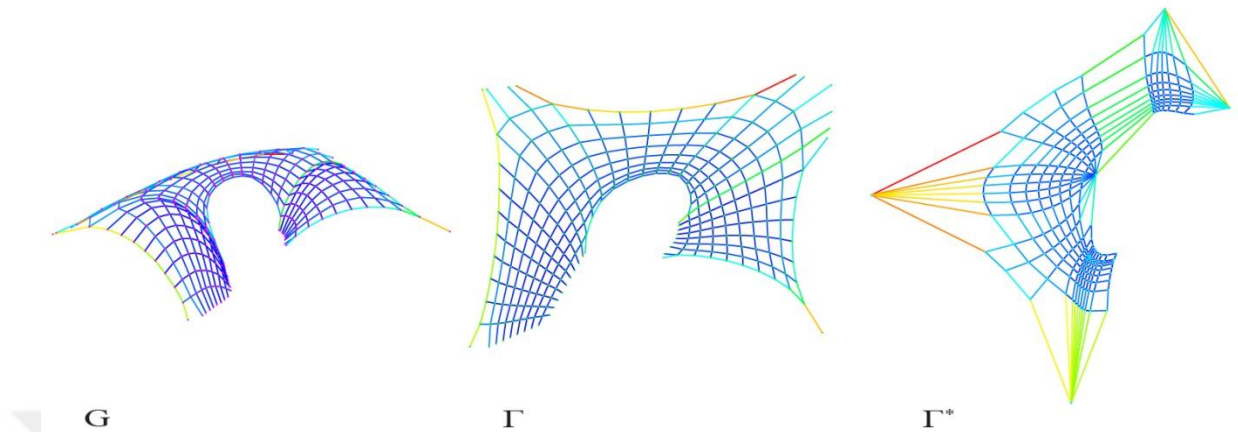


Figure 3. G is Compression only Thrust Network, Γ is Form Diagram and Γ^* is Force Diagram Created by One of the Parametric Tools (Block et al., 2014)

2.1.3. Physical Modelling

Architects have been using physical models to find the correct form for the structures since early times. According to Addis (2014), it is still valid as a means of creating potential geometries for the shell and lattice structures. Robert Hooke's hanging chain law has been largely used as the basis of physical modelling such as hanging fabric model or hanging chain model. These types of physical models form funicular shapes under their own self-weight. The first application of physical modelling was also done by Hooke and his colleague Christopher Wren for the form-finding of St Paul's Cathedral in London. Addis (2014) told about two dimensional models that *"This simple model test would have helped raise Hooke and Wren's confidence that the dome would work satisfactorily as a compression structure and is the earliest known use of a physical model being used to help determine the form of a structure"*.

Hanging chain models were used by many other architects, Antoni Gaudí is the best known of these designers and moreover, according to Huerta (2011), using a space-hanging three-dimensional model idea was, most likely, Gaudí's original. And he explains the hanging model of cross vaulting with Beranek's figure (1988) as *"... each simple arch supports a section of the webs between the cross ribs, represented by the principal chains"*.

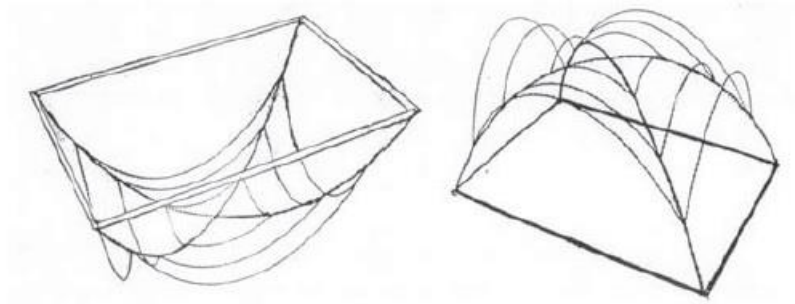


Figure 4. Hanging Model of a Gothic Cross Vault (Beranek, 1988)



Figure 5. Gaudi's String Model with Birdshot Weights Used in the Design of the Colonia Guell (Asmaljee, 2013)

Subsequently, until today, physical modelling has continued to be used for generating optimal forms by other architects such as; Heinz Isler, Frei Otto, and John Utzon. Physical models can be built in full scale or in small scales. Occasionally, physical modelling is examined in two categories; scale-dependent models and scale-independent models. According to this circumstance, physical models can be built in full scale or in small scales. However, West (2006) stated that “*whatever can be built in scaled, working models can be constructed at full-scale*”.

2.2. Computational Design Approach

The basic structural concept of asymmetric shell, which is being discussed in this thesis, is based on compression-only structure systems. Computational design and modelling practices has empowered a novel perspective to the design period of these structures and they present various form-finding methods. With the aid of these methods, complex design forms are generated more easily.

In recent years, computational modelling has become more substantial among architectural designers. Fleischmann and Menges (2012) point out this interest as *“understandable from the perspective of a designer who is seeking a formal exploration of geometric shapes”*. These methods are the shape finding practices where the founded structure is the most favourable static equilibrium shape. The most important thing is to choose the best fit method for the structural type and the parameters.

2.2.4. Interactive Form Finding Methods

Form Finding Process is stated as an advanced process where the variables controlled directly by computational tools to obtain the best shape of a structure. Hence, the geometry is statically in equilibrium with its design loading. Several form-finding methods and computational tools have recently been used to design shells by working up in a relation between performance-related criteria and architectural form of the structure. These methods are all based on different theoretical approaches and have differences in some means such as complexity, usability by designers and requirement of different execution time. The parameters for controlling form-finding process of methods consist of different variables, such as;

- Boundary conditions,
- Supports
- External loads,
- Dead loads,
- Topological properties of the model,
- Forces and their relationship with the geometry.

Methods developed for the form-finding process of the shell systems are discussed in two different ways; geometry-oriented form-finding methods and material-oriented form-finding methods. Geometry-oriented form-finding methods have solved the structural problems of static equilibrium, without depending on the material properties. These methods are *Force Density Method* and *Thrust Network Analysis*. Material-oriented form-finding methods come up with solutions to the problems incorporating material properties or spring stiffness, and solve dynamic equilibrium problems such as *Dynamic Relaxation Method* and *Particle-Spring Systems*.

2.2.4.1. Force Density Method

Force Density Method (FDM) was firstly introduced by Schek in 1974 and it is generally used in engineering to obtain the equilibrium shape of structures comprising of a network of cables with different elasticity properties when stress is applied (Southern, 2011). Thus, this method has demonstrated a precious process to find the appropriate equilibrium for shells designed by using membranes and cable networks. According to Gidak and Fresl (2012), determining the form of pre-stressed cable nets was defined as the process of finding the equilibrium shape to meet the architect. Lewis (2003) explains, in his book '*Tension Structures*', operation of the force density method as "... uses a linear system of equations to model static equilibrium of a pre-tensioned cable net under prescribed force/length ratios". And, it is not only a functional and aesthetic concept, but also fulfilling the engineer in terms of load transfer capabilities and performance.

This method is advanced to avoid the problems faced in the computerization of inverse problems regarding Hanging Chain. Force Density Method in form-finding consists of two parts. Firstly, physical model of the desired geometry is created in accordance with the given boundary conditions by using soap, stretchy fabrics or elastic threads as the material. Then, the desired shape achieved in terms of aesthetic and a numerical model is designed for second part.

Some of the specific properties of FDM mentioned by Southern (2011) are; "... depending only on the force density of the edges and the topology of the network, and the system is sparse, symmetric and positive definite, quickly solved using the conjugate gradient method". Linkwitz (2014) stated the advantages of using the force density method as, "... not required any information about the material for the

later realization of the design. As we are dealing with non-materialized equilibrium shapes, no limitations with respect to material laws exist”. So, any materialization is possible after finding the right equilibrium shape. Linkwitz (2014) also expressed that there are two potentials of being independent for the material properties as; “First, resulting design can be materialized arbitrary, giving the initial lengths of the network in un-deformed state without affecting the final shape. Second, one can simply multiply the loads to any realistic value, and then calculate the internal force distribution, again without changing geometry”.

Although it has been many years since this method was introduced, it is still in use and it is a favoured method for the calculation of the equilibrium state of tensile structures.

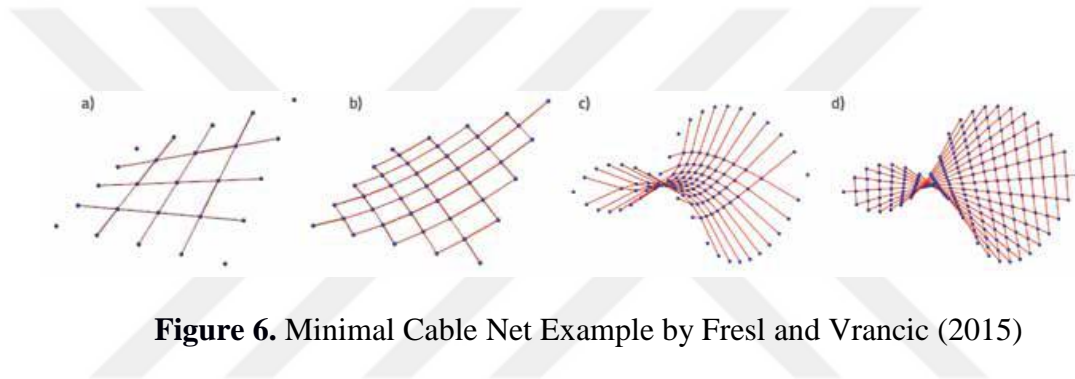


Figure 6. Minimal Cable Net Example by Fresl and Vrancic (2015)

2.2.4.2. Thrust Network Analysis

Thrust Network Analysis (TNA), is described as a graphic statics-based method and it is used for designing compression-only shell structures with complex geometry. Block et al (2014) expressed TNA as “... appropriate for the form finding of compressive funicular shells, thus particularly for any type of vaulted system in unreinforced masonry”. According to Rippmann and Block (2013), using TNA method is advantageous due to having “the inherent, bidirectional interdependency of forms and forces represented in visual diagrams, which are essential for a user-driven and controlled exploration in the structural form-finding process”.

Block (2009) mentioned in his PHD Thesis that, there were four key assumptions to develop TNA for calculating loadbearing structures. They are;

- a. The structural action of the vault is represented by a discrete network of forces with discrete loads applied at the vertices.

- b. A compression-only solution in equilibrium with the applied loads and contained within the vault's geometry represents a valid, i.e. stable, equilibrium state of the vault.
- c. Masonry has no tensile capacity; sliding does not occur; and the stresses are low enough so that crushing does not occur (infinite compression strength is assumed).
- d. All loads need to be vertical.

Block and et al (2014) declare that thrust network was the three dimensional version of thrust line and continue “... *extends discretized thrust line analysis to spatial networks for the specific case of gravity loading, using techniques derived from graphic statics*”. In addition, they examine TNA for the intuitive design of funicular networks, and for a high level of control, they divide the TNA method into three key concepts; *vertical loads constraint, reciprocal diagrams and statically indeterminate networks*. These concepts are examined and defined as follows.

Vertical Loads Constraint: TNA is only studied on vertical loads. Thus, the equilibrium of the horizontal force elements (thrusts) in the thrust network analysis can be calculated independently of the selected external loading. Therefore, the form finding process is separated into two steps;

1. Solving for an equilibrium of the horizontal thrusts,
2. Solving for the heights of the nodes of the thrust network based on: the external loading, the given boundary conditions, and obtained horizontal equilibrium.

Reciprocal Diagrams: Considering Γ as a form diagram, each force transporting is represented by a force diagram Γ^* , in a given scale. A reciprocal relationship, in other words; Γ and Γ^* relate form and force diagrams* are parallel dual graphs. Block and et al (2014) express the reciprocal relationship between form and force diagrams as; “*Branches which come together in a node in one of these diagrams form a closed space in the other, and vice-versa, and corresponding branches in both diagrams are parallel*”.

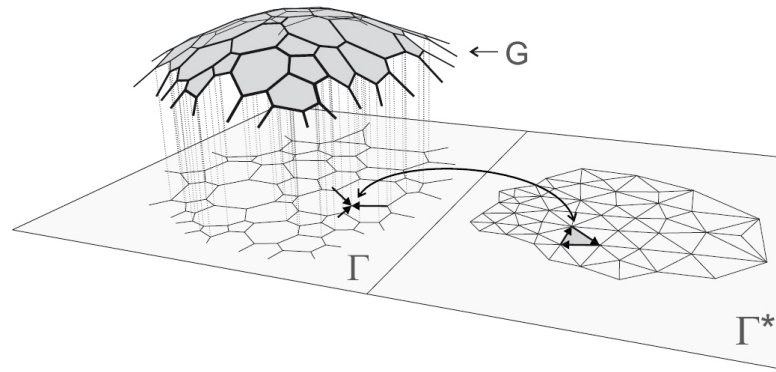


Figure 7. TNA Method Representation by Block (2009)

If the closed polygons of the force diagram Γ^* and the equilibrium of the nodes of the form diagram Γ are all composed clockwise, then it means that; form diagram Γ and the thrust network G will be completely in compression.

Statically Indeterminate Networks: Block and et al (2014) explained statically indeterminacy as, “*For nodes in the form diagram with a valance of higher than three, the network is structurally indeterminate, which means that the internal forces can be redistributed in the structure, resulting in different thrust network for the given form diagram, but for each given form diagram Γ , force diagram Γ^* , and vertical loading P , a unique thrust network G exists*”.

Rippmann and Block (2012) introduced graphical components of the TNA basically as; a form diagram Γ which defines the geometry of the structure and the layout of forces in plan, two possible corresponding force diagrams, Γ_1 and Γ_2 , which represent and visualize two possible distributions of horizontal thrust; and G_1 and G_2 which are the corresponding thrust networks in equilibrium with given (vertical) loading.

2.2.4.3. Dynamic Relaxation Method

Dynamic Relaxation Method (DRM) was first introduced in 1965 by Day and it is a numerical method for form finding. Hüttner et al (2014) describe DRM as “... *an iterative process that is used for the static analysis of structures. DRM is not used for the dynamic analysis of structures; a dynamic solution is used for a fictitious damped structure to achieve a static solution*”. Therefore, the basis of the Dynamic Relaxation can be traced step by step from this fictional damped structure. As well, Adriaenssens et al (2014) summarized the technique of the DRM as “... *traces the motion of the structure through time under applied loads. The technique is effectively*

the same as the leapfrog and Vervet methods, which are also used to integrate Newton's second law through time".

DRM is known to be used generally in the form-finding process of cable and fabric structures. However, Garcia (2012) defines DRM as "... a numerical method usually used in the form-finding of all kind of structures (tensegrity structures, membrane structures, shell structures...) that consists in considering that the mass of the system is discretized and lumped in the nodes; these nodes oscillate about the equilibrium position, and by introducing artificial inertia and damping, the nodes come to rest in the static equilibrium position". Thus, he stated that the method is also applicable to other shell kinds, besides the cable and fabric material.

Dynamic relaxation method was observed as a numerical, finite difference technique in its early periods. First application included analysing shell geometries and after that, it was used for skeleton and cable structures and plates. Lewis (2003) mentioned what basis the dynamic relaxation method was grounded in as "...on a discretized continuum in which the mass of the structure is assumed to be concentrated (lumped) at given points (nodes) on the surface". The system consisted of concentrated mass swings to find the equilibrium position and an equilibrium shape, under the unbalanced forces effect. After a time, the system approaches to the equilibrium position under the influence of 'damping'. Iteration is a process, where the static equilibrium of the system is achieved by simulating.

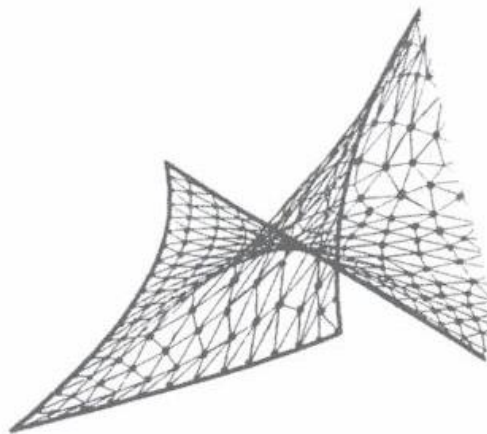


Figure 8. Discretized Continuum that Clarified as the Basis of the Dynamic Relaxation Method by Lewis (2003)

2.2.4.4. Particle-Spring Systems

Particle-Spring Systems (PS) is known to be first announced by William T. Reeves in 1983 and according to Bertin (2011) this technique has long been implemented in the modelling of hair and fabric, in animations, in video games and movies. Fleischmann and Menges (2012) spoke of particle systems as “...a collection of independent objects, often represented by a simple shape or dot. It can be used to model many irregular types of natural phenomena, such as explosions, fire, smoke, sparks, waterfalls, clouds, fog, petals, grass and bubbles”.

For nearly ten years, engineers and architects have used PS and generated simple digital simulations of hanging chain and tensioned membrane models. Otto and Isler can be identified as the examples of the architects and engineers who used this method while designing. Bertin (2011) explained the working process of the method as; “a particle-spring system consists of particles that are given mass and position, and are connected by springs which have stiffness and rest length. Other parameters can be controlled including boundary conditions or anchor points and gravity forces. Once the simulation is started the particles move through space until the forces acting on them are in equilibrium”. According to Lewis (2003), at this point, the working process of the method approximates to a stable configuration similar to the dynamic relaxation method.

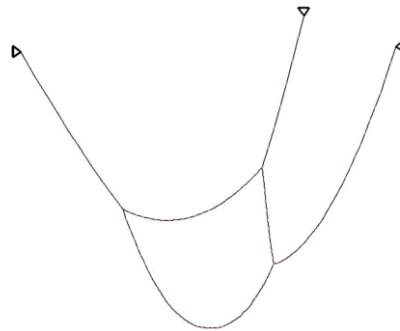


Figure 9. Statically Determinate Funicular Form in 2D Modelled with Particle-spring Simulations (Kilian and Ochsendorf, 2005)

Bhooshan et al., (2014) looks at the PS from another point of view and formulates the process as finding the equilibrium shape of the geometry. Firstly, the topology of the particle spring network with loads on the particles is considered. Secondly, the stiffness and the lengths of the springs are defined. After that, it is attempted to

equalize the sum of all forces in the system. Kilian and Ochsendorf (2005) mention about the particles and the forces applied to these nodes in a spring system as; “*Each particle in the system has a position, a velocity, and a variable mass, as well as a summarized vector for all the forces acting on it. A force in the particle-spring system can be applied to a particle based on the force vector’s direction and magnitude. Springs are mass-less connectors between two particles that exercise a force on the particles based on the spring’s offset from its rest length*”.

Produced design tools, which use the particle-spring systems as the working principle, give users the chance to explore and create new structural forms. When the simulation first starts, the particle-spring system does not work statically in equilibrium. So, the system sets into motion and iterates the particles and springs positions to seek their equilibrium conditions. By using these simulation tools, the user will boost his perception by watching how particles and springs interact and how they move when the system is subject to gravity.

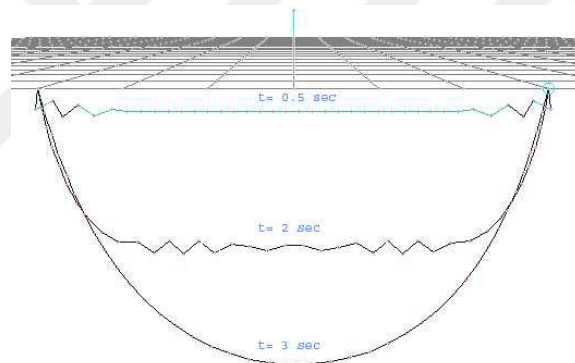


Figure 10. Simulation Process of the System for a Cable with Forty Discrete Masses at Equal Spacing (Kilian and Ochsendorf, 2005)

2.2.5. Comparison of the Form Finding Methods

In the last two decades, computational design and modelling methods empowered a new vision to the design period of complex geometries and enabled the generation of statical equilibrium shapes of these geometries more easily. Different computational interactive form-finding methods are developed for shape finding process of the shell geometries. Designers are able to interfere and see the statical requirements of the geometry simultaneously with design process. Hence, the process and requirements are completed more rapidly in these shape finding practices. The most important

objective is to determine the most suitable method for the desired geometry according to the structural type of the shell, material and the parameters.

In this chapter, these methods are discussed and compared with each other in order to find the right method. In this comparison, two questions are answered:

- How these methods are different from each other?
- Are they applicable to all kinds of structures and materials or not?

There are very few researches comprising the form-finding methods of shell structures. While preparing this chapter, Veenendaal and Block's articles, books and their disquisitions are reviewed.

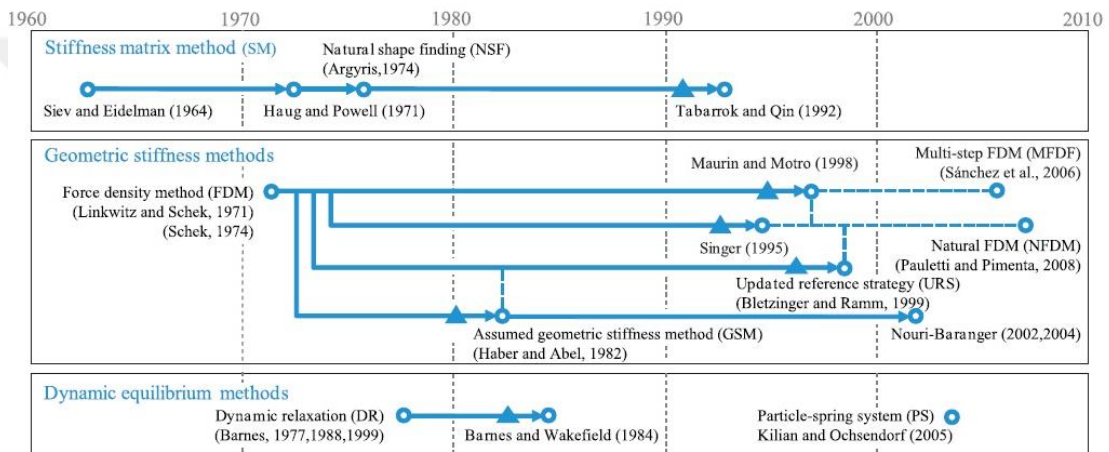


Figure 11. Timeline of Form Finding Methods - Development and Categorization (Veenendaal and Block, 2012)

In the figure above, the form-finding methods and the related information has been shown with decades by Veenendaal and Block (2012) and, FDM has been specified as a geometric stiffness method and DRM and PS are as dynamic equilibrium methods.

Geometric Stiffness methods are defined by Veenendaal and Block (2014) as; “... are material independent, with only a geometric stiffness. In several cases, starting with the Force Density Method, the ratio of force to length is a central unit in the mathematics”. Furthermore, TNA is also accepted as a geometric stiffness method and an extension of FDM. Moreover, this method is free of material kind and it just concentrates on the collaboration of forces rather than force magnitude. Dynamic Stiffness is generally affirmed as outlining the shape equilibrium to obtain a stable-

state result which is corresponding to the static equilibrium. Dynamic Relaxation Method and Particle-Spring Systems are in this category.

As a study to compare these methods, finding similarities and differences; Veenendaal and Block (2014) applied all the methods mentioned on a simple example using the same data structure to develop the shape of a simple shell. Thereby, firstly it is highlighted that every form-finding method consists of at least the following parts;

1. A discretization to describe the (initial) geometry of the shell. The discretization can be made up of line elements, or surface elements such as triangles or quadrilaterals.
2. A data structure that stores the information on the form (geometry), connectivity of the discrete elements and forces within the shell.
3. Equilibrium equations that define the relationship between the internal and external forces. A shape resulting from form finding represents a system in static equilibrium. The internal and external forces add up to zero. Additional constraints might be placed in the equilibrium equations influencing how they can be solved numerically.
4. A solver, or integration scheme, which describes how the equilibrium equations are solved. If the system of equations is nonlinear, one typically tries to solve this system incrementally. The solver includes stopping criteria and means to measure convergence. Applicable solving methods may differ in how fast they converge or how stable they are, but assuming that they do converge, should result in the same solution if the problem and its boundary conditions are identical.

The properties, related to the chosen method, which are needed to be provided, are mentioned as follows;

- Coordinates of the supports,
- Topology, connectivity of the networks,
- Prescribed loads (or mass densities for shape dependent loading)
- Convergence tolerance (for iterative methods.)

At the end of the study conducted by Veenendaal and Block (2014), it is reported that the input for FDM and TNA were reduced to a bare minimum. This was an advantage, though as discussed, force densities were physically not meaningful and therefore difficult to control. Nonetheless, it is mentioned that this situation is not same for dynamic methods. The drawback of these methods is explained as “... methods such as DR and PS, in this respect are the much larger number of parameters necessary for their control. However, in DR these parameters (for example; axial stiffness, bending stiffness, initial coordinates, or lengths) are either fictitious values, chosen for their effect on convergence or on the resulting shape, or they are related to the material and physical properties of the structure”.

Method	User-prescribed quantities
FDM	force densities
TNA	projected coordinates thrust distributions (from) scale factor
DR	axial stiffness bending stiffness (for splines) initial coordinates, or lengths damping factor (for viscous damping) time step
PS	spring stiffness initial coordinates, or rest lengths damping coefficient drag coefficient time step

Figure 12. The Values Which are Needed to be Prescribed by user for Each Method (Veenendaal and Block, 2014)

It is mentioned that, once an equilibrium state was found, material or physical properties could be changed regularly without disturbing shape or equilibrium. Veenendaal and Block (2014) explained this as; “... combined with the ability to manipulate the internal forces (through force density, elastic stiffness or spring stiffness, as well as loading), suggests that these methods are theoretically interchangeable”.

Furthermore, cases, in which any compression-only shape of static equilibrium is acceptable, are undertaken more easily through purely geometric methods (e.g. FDM

and TNA), however for the cases, in which initial geometry and descendent deformation have meanings and material properties are known, DRM is more clear and appropriate. The bare integration schemes in DR and PS also do not need matrix algebra, which may be an advantage in terms of a simple implementation.

2.3. Computational Structural Analysis and Finite Element Method

The practice of designing a structure, which is based on scientific rules, is a recently advanced technic. In old times, construction of a building was done without any computations and theories. Still, there are a lot of examples of significant historical buildings which survive to this day. Having experience and practical training, masters of these buildings found out how to handle the material and how to design the building architecturally. From these old times to the present, in the light of the discoveries of these masters and engineers, calculation methods and equilibrium equations are developed for analysing the structures according to structure type, material kind and its properties. Thus, structural analysis methods are improved.

Kaveh (2013) clarifies the structural analysis and structural design as; “... *the determination of the response of a structure to external effects such as loading, temperature changes and support settlements. Structural design is the selection of a suitable arrangement of members, and a selection of materials and member sections, to withstand the stress resultants (internal forces) by a specified set of loads, and satisfy the stress and displacement constraints, and other requirements specified by the utilized code of practice*”. He stated that, the cycle of structural analysis and design has to be applied over and over again to find the effective solution for settled requirements such as the weight or cost of the structure.

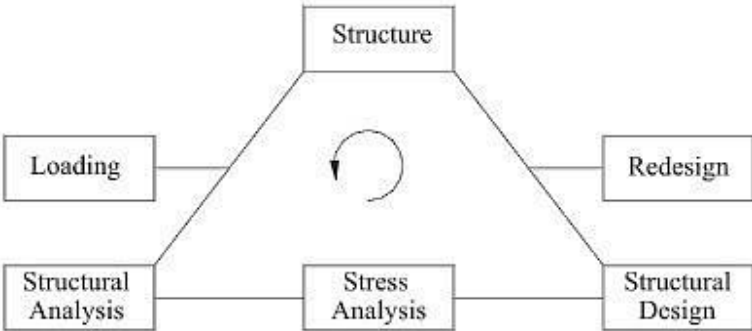


Figure 13. Cycle of Structural Analysis and Design of a Structure (Kaveh, 2013)

Resulting geometry of a structure must fulfill the requests of equilibrium, compatibility and force-displacement relationship. In other words, the external and internal loads applied to a structure must be in equilibrium for each node, nodes of the structure must deform hence that they all fit each other, and the internal loads and deformations must satisfy the relationship between stress and deformation of the nodes. Two basic methods have been used for structural analysis; these are *force method* and *displacement method*.

Regarding the force method for structural analysis, several internal forces and responses are obtained as unnecessary. Deformations of the members concerning external and unnecessary forces mentioned before are defined according to the relation between stress and strain. A set of linear equations calculate the values of the unnecessary forces by providing the suitable conditions for the deformed members to be fitted together. The stress results in displacements at the particles in the direction of external forces.

Concerning the displacement method, firstly, the displacements of the particles, which are required to define the structures' deformed state, are described as unknowns. Secondly, the calculations of the deformations on the nodes are done in terms of these movements, and by using the relation between stress and strain, the internal forces are included in these calculations. Finally, the solution resulted in the unknown nodal displacements is achieved by handling the linear equation set for finding equilibrium of each node.

In 1950s, Finite Element Method (FEM) was established in engineering field and it was defined as "*a method of analysis for highly redundant structures which is particularly suited to the use of high-speed digital computing machines*" by its inventors, Argyris, Clough and Zienkiewicz. The term FEM classifies a wide set of techniques that share common features in engineering. In combination with computers, FEM has run to modern computer-aided mechanics of which structural analysis is a part. Pedron (2006) mentioned about the performance of FEM through digital computing that "*... non-trivial calculations concerning dynamics, collapse mechanisms, materials and geometrical non-linearity as well as ultimate loads could also be routinely performed*".

The use of FEM is clarified as answering a set of related calculations by approximating iterating field variables as a set of field variables at particles. Structural problems are related to equilibrium equations, and the field variables are nodal displacements and loads. Finite Element Method is in use of engineers, to analyse physical systems and it is commonly known as finite element analysis FEA.

Pedron (2006) points out how the development of Finite Element Method has changed the structural analysis as follows; *“Until the mid-20th century, despite the use of simplified calculation methods like the force method, the displacement method and the Hardy-Cross method, it took a long time to analyse structures even of medium complexity, mainly due to the difficulty of solving linear equation systems. In the late 1950s the advent of computers and the development of the Finite Element Method (FEM) completely revolutionized structural analyses”*.

Thanks to the improvements in computer graphics, FEM computer programs can be easily found. Engineers model with the program of FE and designate the external loads to be carried by the model. Consequently, computer will calculate the internal forces and matching stresses. If the results do not meet the safety criteria, the computer can make alterations until safety criteria are fulfilled. Programs based on FEM analysis are suitable for determining stresses, deflections, and dynamic behaviour for complex geometries using very complicated techniques.

FEM is a very powerful program for engineers and architects to analyse complex structures and mechanical systems. Thus, FEM assists users in solving the problems for which analytic or mechanical methods are difficult to use. The teamwork of FEM analysis tools with architectural design packages focuses on the field of structural design, and can forward information to structural analysis tools using this method. Furthermore, using FE analysis tools is also interacting with other fields such as building physics and energy efficient design as a part of architectural design.

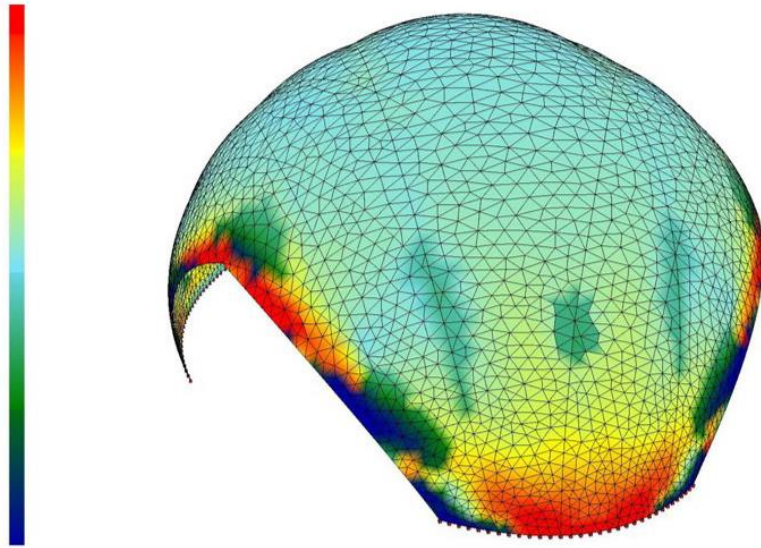


Figure 14. FE Analysis of a Composite Shell by ICD/ITKE, Research Pavilion
(2014-2015)

CHAPTER THREE

AUTOCLAVED AERATED CONCRETE (AAC) AND PROPERTIES

Autoclaved Aerated concrete (AAC) is a material that is obtained from lightweight construction elements with improved technology (Kömürlü and Önel, 2007). It is a concrete construction material that is produced by a chemical curing method, located within pores, can be easily shaped according to the application purpose, light, static strength in certain levels and has insulating properties. It was first produced nearly 100 years ago and has been improved in time. AAC is an alternative to the masonry building materials, stones and bricks and carrier featured concrete, both in terms of insulation and structure in architectural constructions. Narayanan and Ramamurthy (2000) point out the outstanding advantage of AAC as “... *is its lightweight, which economizes the design of supporting structures including the foundation and walls of lower floors*”.

Ferretti et al. (2014) emphasise that “*In recent years, autoclaved aerated concrete (AAC) has been widely recognized as a high quality, innovative material that has been extensively used for the realization of residential, commercial and industrial buildings*”. However, it has not taken too much part in present applications such as vault and shell structures. The scope of the thesis is to research the appropriateness of AAC and related materials for asymmetrical masonry shell geometries in architectural and physical zone.

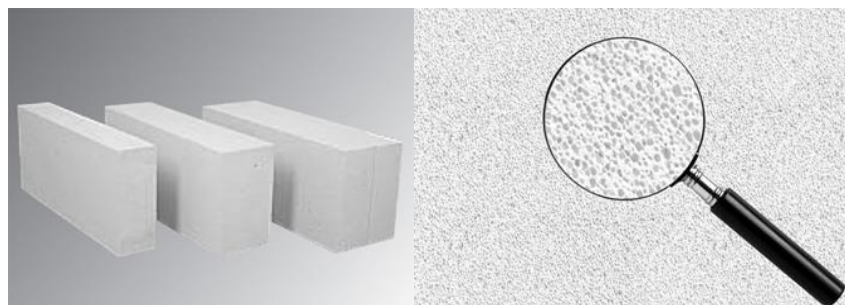


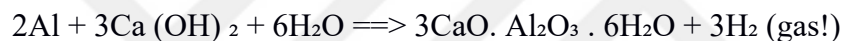
Figure 15. A Symbolic Image of AAC (Retrieved 02.11.2016 from <http://www.akg-gazbeton.com/wall-blocks>)

In this chapter, AAC is examined and reviewed in two steps; *'Material Properties'* and *'Production Process and Application areas'*.

3.1. Material Properties

3.1.1. Chemical Characteristics of AAC

Aerated concrete material is expressed by Narayanan and Ramamurthy (2000) basically as “... a mortar, with pulverized sand and/or industrial waste like fly ash as filler, in which air is entrapped artificially by chemical (metallic powders like Al, Zn, H₂O₂) or mechanical (foaming agents) means, resulting in significant reduction in density”. Few different methods can be used to shape air pores in aerated concrete, but the mostly by the addition of aluminium powder, which is added to the mixing ingredients is used (Holta and Raivio, 2005). The reaction equation of the pored material is as shown below.



(Neville, 1981)

Being non-autoclaved or autoclaved of aerated concrete depends on the curing technique. Steam of the aerated concrete is cured in a high pressure autoclave, in autoclaving. Ioannou et al. (2008) explained the manufacturing process of AAC as; “Adding aluminium powder to the slurry before decanting the mix into a mould and allowing it to ‘rise’ in an oven develops its cellular or foamed character. After the slurry has achieved sufficient mechanical competence it is removed from the mould and autoclaved at about 180 °C for 10 to 16 h”.

The elemental composition and the curing method control the microstructural form and the density of the material, and directly the physical and mechanical properties are influenced.

3.1.2. Physical Characteristics of AAC

70% - 80% of the AAC's volume comprises from macro-pores and micro-pores, due to this feature, it is a low-density lightweight material and there is an inverse proportion between the pore ratio and the density of AAC. Most of the physical characteristics of AAC are determined by its density. Pehlivanlı (2010) mentions that

the density was due to its porosity form, between 300 and 800 kg/m³, the density without voids were approximately 2600 kg/m³. Increasing amount of porosity of the AAC brings about a decrease in a desired manner in thermal conductivity and dry density. However, in parallel, the mechanical strength of the material also decreases. Ünverdi (2006) remarks on the mechanical strength of AAC that “*porosity and the pore dimensions determine the strength, conductance and shrinkage of the material*”.

Depending on the type of the siliceous raw material used, it can have a white, grey or pink color. It can be shaped very easily and depending on the cutting machine, the blocks’ surfaces can be smooth or stripe-rough.

3.1.3. Mechanical Characteristics of AAC

Compressive strength of the AAC is determined according to its dry weight per unit of volume (d) and the amount of moisture it contains, and it has a high compressive strength in connection with its unit weight. The density and the strength of AAC can be used to meet the definite structural requirements. Generally, compressive strength and density increase linearly. And in compression reports, the modulus of the elasticity of AAC is formulated as a function of the compressive strength. It is also related to the density and, they have a linear proportion, too.

The relation between density, compressive strength, modulus of elasticity and thermal conductivity is shown in a table below.

Dry density (kg/m ³)	Compressive strength (MPa)	Static modulus of elasticity (kN/mm ²)	Thermal conductivity (W/m ^{°C})
400	1.3-2.8	0.18-1.17	0.07-0.11
500	2.0-4.4	1.24-1.84	0.08-0.13
600	2.8-6.3	1.76-2.64	0.11-0.17
700	3.9-8.5	2.42-3.58	0.13-0.21

Table 1. Mechanical Characteristics of AAC (Ünverdi, 2006)

Korkmaz et al. (2014) studied on different materials structural behavior in Masonry constructions in their “*Effects of Different Structural Material Properties on Masonry Building Structural Behavior*” thesis. According to this research, most of

the masonry materials and AAC's Poisson Ratio are given as 0, 2.

3.1.4. Functional Characteristics of AAC

Narayanan and Ramamurthy (2000) express the Water absorption of Aerated concrete as; *"...being porous, there is a strong interaction between water, water vapour and the porous system and there exists various moisture transport mechanisms. In the dry state, pores are empty and the water vapour diffusion dominates, while some pores are filled in higher humidity regions"*. According to Cicek (2002), AAC material is known as, even in the waterlogged situation, having 60% dry pores.

Furthermore, according to Pehlivanlı (2010), the most important and superior property of AAC is having low thermal conductivity. AAC is more homogeneous than an ordinary concrete. As a consequence, the thermal conductivity of AAC is better or at least as good as ordinary concrete. Just like thermal conductivity, the fire-resistance of AAC is better or at least as good as ordinary concrete. Closed pores of AAC plays the role of divider in the material and it prevents heat transfer and enhances the degree of fire-resistance.

Moreover, Acoustical absorption of the AAC is stated twice as better than ordinary concrete. This is related to the transformation of sound energy into heat in the capillary of the material. However, there are also negative statements, as AAC does not have any unique or significant sound insulation characteristics.

3.2. Production process and Application Areas

Manufacturers have slightly different formulas for producing AAC. However, basic raw materials are common for every product. They are Portland cement, limestone, aluminium powder, powdered silica sand, water and for some of the producers fly-ash. With the reaction between calcium hydroxide and water, white heat is liberated. Aluminium powder also reacts with this mixture and thus it produces hydrogen gas which causes the mixture to produce millions of microscopic disconnected aerated cells. Also, adding foam or whipping the mixture is a method used for producing gas. The mixture is then transferred into moulds and it rises and sets. Hardening process starts as calcium hydro silicate and aluminium hydro silicate are added to the mixture. When a certain degree of strength is obtained, it is cut into the desired size

with the help of special steel wires in an automatic cutting machine. Finally, after cutting, AAC blocks are steam-cured in an autoclave under approximately 190°C, at 10-12 atmospheres, from eight to 14 hours, depending on the product formula.

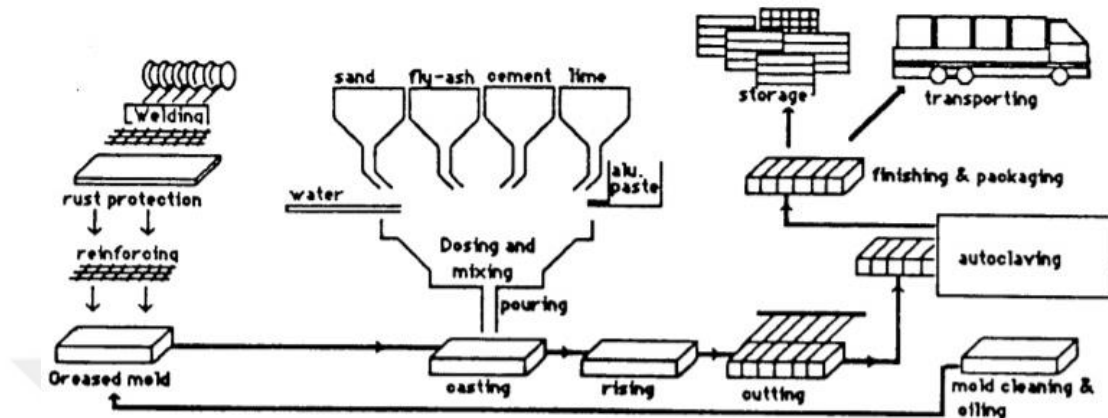


Figure 16. Producing Process of AAC (Wittmann, 1992)

Thanks to its properties, AAC material has been used for many building constructions such as residential homes, commercial and industrial buildings, schools, hospitals, hotels and many others. Beyond the existing AAC market mentioned before, there is also a rising worldwide call for integrated building solutions of AAC and designers, architects and engineers have been working on finding new application areas. This study is also an example for the ones who are working on new areas for using AAC material.

CHAPTER FOUR

DEVELOPMENT OF THE PARTICLE BASED COMPUTATIONAL ASYMMETRIC SHELL MODEL

In the previous chapters, major characteristics and principles of vault and shell structures and AAC material have been examined. In the light of these examinations, this chapter introduces the integration of AAC material to the shell systems designed by using computational methods.

Researches enlighten that there is very little information about a constructible shell made of AAC blocks and studies are very limited. In this chapter, the examinations and tests which have been conducted to find the structural stability constraints for a masonry freeform shell designed by using AAC material. Found parameters are important and needed for the form-finding process of the shell.

4.1. Designing Considerations of Developed Asymmetric Shell Model

Finding similar examples, most useful studies of shell design are made by Block Research Group. One of the books, which is edited by a member, "*Shell Structures for Architecture: Form-finding and Optimization*" represents current approaches to designing shell systems, consists of form-finding and structural optimization methods, and provides computational techniques. The book divides form-finding and structural optimization processes of the shell structures into different types according to the form-finding methods. Regarding the book, the ideal design parameters depend on the loads and their combination, boundary conditions and material. In accordance with the researches, two-dimensional hanging chain tests are considered to be done to find the structural stability information about AAC material and generate the direction for the design process of the shell model.

4.1.1. Hanging Chain Tests

It is seen in the researches that the structural design and analysis of the masonry structures first start in two-dimensions. In parallel, designing process of the thesis shell also starts with the tests of two-dimensional slices. Inverted hanging chain is used as the method of two-dimensional tests. It is the simple way to find a compression only form for an arch with uniform thickness. According to the Block (2009), it gives more controlled results, widely discounting the complete three-dimensional system.

For the tests, 1:20 scale is determined as the scale of the 2 dimensional models. 1:20 scaled blocks are prepared by using gypsum to use instead of AAC blocks for the models. The mortar of the blocks is made by mixing the gypsum and water, and a 1:20 scaled timber form is constructed. The mortar is poured into the mould which is in timber form and it is left to dry. One day later, they are removed from the mould and seen that the dried blocks are as light and rough as AAC blocks. Furthermore, a 45° angled backboard is prepared from cardboard and fixed to the table. Sandpaper is placed on the bottom of this backboard to prevent the blocks from slipping. For every funicular arch, the base plates are prepared on the backboard and the arch models are tested on these base plates.

By using these prepared 1:20 scaled blocks, the inverted hanging chain models are started to be checked. The maximum and minimum span and height level of two dimensional funicular arches, which are constructed from these blocks, are to be found by the tests. Span, heights and lengths are estimated as *unstable*, *feasible* or *not feasible* according to the case situation of the arches in the tests.

The tests are done according to funicular arches' lengths. Thus, six different arches are considered regarding their lengths and block numbers. The lengths of the arches are changed by adding or removing blocks. So firstly, the block numbers and the lengths of the arches are determined. Arches of 6 blocks, arches of 7 blocks, arches of 8 blocks, arches of 9 blocks, arches of 10 blocks and arches of 11 blocks are tested. For every different funicular arch, a base plate is prepared on Grasshopper and printed.

The tests began with funicular arches of 6 blocks, for that matter, arch length is also fixed and it is 18cm. For every span between 1cm. to 17cm., different funicular arches are drawn and used as base plates. The blocks are placed repeatedly on base plates and in which span the arches are stable and unstable are examined. The changes between variables, the span and correspondingly the height of the arches, are monitored.

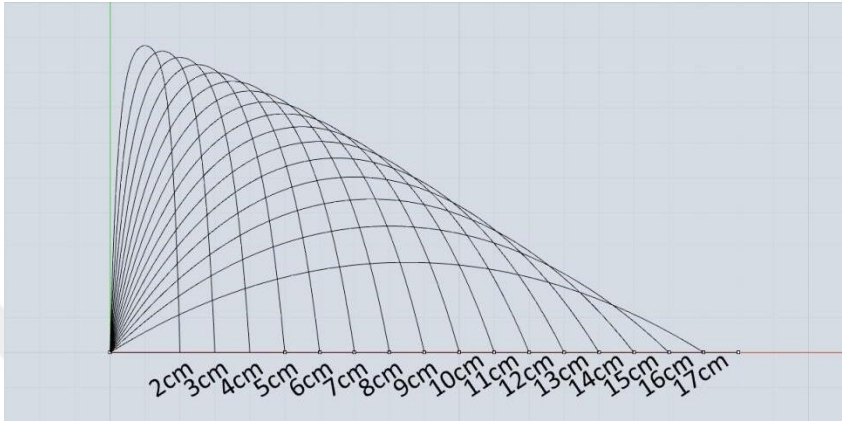


Figure 17. The Baseplate Catenary Arches for 6 Blocks,

The blocks are placed starting from 2 cm. span length, and it is seen that they are stable for these minimum spans. However, for 2cm. and 3cm. of span length, the shape of the arches of 6 blocks do not become funicular. The arches are observed to become funicular after 4 cm. of span length. The arch is feasible and the shape is more proper to be funicular. So, 4cm. is accepted as the *minimum span length*.

In the range of 5cm. and 15cm. of span lengths, the arches are modelled feasibly and give funicular shapes. For every increased span after 15cm. (16cm and 17cm.), the arches collapse. So, 15cm. is accepted as the *maximum span length*.

Moreover, while the span changes, the heights of these arches also differ proportionally with fixed block numbers and fixed arch lengths. In the Figure 18, for 6 numbers of blocks, the changes in height can be seen as span length changes.

Furthermore, to achieve an approximate span length - arch length relation for stable funicular arches constructed with AAC material, the tests are repeated for other arch lengths and block numbers that were determined at the beginning of the tests. New tests are done for the arches of 7, 8, 9, 10 and 11 blocks. The minimum and the maximum span lengths of these funicular arches are found and the curve lengths of

these arches are determined (see Appendix 1). Table 2 and Table 3 show minimum and maximum rates between span length and curve length for arches of the certain block numbers.

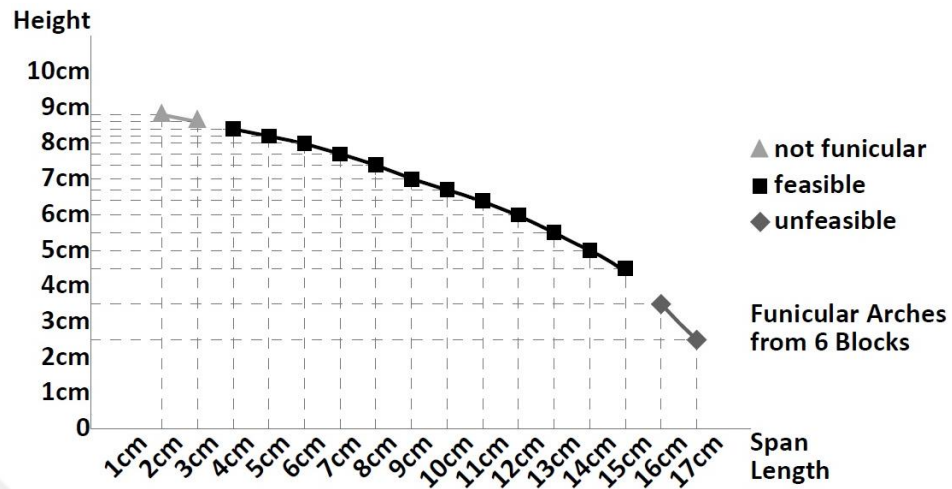


Figure 18. The Height and Span Length Relation of Arches from 6 Blocks

Number of Blocks	Span Length	Arch Length	Minimum Rate
n: 6	4 cm	18 cm	0,22
n: 7	4 cm	21 cm	0,19
n: 8	5 cm	24 cm	0,20
n: 9	5 cm	27 cm	0,18
n: 10	5 cm	30 cm	0,16
n: 11	6 cm	33 cm	0,18

Table 2. Min. Span Length and Min. Curve Length Rates According to Block Number

Number of Blocks	Span Length	Arch Length	Minimum Rate
n: 6	15 cm	18 cm	0,83
n: 7	18 cm	21 cm	0,85
n: 8	19 cm	24 cm	0,79
n: 9	20 cm	27 cm	0,74
n: 10	22 cm	30 cm	0,73
n: 11	23 cm	33 cm	0,69

Table 3. Min. Span Length and Min. Curve Length Rates According to Block Number

In agreement with the tables, it is found out that every minimum and maximum rate has approximate values. Thus, minimum rate average and maximum rate average for designing stable funicular arches of AAC blocks are specified.

	Average
Minimum Span Length / Arch Length	0,192
Maximum Span Length / Arch Length	0,775

Table 4. Span Length / Arch Length Averages

As a conclusion of these tests, the values which are important for structural stability of the shell that constructed with AAC blocks as material are found. These parameters are about the span length, arch length, blocks number and their relations. The maximum and minimum span values are found for different fixed block numbers and arch lengths. From the values that are found for funicular arches, the minimum rate of span length to curve length average and maximum rate of span length to curve length average are determined. In other words, values obtained from 2 dimensional tests are basics for the compression only freeform shell design with AAC material. These parameters are used as the stability constraints of the arches for generating the computational model with Grasshopper plug-in.

4.2. Computational Process

Computational process is the step of applying the determined variables to the design process of the shell geometry. Interactive methods innovated for the form-finding process of the shell structures have been reviewed in Chapter 2. And these methods have been discussed in two approaches; these were geometry oriented form-finding methods and material oriented form-finding methods.

In this study, the shell is considered to be material oriented and, a material oriented form-finding method *Particle-Spring Systems* is preferred for computational design process of the geometry. ‘Kangaroo’, which is an add-on for Grasshopper/Rhino and being used for particle based form-finding, is chosen as the design tool for the Particle-Spring Systems. Furthermore, *Finite Element Method* has been decided to be used for Structural Analysis of the designed geometry. And, ‘Millipede’ component in Grasshopper is also worked for FEM analysis. The two tools which are selected

are those that are simple to use and architects or designers can manipulate the geometry and be in interaction while designing and analysing.

4.2.2. Particle Based Form Finding

Kangaroo Physics add-on is a particle based form finding tool for Grasshopper/Rhino and it has been created by Daniel Piker. In the manual of the tool, this physics system is clarified as “*Kangaroo is an add-on for Grasshopper/Rhino and Generative Components which embeds physical behaviour directly in the 3D modelling environment and allows you to interact with it 'live' as the simulation is running. It can be used for various sorts of optimization, structural analysis, animation and more*”. Bertin (2013) remarks on being live of this interactive tool as “... allowed users poke, prod and tweak the model while watching the effect changes the model instantaneously”.

The particles which are dealt with in this tool are an abstraction, also can be called as sketch drafts, but have a strong connection in understanding how the real world works at a fundamental level. Naturally in the real world, objects are solved by more particles than we have used in this simulation, but if some attention is drawn to how the points are distributed and their masses, quite good approximations of real physical behaviour using this method can be taken. The great advantage of particle-spring system when compared to more intricate form-finding methods is that it is easier to understand to control the geometry with it in design process. Therefore, the producer claims that this conceptual simplicity made it possible for designers to apply and manipulate the physics simulation in a very direct way, without the need of specific technical knowledge.

Kangaroo physics works with the definition of anchor points, force objects and plan geometry on the kangaroo engine and it simulates and finds the equilibrium shape for the desired geometry. It contains various techniques of force generation which affects the particles in the simulation. All of these are plugged into the 'Force Objects' input and it can consist of various sources such as the response of materials to deformation (elasticity), user input, and geometric constraints.

For this study, curve pulls, springs from surface curves and vector force, which are applied to a point, are used as force objects. By considering all of these within the common way of force vectors, Kangaroo tool allows a live contact between them. To

determine these forces, the plan of the geometry is determined as a mesh object and it has been divided to its quads and edges. Edges that have been used for spring forces and quads are used as vertices for vector forces. The applied force at this point has been calculated in accordance with the volume of the geometry and the dry weight (d_{dw}) of the material. Thereby, a material orientation has been provided in the Form-finding step.

In addition, curve pulls represent the open edges of the geometry. To create the force object on the curve pulls, open edges should be determined in the shell. Catenary curves are determined in these places by naked vertices of the plan geometry, by using the range of span and curve length founded as a result of the hanging chain test. Pull points corresponding to these catenaries are defined as curve pull forces to the Kangaroo engine.

Anchor point is used to define fixed points at a certain place and regardless of the force applied to them, they will not be moved by Kangaroo engine. But, the designer can manipulate them in Rhino/Grasshopper to cooperate with the simulation while it is still iterating.

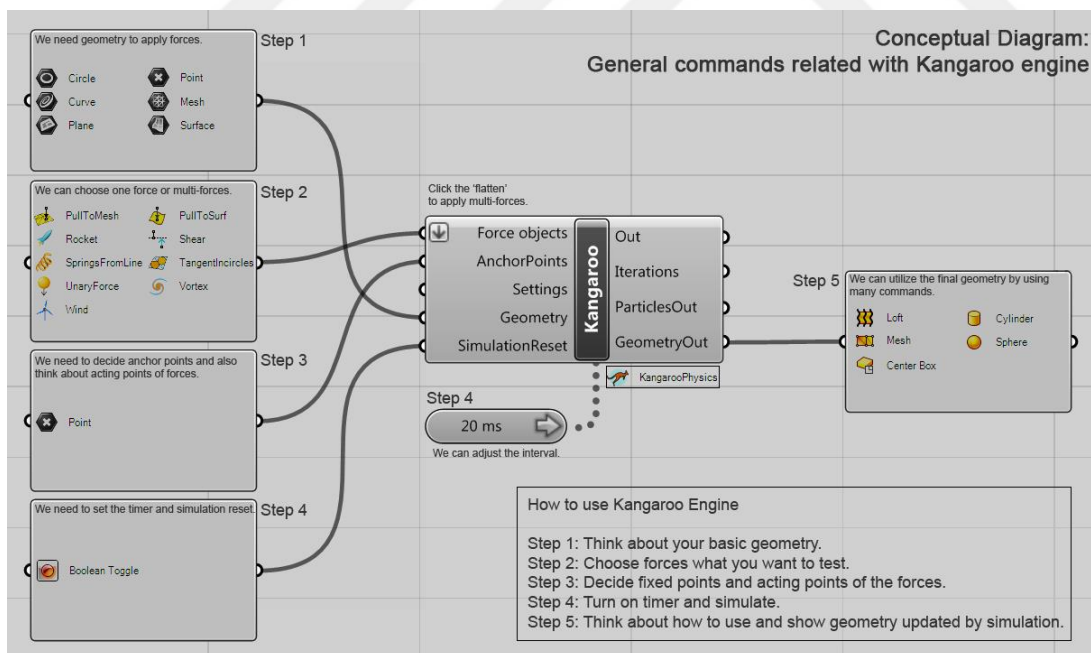


Figure 19. Conceptual Diagram of Kangaroo Physics

(<https://sites.google.com/a/umn.edu/digitalresources/tutorials/kangaroo>, last seen on 20.12.2016)

The plan geometry (topology), which is drawn at the beginning of the design process, is defined as the user input for the geometry to transform on the engine. Simulation reset and the timer are fixed for every design and after plugging them to the engine, the iterations for finding the equilibrium position of the particles will start. Finding the equilibrium particles also leads to the outcome of the equilibrium shape for the compression only vault.

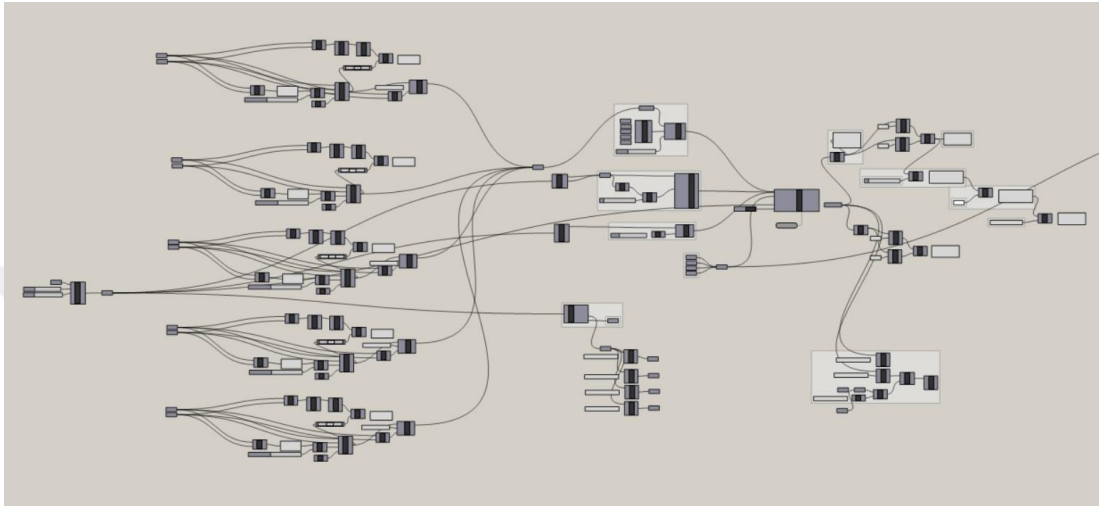


Figure 20. Form-Finding Definition in Grasshopper

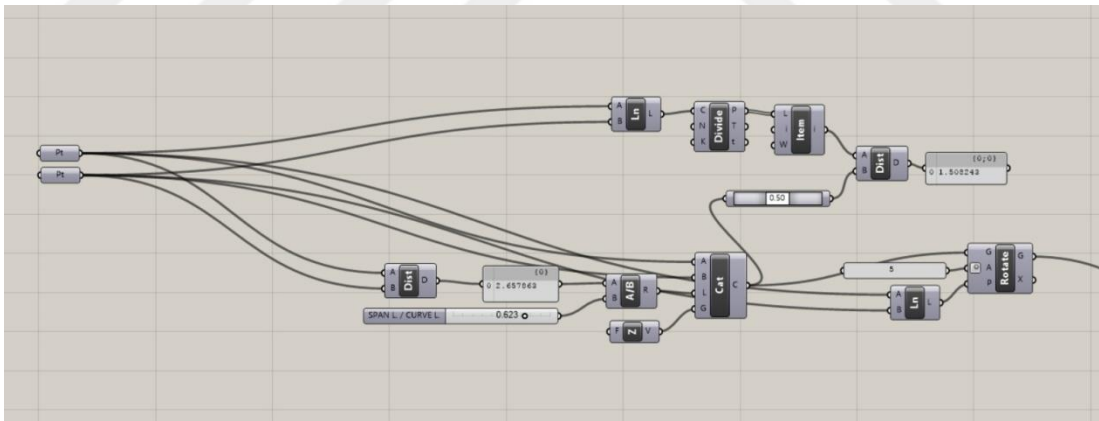


Figure 21. Catenary Definition with Span Length / Arch Length range

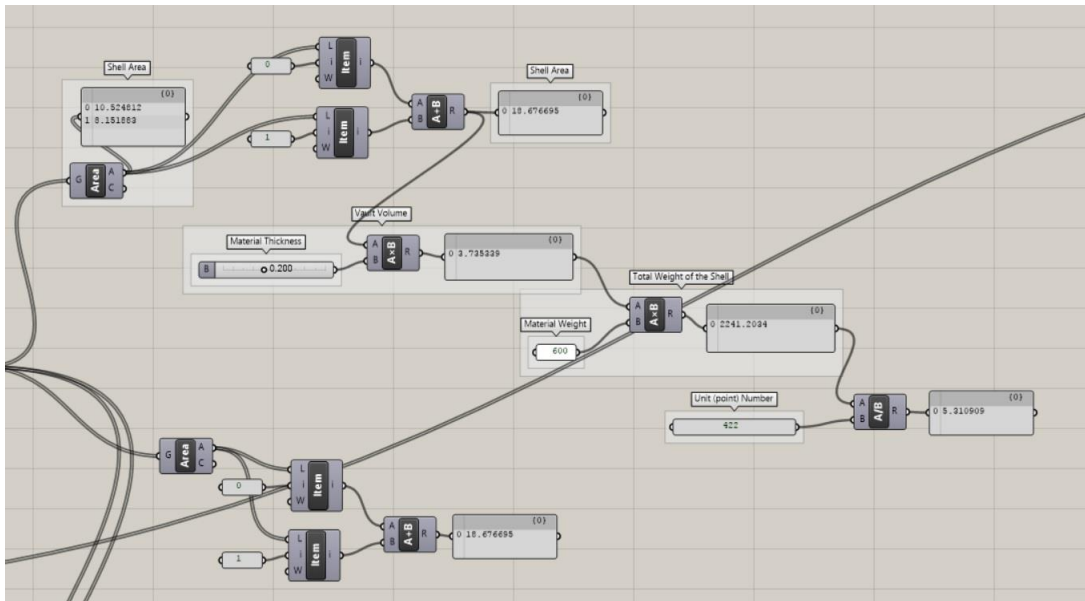


Figure 22. Unary Force Calculation According to the Material Unit Weight

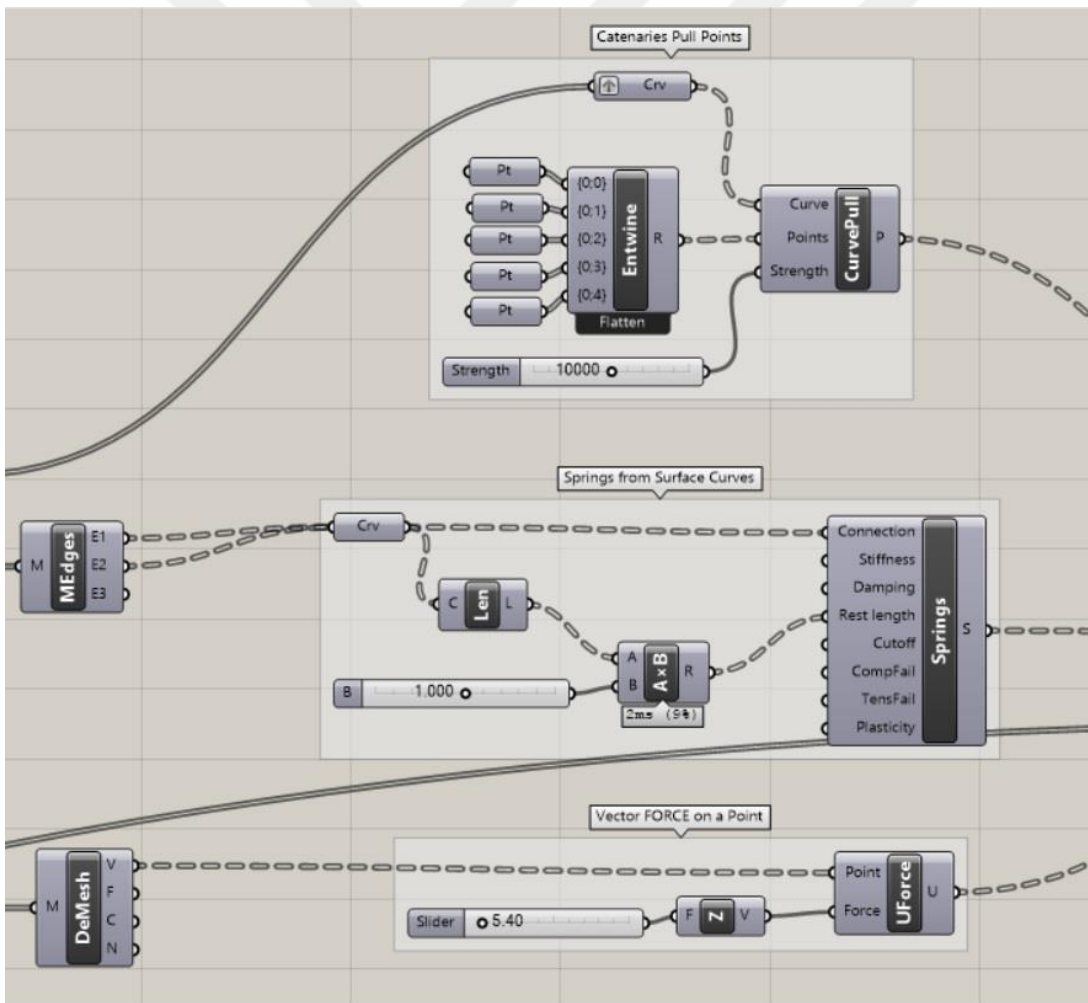


Figure 23. Force Objects Defined in Kangaroo Engine

4.2.3. Structural Analysis

Millipede is a component used in Grasshopper/Rhino for analyzing and optimizing the geometry structures, and it uses FE method. This tool is a functional software used commonly for architectural design practices, provides “material computation” techniques and produces experimental analysis for designers. The producers also remark that combining with Galapagos, this tool can be used for solving generic form-finding problems.

A basic definition is built for analysing the shell structure in grasshopper with this component. For this, shell elements of the tool are concentrated. Firstly, an FE mesh is created; the mesh surface generated with Kangaroo physics before, decided material and its thickness are determined, and thus a quad shell is defined for each face in the mesh. The custom material specification is done with Isotropic material component, and mechanical properties such as elasticity modulus, Poisson’s ratio, density and compression strength are described in this component. The second step for the FEA is to demonstrate boundary supports and point supports of the shell. Thereby, the quad shells defined before and these boundary conditions are defined to the FEA system, and a finite model is generated. FE Solver for FEM analysis is activated after that, and this is the slowest step in the process. The quad optimization part in this solver determines how much iteration should occur to solve the model.

After the iterations are completed, shell element visualization is used for showing the distributions of forces and stresses over the shell element. Visualization is shown with a coloured mesh and with stress pattern. Several color modes are available for showing deflection of the shell geometry. Color mode is defined by right-clicking on the component and selecting one of the available color modes. The color modes correspond to;

- von Mises Stress: Rainbow visualization of the von Mises stress
- von Mises Stress max: Rainbow visualization of maximum projected von Mises stress
- Bending Moment: Rainbow visualization of the maximum bending moment applied at the center of each quad.

- Principle Stress: Red/Cyan visualization subjected on whether the main effect is local or not and whether it is tension or compression at the specified layer.
- Yield: Stress ratio to yield stress for the specified material. Red explains that the stress is more than the yield stress threshold.
- Deflection: Rainbow visualization of the dispersion of deflections.
- Normal Displacement: Rainbow visualization of normal displacement at the midpoint of each shell quad.
- Stiffness Factor: This is the calculated optimization factor that defines the relative strength, density or thickness of the shell.

Normal Displacement and Principle Stress visualizations are noted for this study, and their results are taken.

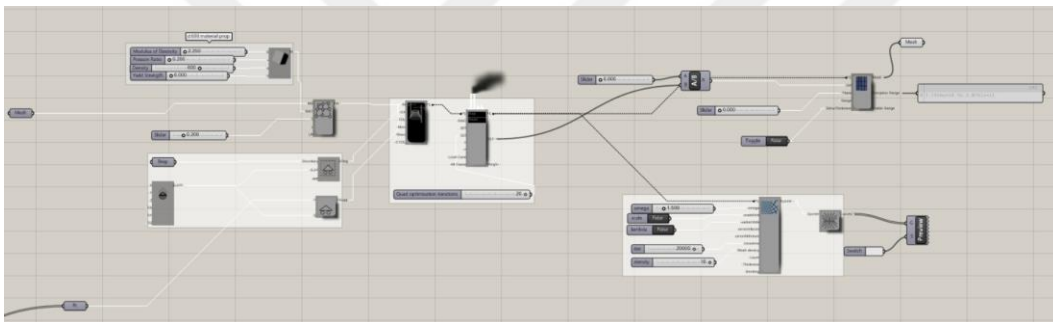


Figure 24. Structural Analysis Definitions with Millipede Component

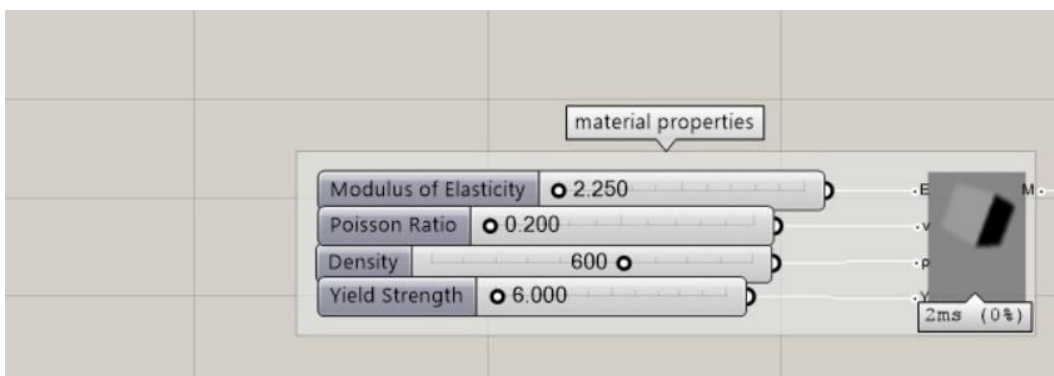


Figure 25. Custom Material Definition with Isotropic Material Component

4.3. Structural Performance Examinations

Masonry structures are unreinforced structures, and they are mostly constructed of materials such as unsupported bricks, or stone blocks called voussoirs. These structures are standing by their specific structural forms and thicknesses. On account

of being in a static equilibrium and all the forces' being in compression, these structures do not need any other supporting.

Designing of complex masonry structures is difficult to challenge, and it needs structural knowledge. However, there is not any information about AAC materials' structural behavior in complex structures and asymmetrical vaults. In accordance with this statement, the structural performance of vault models designed with AAC material is examined in this chapter.

4.3.1. Examining Symmetric Vault Models

In this study, the structural behavior of the shells designed by using AAC material is examined with symmetrical models. Thus, there is an attempt to find out and specify the structural knowledge of AAC material from simple geometries. In the examination, Rhinoceros3D with Grasshopper software and some plug-ins are used for the computational design of the models. A Grasshopper definition is presented for each model by using the value found as the result of *hanging chain tests* conducted in the previous section. Kangaroo plug-in, which uses the particle-based form-finding method as its working principle, is used to find equilibrium structure and Millipede plug-in is used for analyzing the models.

The examination is arranged as designing models, that each model have different number of supportors, in the same area. In other words, a circular area is defined and models supported by different number of foots are designed in it. Firstly, the physical properties of these models are compared such as the height of the catenary arches and the surface area of the models. After that, structural behaviors of these models are analysed by Millipede plug-in, which uses FEM as the analyzing method.

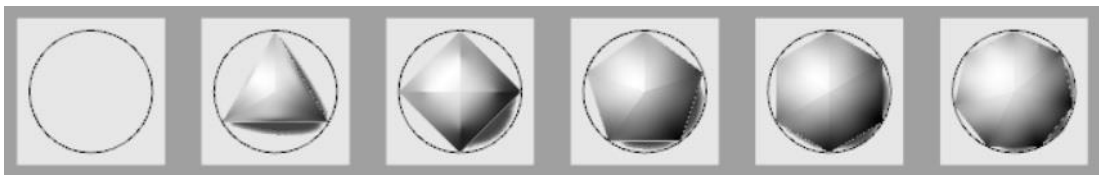


Figure 26. Circular Area and Designed Symmetrical Models

The examination starts with drawing a circular area and equilateral polygons into this circle as a triangle, a tetragon, a pentagon, a hexagon, and a heptagon. These equilateral polygons are defined as equilateral meshes on Grasshopper, and with

Kangaroo physics engine, these meshes get relaxed under applying their self-weight. For each model, the same material properties such as thickness and density are implemented. Moreover, with this simulation engine, the motion and the changes of these models are monitored.

d_{uw}	Thickness
400kg/m ³	10cm

Table 5. Material Properties for AAC Material Used in Kangaroo Plug-in

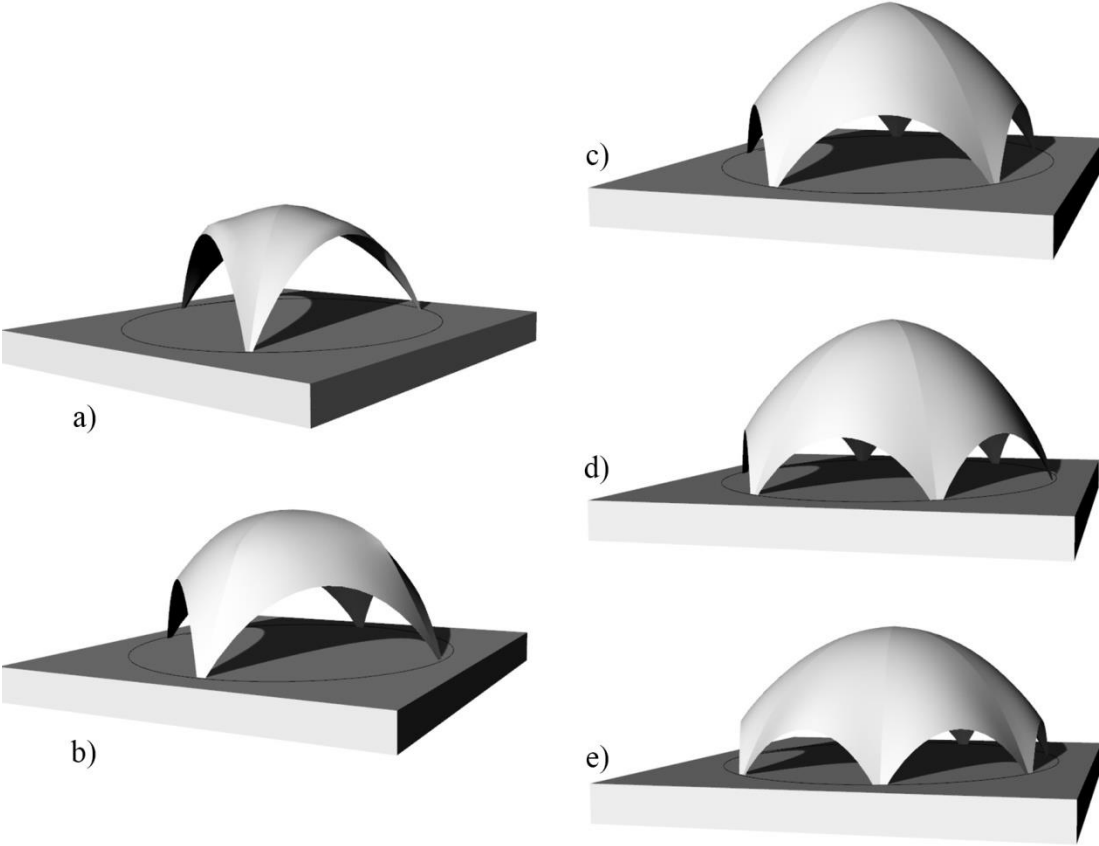


Figure 27. Relaxed Shape of 3D Models. a) Triangular shell b) Tetragonal shell
c) Pentagonal shell d) Hexagonal shell e) Heptagonal shell

Subsequently, the height of the catenary arches and the surface areas are calculated again in Grasshopper for the physical comparisons of these shells. As a result of these physical comparisons, it can be said for equilateral symmetric vaults from AAC material that as the number of supporter increase (number of the foots increase), the height of the catenary arches reduces. Furthermore, correspondingly to the increasing number of edges, the surface areas of the shells also increase.



Figure 28. The Elevations of the Models after Relaxation

Thereafter the physical comparisons of the models, all these shells are analyzed in Millipede plug-in to observe the structural behaviour of AAC material. All 3D models, found out by Kangaroo Engine are add to the FE system of the Millipede and defined as a mesh. AAC material is not in the library of materials in Millipede. Therefore, a custom material is prepared by using mechanical properties of AAC material according to reviews on chapter three.

Static modulus of elasticity (Gpa)	Poisson Ratio (v)	Density (kg/m ³)	Compressive strength (MPa)
1.650	0.2	400	2.8

Table 6. Material Properties for AAC Considered for FE Analysis.

The statical examination begins with Normal Displacement analysis. With the given results of these analyses, the displacement at the centre of each quad is visualized. According to the analysis, it is seen that for each shell, the displacement is mainly seen on the foots of the models. Furthermore, as the number of supporters increases on the shells, the surfaces' forming foots also increase. Thus, the displacement spread to these surfaces and the displacement density on foots decrease.

Moreover, according to stress analysis results and visualization, it is experienced that the high-stress concentrations are located around the foots and open arches of the shells. According to visualization results of the stress analysis, as the number of supports increase, stress curves are drawn near to these supports and become more regular. The stress on the other places of the shell decreases. As a consequence of these analyses on a defined circular area, it is acceptable that the more foots there are on the shells, the more durable the structure is modelled.

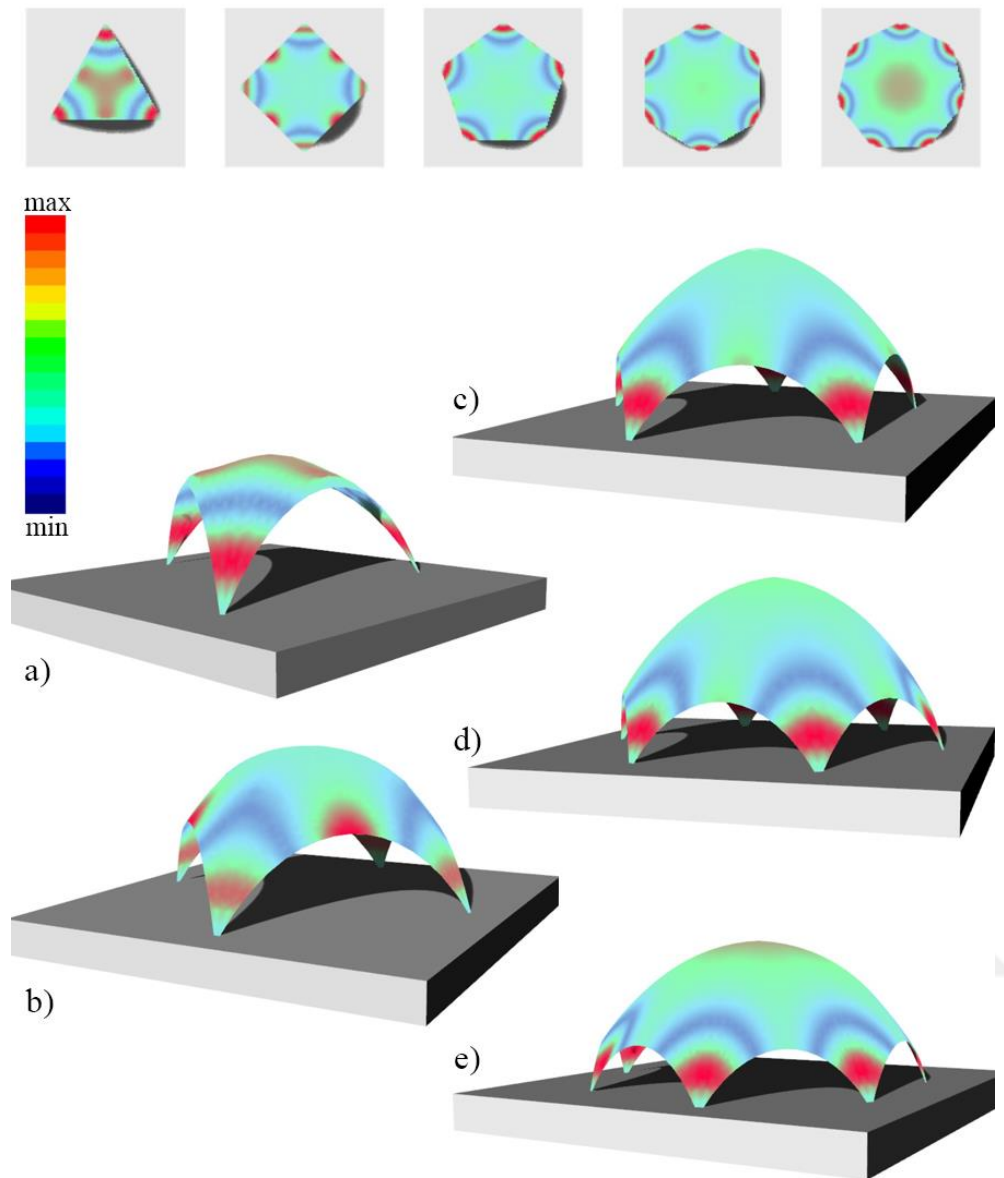


Figure 29. Colored Visualization of Normal Displacement Analysis on Millipede

- a) Triangular shell b) Tetragonal shell c) Pentagonal shell d) Hexagonal shell
e) Heptagonal shell

	Displacement Result (m):
Triangular shell	0.000088 to 0.001516
Tetragonal shell	0.000021 to 0.001211
Pentagonal shell	0.000033 to 0.000999
Hexagonal shell	0.000002 to 0.000670
Heptagonal shell	0.000024 to 0.000591

Table 7. Normal Displacement Values of the Shells

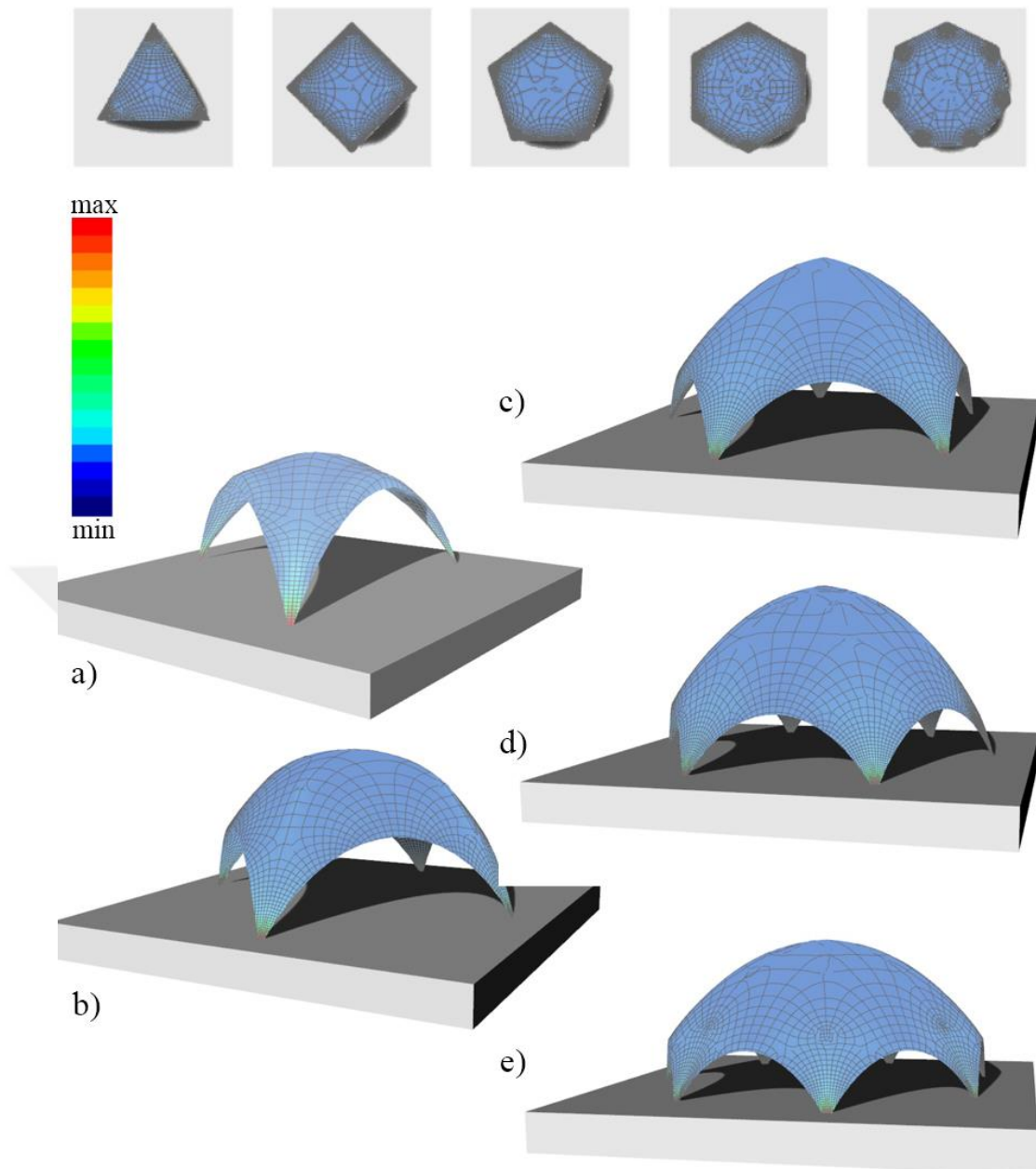


Figure 30. Colored Visualization of Principle Stress and Stress Pattern Analysis on Millipede a) Triangular shell b) Tetragonal shell c) Pentagonal shell d) Hexagonal shell e) Heptagonal shell

4.3.2. Examining Material Thickness of AAC

In this examination, structural behaviours of vaults are observed depending on thickness of AAC material. Thus, there is an attempt to find out the structural effect of the material thickness on these shell structures. Hence, a tetragonal shell is prepared as in the previous examination, on Grasshopper and for relaxation part by

the Kangaroo plug in; material thickness of the shell is changed for each shell manually. For the first shell, the thickness is considered as 10cm, 20cm for the second and 30cm for the last shell. Depending on the changes in thickness, the self-weight of the shells and the applied force from one particle are revised directly. At the end of all these modifications, according to the thickness, the equilibrium shape of the shells are found as shown in the Figure below. Millipede plug-in is used for analysing structural behaviours.

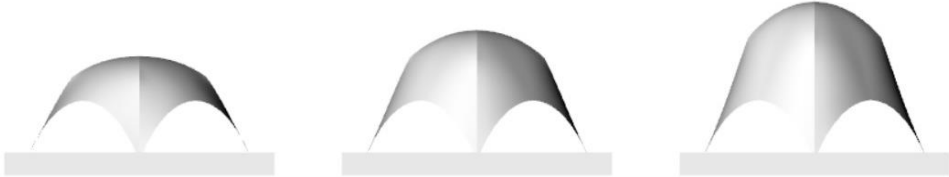


Figure 31. The Elevations of the Models after Relaxation a) Thickness: 10cm. b) Thickness: 20cm. c) Thickness: 30cm.

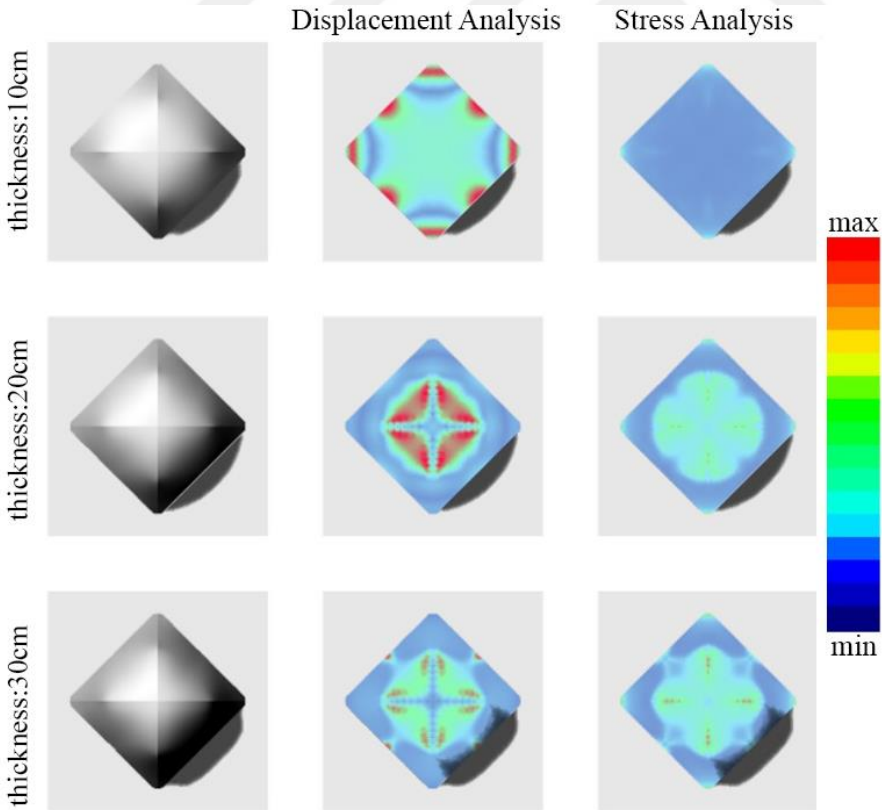


Figure 32. Colored Top View Visualizations of Displacement and Stress Analysis on Millipede

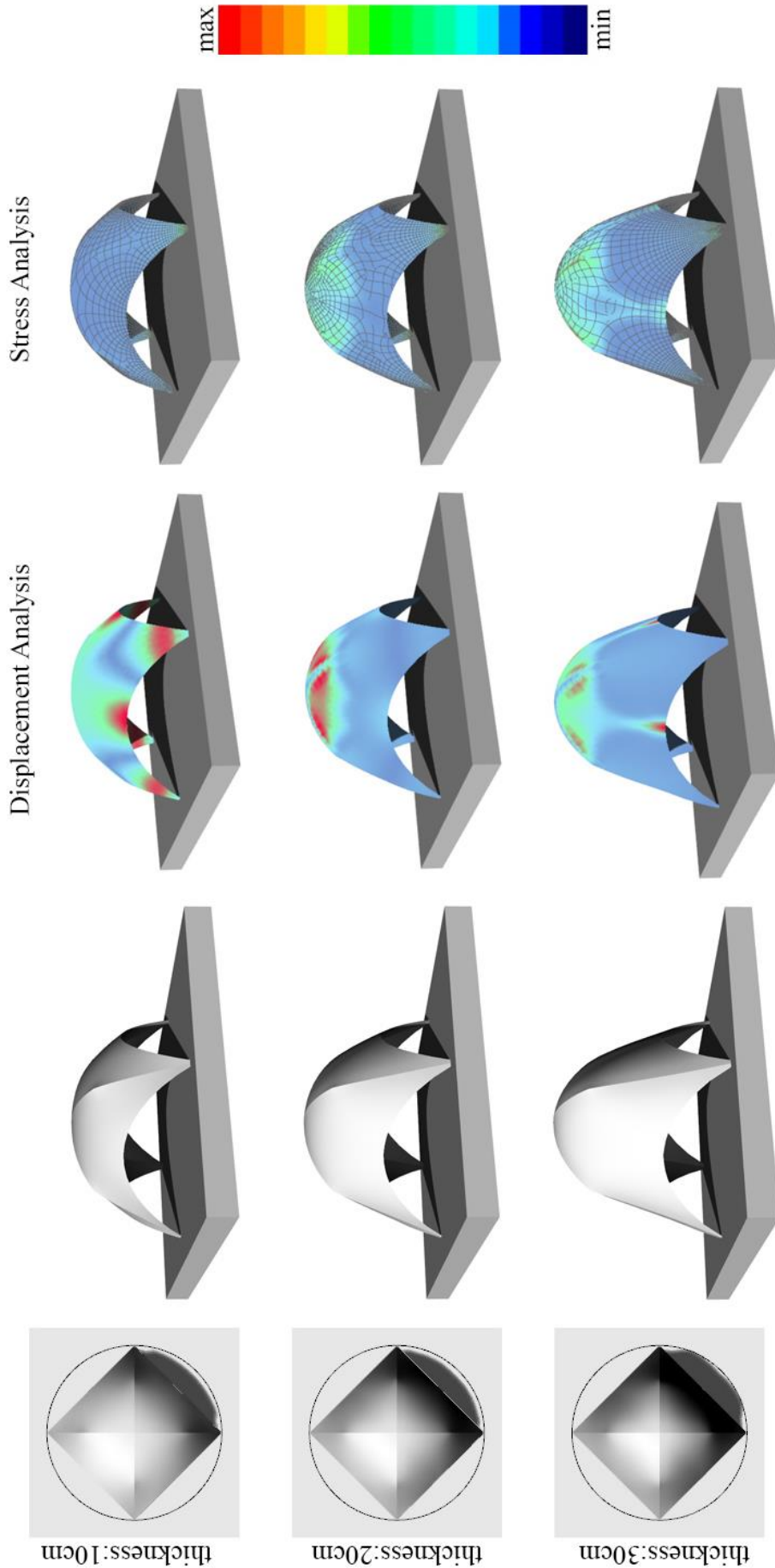


Figure 33. Colored Visualizations of Displacement and Stress Analysis on Millipede

Shell Thickness	Displacement Result(m):
10cm	0.00003 to 0.001275
20cm	0.000029 to 0.00328
30cm	0.000068 to 0.00394

Table 8. Normal Displacement Values of the Shells

As the result of the visual shell analyses, it is seen that for 10cm thickness of AAC material, the displacement is seen mainly on supporters and keystones of the structure and stress patterns are also concentrated here. As the material thickens, the displacement and the stress density on the shells increase. However, displaced areas are changed for each thickness of material, for 20 cm. thickness material displacement is seen only on the top of the shell, and for 30 cm. thickness displacement is seen on the top and on the keystone of the arches. In light of these examinations, as a result, it can be said that for constructing shells' foots or the supporters on where it is grounded, thick AAC materials are more suitable for structural aspects. And, slimmer materials are more appropriate for the body and the top of the shells.

4.3.3. Examining Unit Weight of AAC

This study is conducted to examine the structural behavior and displacement changing of the shells, depending on the unit weight (density) of AAC material. Thus, the structural effect of the density on the masonry shell structures is found out. Three different unit weights are considered to examine. They are as d_1 : 350kg/cm³, d_2 : 450kg/cm³ and d_3 : 600kg/cm³, and according to these density values, similar tetragonal equilibrium shells are prepared on Grasshopper and Kangaroo as the previous examination. Depending on the changes in density, the weight of the shells and the applied force from one particle is revised directly. The equilibrium shapes of the shells are found as shown in the Figure below. It is seen that, as the density of the material increases, the weight of the shells increases. And dependently, for distributing the increased compression and loads balanced over the shape, the height and the volume of the shells increase.

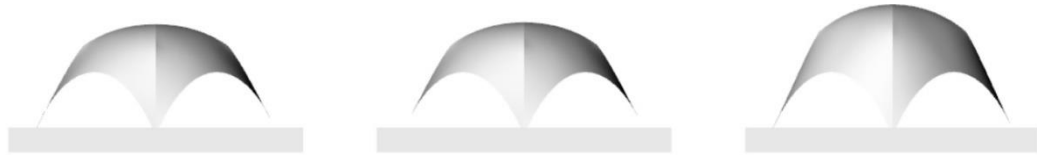


Figure 34. The Elevations of the Models after Relaxation

Finding the equilibrium shape of the shells according to the density, Millipede is used for analyses. According to the reviews done in chapter three and the Material Performance Certificates of the producer companies, the approximate values of each density needed for the FEM analyses are specified as shown in the table below. Thus, displacement and stress analyses are carried out for each shell.

Density (d_{uw})	Poisson Ratio	Static modulus of	Compressive
350	0.2	1.350	2.450
400	0.2	1.650	2.800
600	0.2	2.250	6.000

Table 9. Material Properties for AAC Considered for FE Analysis

As a result of the analysis, it is seen on the visualization that the displacement is decreased while the density value increases. And, as the density intensifies, the displacement moves from foots to keystones. For 350 kg/m³ density, the stress pattern is observed to be more regular. However, as the density increases, the regularity deforms and concentrates on the supports.

Material Density	Displacement Result (m):
d_{uw} : 350kg/m ³	0.000019 to 0.000791
d_{uw} : 400kg/m ³	0.000017 to 0.000870
d_{uw} : 600kg/m ³	0.000011 to 0.000407

Table 10. Normal Displacement Values of the Shells

Therefore, it is deduced that 600kg/m³ dense AAC material is proper for foots and supporters of the shells where they are grounded, 350kg/m³ dense material is proper for the keystone parts of the arches, the top of the shells and 450 kg/m³ dense AAC material is proper for the middle area between top of the shell and its supporters.

According to numerical results of the Normal Displacement analysis, it is noticed for a complete shell that, the minimum displacement is seen on the shell designed with AAC material of 600kg/m³. So, if minimizing the mass is not targeted, this material density is verified as more suitable to be used in the design of a complete shell.

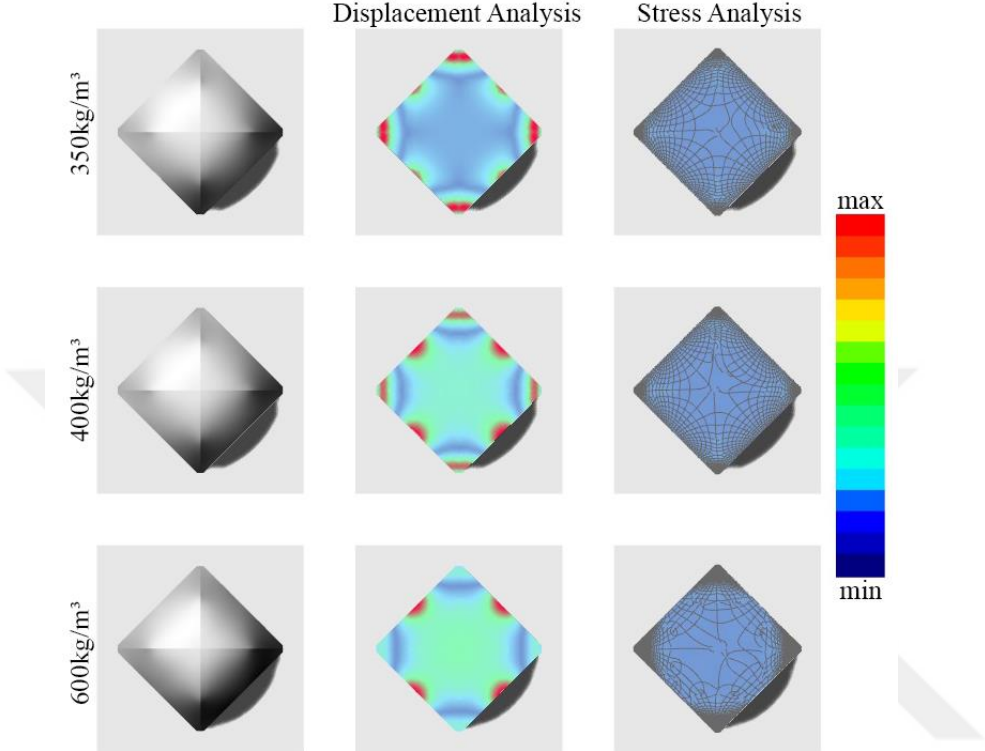


Figure 35. Colored Top View Visualizations of Displacement and Stress Analysis on Millipede

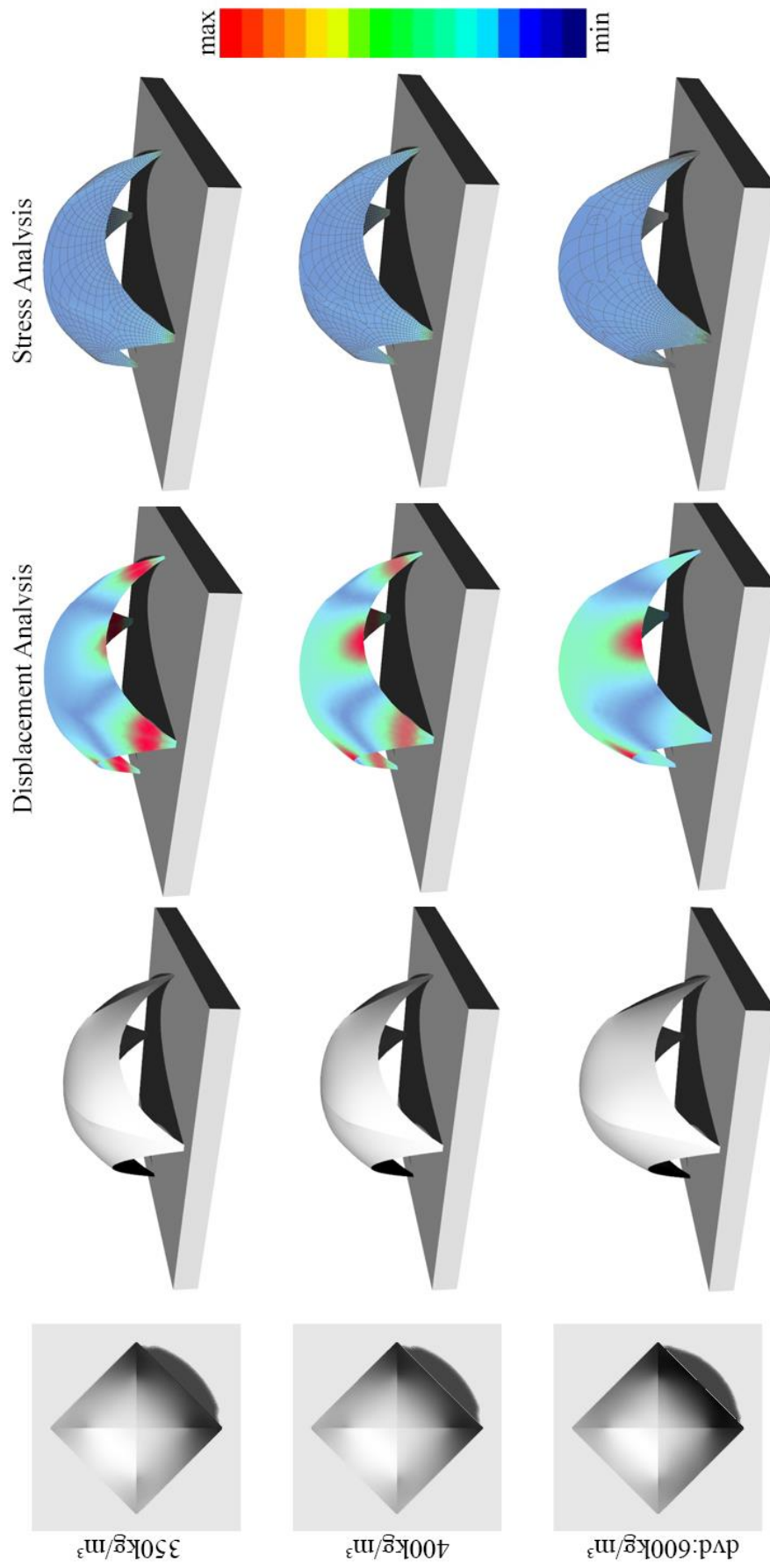


Figure 36. Colored Visualizations of Displacement and Stress Analysis on Millipede

4.3.4. Examining Maximum Span of Arches

In section 4.1, hanging chain tests are done, and maximum and minimum *Span Length - Curve Length Averages* are defined as a constraint of funicular arches that are used in open edge determination part of the shells' design processes. By using the range of these averages, the funicular catenary arches work in equilibrium and stand in compression with their own weight. Thus, the funicular arches designed considering this range are stated to be stable all the time.

However, in this study, the maximum limit of the span that the arch can pass between two supporters of the geometry is requested to find out. Again, depending on the tests done in section 4.1, the minimum span length of a funicular arch is accepted as 4 cm in scale 1/20, in real scale (1/1), its true size subtends to 80cm. This examination is done for determining the maximum span length of arch passes that is built with AAC material.

For the examination, six tetragonal shells are prepared computationally in respectively L: 13m., L: 15m., L: 17m., L: 18m., L: 19m., and L: 20m. Span lengths and deformations on the shells are analysed by Normal Displacement and Principle Stress analysis. Reviewed examples take the serviceability criteria into account for buildings to settle upon whether the deformation results are acceptable or not. In order to check the examination results, these serviceability criteria are determined to be used as in the examples. According to the criteria, to prevent damage, deformation must result in less than 1/500 of the span. So, if the displacement is less than 1/500 of the span, it is an acceptable result for the structural strength. The numerical results of the shell are as in the table below. The formulation of displacement criteria is as;

$$d < \frac{L}{1/500}, \text{ result is acceptable}$$

$$d \geq \frac{L}{1/500}, \text{ result is unacceptable and structure can be damage.}$$

d: Displacement result, L: Span length

Span Length	Displacement Result (m):
L:13 m.	0.000073 to 0.00275
L:15 m	0.000020 to 0.00312
L:17 m	0.000113 to 0.015023
L:18 m	0.000176 to 0.025487
L:19 m	0.000214 to 0.034836
L:20 m	0.00057 to 0.053958

Table 11. Normal Displacement Values of the Shells

According to structural analysis results, all the spans are controlled whether they are acceptable or not depending on the displacement criteria. The results for L: 13m., L: 15m., L: 17m., L: 18m., and L: 19m. span lengths are acceptable. But the outcome of L: 20m. displacement result is found higher than $1/500$ of span, so this is unacceptable. In this case, L: 19m. span length is determined as the maximum span length of the arch can be constructed with AAC material.

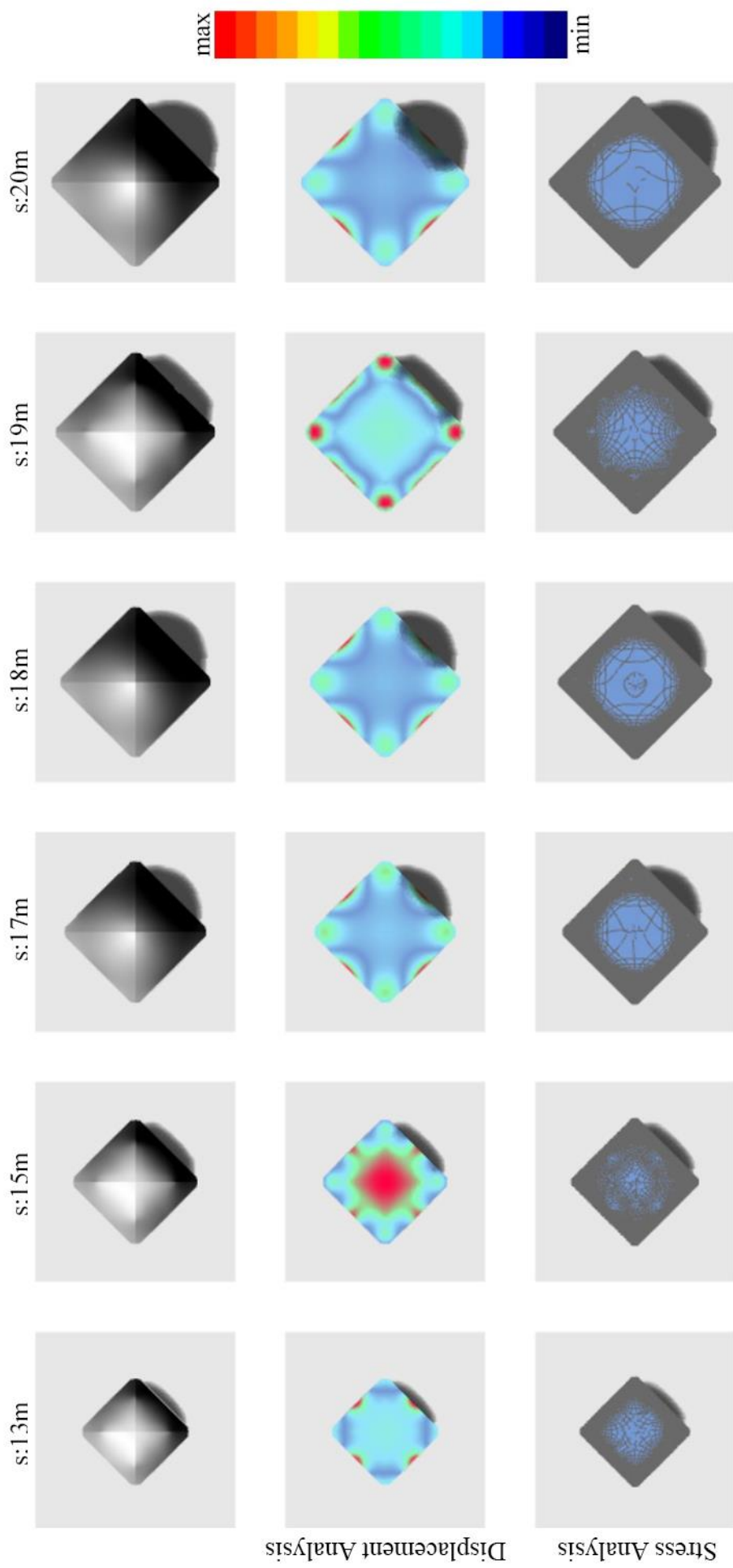


Figure 37. Colored Top View Visualizations of Displacement and Stress Analysis on Millipede

4.3.5. Result of the Examination for Structural Performance

Building a masonry shell is an old technique. However, it is often designed with bricks or stone blocks. The formal complexity of this thesis is to use a new material for this form of masonry. Therefore, to understand the structural behavior of the AAC material in masonry structures, examinations that have been carried out so far are explained in this section. As a result of these examinations, it is found out that;

The supporter foots of the shell on the ground are very important for structural strength. And it is demonstrated that if the number of these foots or the area of these supporters grounded on increases, the structure of the vault becomes more durable.

In the material thickness examination, it is realized that the displacement is seen on different areas for each material thickness. For 10cm AAC material, the displacement is seen mainly on the foots and the keystones of the structure, for 20 cm. thickness material the displacement is seen only on the top of the shell, and for 30 cm. the displacement is seen on the top and on the keystone of the arches. Therefore, the result generated for material thickness is that the shell is divided to three regions and for each region, material thickness is changed. 10cm thick AAC material for the top region, 30cm thick AAC material for foots and supporters on the ground and 20cm thick AAC material for the middle area between the top and the foots, are determined to be more proper.

Depending on the unit weight of the material examination, just as thickness changes, it is seen that as the density of the material changes, the displacement area also changes and it is seen for each density on different regions. It is deduced that a more dense AAC material is more proper for the foots and the supports of the shells, and lighter materials are proper for the keystone parts of the arches and the tops of the shells. For the material densities used in examination, the result is as; 600kg/m^3 dense AAC material is proper for the foots and the supports of the shells located on the ground, 350kg/m^3 dense material is proper for the keystone parts of the arches and the tops of the shells and 450 kg/m^3 dense AAC material is more proper for the middle area between top and foots. Also, according to the numerical results, it is seen that the displacement on the shell designed with 600kg/m^3 dense AAC material is less than others. So, it is found out that this material is more suitable for using in the design of a complete shell if it is not subject to minimize the total mass of the shell.

According to the test done in section 4.1, minimum and maximum *Span Length - Curve Length Averages* are defined for the constraint of funicular arches that are used in the open edge determination parts of the shells' design processes. Correspondingly, the maximum span of arches examination is done to bring a constraint to the determination process of funicular arches of the geometries designed with AAC material. Thus, with the work done, the *maximum span length* is determined and it is L: 19m.

With these examinations, suitability of the material properties and the structural characteristics knowledge of AAC material are observed through symmetric masonry shells. Thus, design complexity of the asymmetrical shell of AAC material is managed to challenge.

4.4. Development of the Generative Model

In light of the examination done on the previous section, material properties and the constraints of a designed shell by using AAC material are determined. Thus, knowledge about structural behavior of this material used on symmetrical geometries is obtained. This information is also adaptable for complex structures and asymmetrical shells. Upon adapting this information to asymmetrical shells, the design process starts. The design process of generating the model contains three steps; topology and open edge determination, shape optimization and structural analysis.

For the first step, a topology surface is determined and the initial conditions of the shell are defined. This surface is subdivided to its quads in U and V direction. During the subdivision, the unique set of faces and edges are partitioned. Thereby, for every vertex a vertex point and for every edge an edge point is added. Thus, all vertices of the surface are designated. Naked vertices are used for drawing catenaries for the open edges. Arches of the catenaries are designed by using the range of span and curve length founded as a result of the hanging chain test on the previous section.

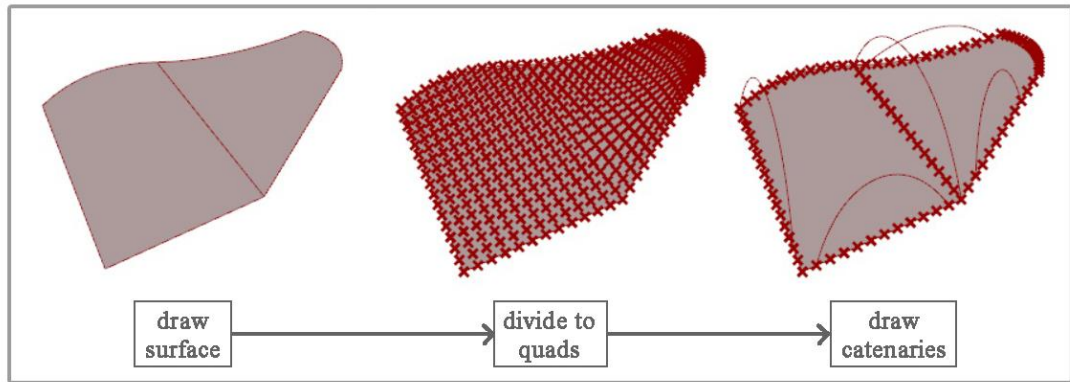


Figure 38. Topology and Open Edge Determination

The shape optimization is fulfilled by using particle based form finding Kangaroo plug-in in Grasshopper. Bhooshan (2014) stated about principle of the particle system as *"finding structures in static equilibrium and, achieved by defining the topology of a particle spring network with loads on the particles, the masses of the particles, the stiffnesses and lengths of the springs, and then by attempting to equalize the sum of all the forces in this system"*. In view of Bhooshan's statement, force objects of the system are arranged, these are pull points of the catenary arches, springs from edges of mesh and unary force applied from each particle. Vertices designated in first step are used in regulating force objects. Furthermore, anchor points, which are the points on the ground, are determined before the iteration process of the system started.

For each design on Kangaroo, unary force that is applied from the particles is updated. At first, an estimated value is given for the unary force, from the first iteration of the system, the volume of the shell is checked out and the gravity load is updated. It is calculated based on the self-weight of the shell. With computing volume and unit weight of the material, the self-weight of the shell, which is the total load applied to particles, is found. By the division of this value to the particles number, applied load from one particle is calculated. Thus, form finding is performed by updating calculated value for each particle. Iteration continues until the changes of the geometry gets smaller and the equilibrium shape of the shell is found.

Subsequently, upon finding the equilibrium shape of the shell, structural analyses are commenced. They are performed by using FE method in Millipede plug-in for Grasshopper. Depending on unit weight of the material examined, it is seen that the displacement on the shell designed with 600kg/m³ dense AAC material is less than others and this dense material is chosen to be used for the shell. The mechanical properties of the chosen AAC material obtained from the literature are used to perform the analysis.

Density (kg/m ³)	Poisson Ratio (v)	Static modulus of elasticity (Gpa)	Compressive strength (MPa)
600	0.2	2.250	6.000

Table 12. Material Properties for AAC Considered for FE Analysis

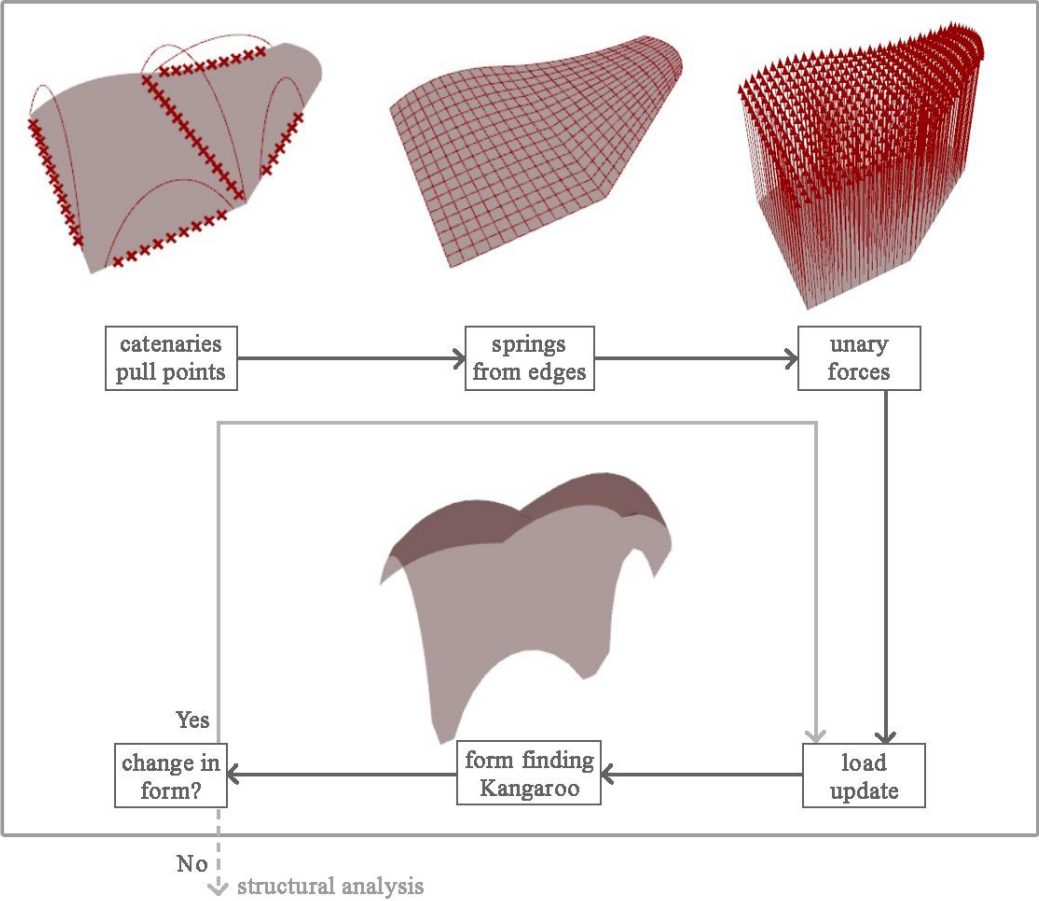


Figure 39. Shape Optimization by Particle-based Form Finding with Kangaroo.

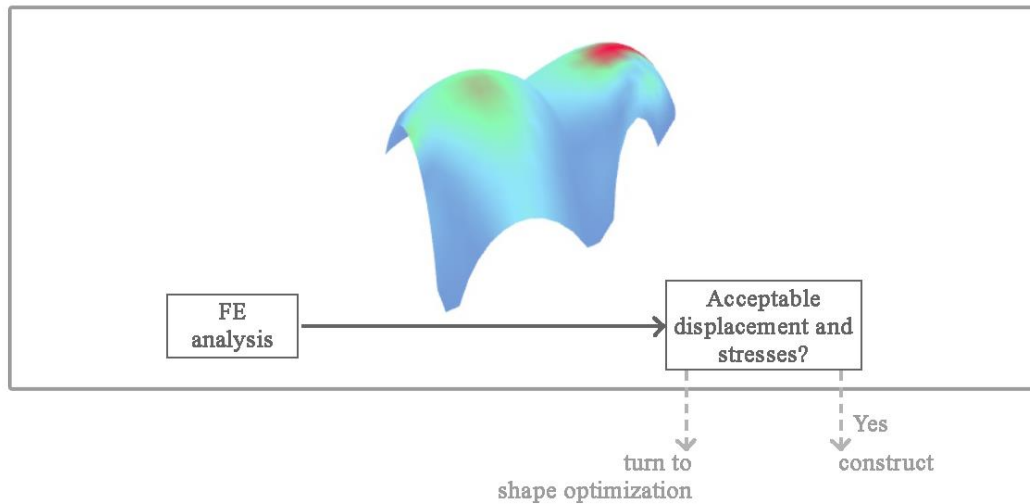


Figure 40. Structural Analysis by FEM in Millipede

In the FEM analysis program, Normal Displacement and Principle Stresses of the shell are analysed and the values are controlled whether they are admissible or not. If the analysis results are acceptable and feasible, the structure can be constructed. However, if the results are more than expected, form of the shell should be changed. For such circumstances, the flow is turned to the shape optimization part and all the steps are done over again. The steps of the design process are all interchangeable with the previous steps.

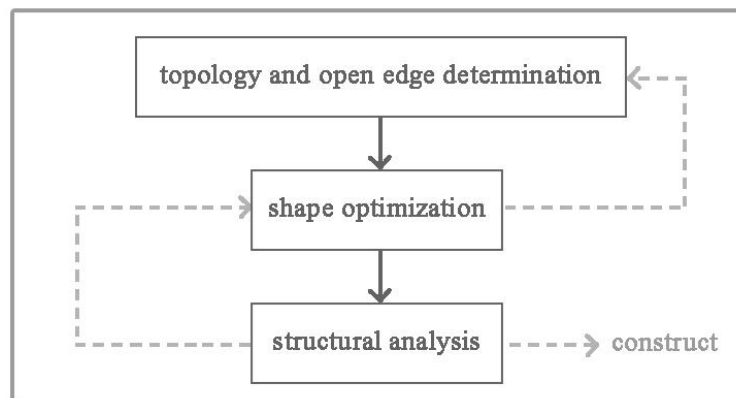


Figure 41. Flowchart for the Three Design Process Steps

As a result, it can be said that the purpose of this study is to design an asymmetric shell system which is in static equilibrium in order to achieve a constructible structure. This objective has been reached by carrying out the progress of the particle- spring system method. After the particle based shape optimization of the model, the structural analyses have been done for controlling the shape in respect of

displacement and stress formation. Thus, the generated system has been checked once again in terms of structural stability.

The maximum span of the overall area that the generated shell topology settled in is 5 meters. Regarding the considered serviceability criteria, the maximum deformation on the shell is essential to be less than 1/500 of this span. Calculations have been made and it has been found out that the maximum displacement on the model resulted less than the 1/500 of the maximum span. It has been verified that, the generated shell model performed an acceptable result in terms of displacement. The displacement results are as in table 14.

	Displacement Result (m):
Design	1.0389e-7 to 0.00116

Table 13. Normal Displacement Values of the Shell

Regarding the visualization of the structural analysis, red colour refers to maximum displacement and maximum principle stress and dark blue refers to minimum displacement and minimum principle stress. According to the displacement visualization on Figure 44, it is seen that the displacement is concentrated on a region on the top of the equilibrium shape and the other regions of the shell do not have a serious deformation. Hence, it is interpreted that the regeneration of the model could be performed for this region of the shell which shows high displacement values.

With respect to the stress visualization taken from structural analysis, it can be said that the coloured stress visualization does not correspond to only tension or only compression effects, instead it refers to both of them and visualize the colours according to the dominance of the effect that they show. So, it is understood from the stress visualization diagrams that principle stress has a more dominant effect on higher regions of the developed generic model. Also, stress pattern indicates the stress directions on the shell. Designers can control the scale of these patterns according to the project requirements (Michalatos and Kaijima, 2014). Stress patterns give the chance to designers to control the laying pattern of the blocks, and also the shape of the blocks can be determined subject to stress directions on the model. Thus, remarkable design opportunities can be created. In this study, the shape of the blocks and laying pattern are taken out of scope for this reason, stress patterns have been only used for observing the stress directions.

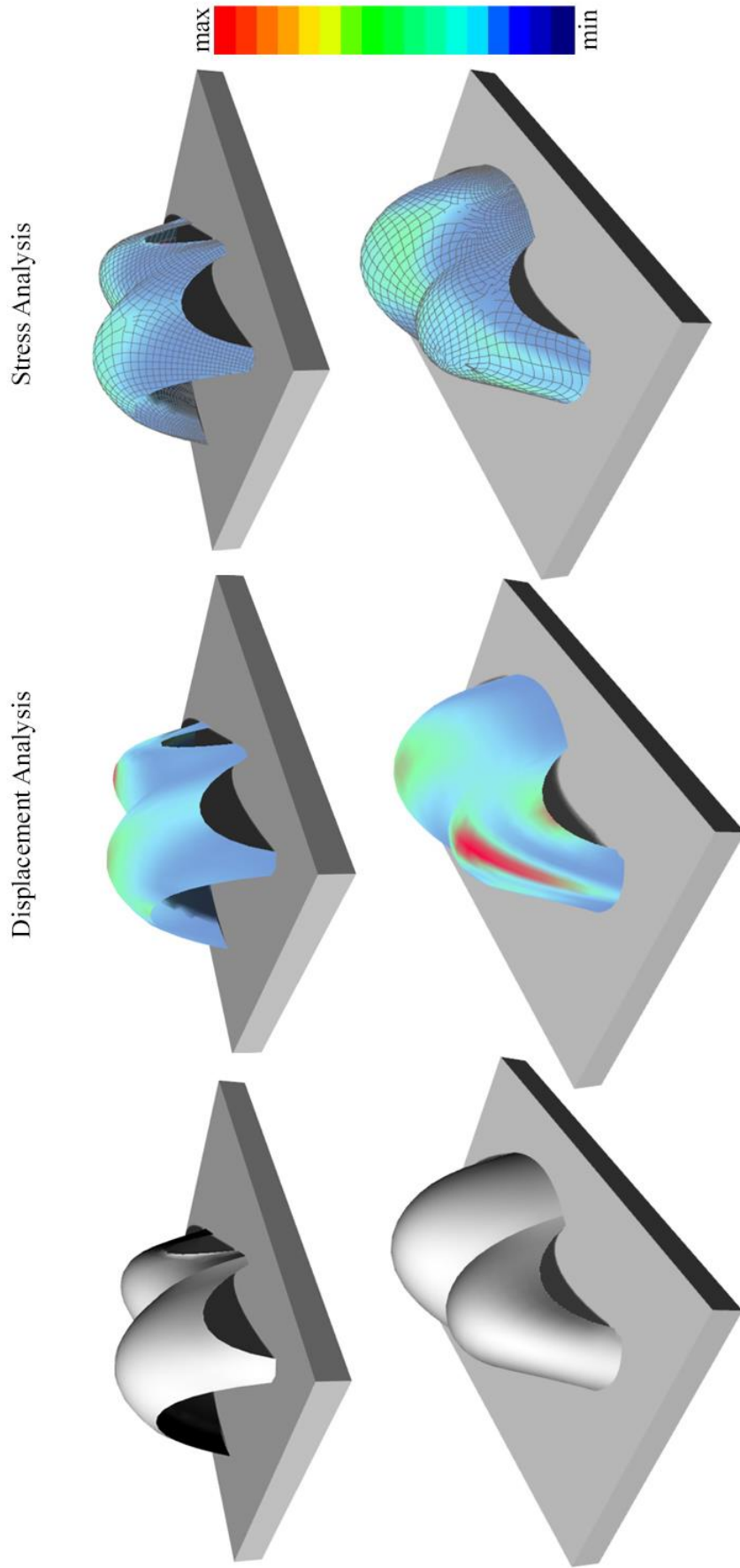


Figure 42. Colored Visualizations of Displacement and Stress Analysis on Millipede

CHAPTER FIVE

CONCLUSIONS

5.1. Summary

Masonry shells are significant structural systems in architecture among the buildings of world's architectural heritage. Today, they are still a part of Contemporary Architecture. The main goal of this study was making research and examinations on developing masonry shell models by using AAC blocks as the material and preparing a digital sketch model in light of the information obtained from the research. Hence, throughout the study, it has been seen that the design of shell systems are considered as difficult efforts due to the requirement of deep structural knowledge.

More complex structural requirements with complex shapes and geometries are requested and regarding to this request, an exploration and validation of the structural system within this complex geometry are needed. In the reviewed literature, it has been noticed that the current situation of computational modelling techniques have empowered a new approach to these structures in architectural design field, for the last two decades, and have opened the way of new remarkable methods in analysing and making complex geometries.

The conclusion of the thesis is reached by the questions grounded on the knowledge that have been obtained from the literature review, the knowledge that have been developed by experimenting the material properties and the knowledge expanded through the design process of the model generated. Consequently, research questions are being concluded as follows in below:

What are the main methods for understanding and calculating the structural principles of shells?

To find the main methods in understanding and calculating the structural principles of shells systems, *Basic* and *Interactive* form finding considerations have been surveyed. Firstly basic form finding methods have been studied in three titles;

1. *'Hooke's Hanging Chain'*,

2. *'Graphic Statics'* and

3. *'Physical Modelling'*.

It is noticed that, the theoretical methodology of computational design approach of shell structures is based on the basic methods above. Regarding to being mentioned earlier, interactive form finding approaches have also been reviewed. In literature review, it has been seen that interactive form finding methods for shell structures were;

1. *'Force Density Method'*,

2. *'Thrust Network Analysis'*,

3. *'Dynamic Relaxation Method'* and

4. *'Particle-Spring Systems'*.

Being material oriented, suitable form finding method is chosen as *'Particle-Spring Systems'* and a particle- based modelling has been assigned to be applied. The objectives of particle based modelling are determined based on Hooke's hanging chain law, material properties and behaviour on shell structures. Therefore, these issues have also been studied in the thesis.

What are the material properties of ACC in designing a shell structures with load bearing principles?

AAC is a high-efficient material for application and construction in terms of being compressive stress resistant, lightweight, porous and, high efficiency heat impermeable. It is an alternative material for the masonry building of architectural constructions, in terms of insulation and structural characteristics and so, it is a promising material of the future. However, according to the literature research, there are limited studies have been made on ACC in the field of designing or constructing masonry shell structures. For this reason, this innovative material has been examined in this study for a new area of usage.

The material properties of ACC in designing shell structures with load bearing principles has been researched in relation with its chemical, physical, mechanical and functional properties. Thus, the requirements, depending on the material properties, needed for the particle based form finding process have been achieved. Furthermore, to control the geometry within the context of gravity loading, Hooke's hanging chain law has been examined on AAC material. For the tests of funicular shaped arches, results have been found and minimum- maximum, span length-arch length ranges have been calculated. The range calculated by the relation between span lengths and curve lengths of the tested arches has been used for the determination process of open edges of the model geometry. Thus, it has been provided that the three-dimensional system gives more controlled results within the context of loads.

The designing of complex masonry structures is hard to be challenged and needs a deep structural knowledge. Under these circumstances, four structural performance examinations have been done on AAC material by designing very simple shells to determine the structural behaviour of the material in load bearing geometries. Examinations are as follows;

- 1. Examining Symmetric Vault Models*
- 2. Examining Material Thickness of AAC*
- 3. Examining Unit Weight of AAC*
- 4. Examining Maximum Span of Arches*

As mentioned before, the structural behaviour of AAC material is not a known process for complex structures or asymmetrical shells and this creates a formal difficulty for this thesis study. With the applied examinations, the structural behaviour of the AAC material on shell geometries has been understood. Thus, the parameters of the material for the design of asymmetric shaped shell model have been developed.

How a computational generic model could be applied as a support tool for designers and architects in order to design a material and structural based model?

Taking everything found by the tests and examinations done into the account to understand the structural performance of AAC material, a computational generic model has been developed. Three steps have been applied to the design process of generating the model. They were as follows;

- 1. Topology and open edge determination,*
- 2. Shape optimization and*
- 3. Structural analysis.*

Topology and open edge determination has been completed by the results of Hanging Chain tests and Structural Performance examinations. An optimum shape has been formed by the particle based form finding engine. Designing the model with the aid of these steps, it is known that the found generic model is a controlled structure within the context of gravity loading. But also to control and analyse these steps, a structural analysis has also been done. With this analysis, the maximum deformation over than 1/500 of the span has been controlled and how principle stress and stress directions occur on the structure have been observed. If the result does not satisfy the designer or ‘serviceability criteria’ , the regeneration of the model is optional for the development of the shape. In such circumstances, application of this computational generic model tool is proper and gives the opportunity of turning back to the previous steps for the redevelopment of the shape. In Figure 47, the design process steps are visualized.

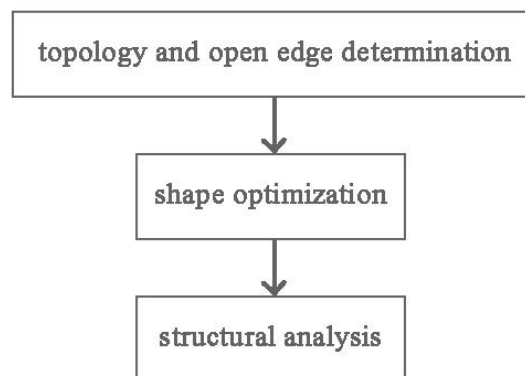


Figure 43. Computational Design Process of the Developed Generic Model

In this study, computational generic modelling is applied as a high level digital sketching tool for a shell structure and a model proposal is developed. In accordance with the research goals and questions asked at the beginning of the study, form finding methods, AAC materials properties, structural performance of AAC material have been studied and examined. Most importantly, Hooke's hanging chain law has been determined as the design consideration and load bearing criterion of this study. The model has been built based on hanging chain average of AAC material.

Regarding to the application of hanging chain average to the design process, the model and its load bearing arches have become safer in terms of structural stability. If hanging chain law had not been used, the structure would be less safe. So, by determining the open edges of the model, one of the structural decisions is given in the first step of the design. In addition, the first step of the design process, including topology and open edge determination, is not only done based on hanging chain law, but also on structural performance examination of AAC material and designers' intuitive. In this way, the designer's authority over the model has been increased.

The objective of the study was determined as finding a statically equilibrium shape for the shell model. This objective has been fulfilled at the second step of the design process. The simulation has been iterated until the shape got relaxed and the changes in the geometry got smaller. By the help of relaxation, the equilibrium shape has been found. With relaxation, loads on the structure have made the shape move in a balanced manner. In this case, it has been seen that a structure, with the same topology and open edge properties, swell and rise when the applied loads increase. The reason is the same about loads, to distribute the increased compression balanced over the shape.

As the third step, structural analyses have been done to the relaxed equilibrium shape. Thus, the shape has been analysed in respect of displacement and stress formation with FE method. This method is highly suitable and mostly preferred for shell structure analyses. With this analysis, the structural stability of the generated model has been double checked. For the displacement analysis, the result has been considered to be less than 1/500 according to the focused serviceability criteria. Regarding this criterion, the generated shell model has performed an acceptable result in terms of displacement. If the displacement result becomes more than the serviceability criteria, a regeneration of the model is required. Because according to

this situation the generated model is not durable according to the load bearing principles and, too much deformation might be occur.

As reported by the stress visualization results in the structural analysis, it has been noticed that the dominance effect of principle stress resulted on higher regions on the developed generic model. However, the colour of this dominant region is light green, and it refers to the effect of principle stress on the places where the effect is not so dominant. Furthermore, for this study, stress patterns have been only used for observing the direction of stresses and how they occur. Stress pattern and directions show the way for finding the laying pattern of the blocks on the shell and also to find the shape of all block individuals. How the shape of the blocks and laying pattern would be in construction is not taken in the scope of this study. If they are focused, they would be obtained by using the stress pattern taken from the structural analysis.

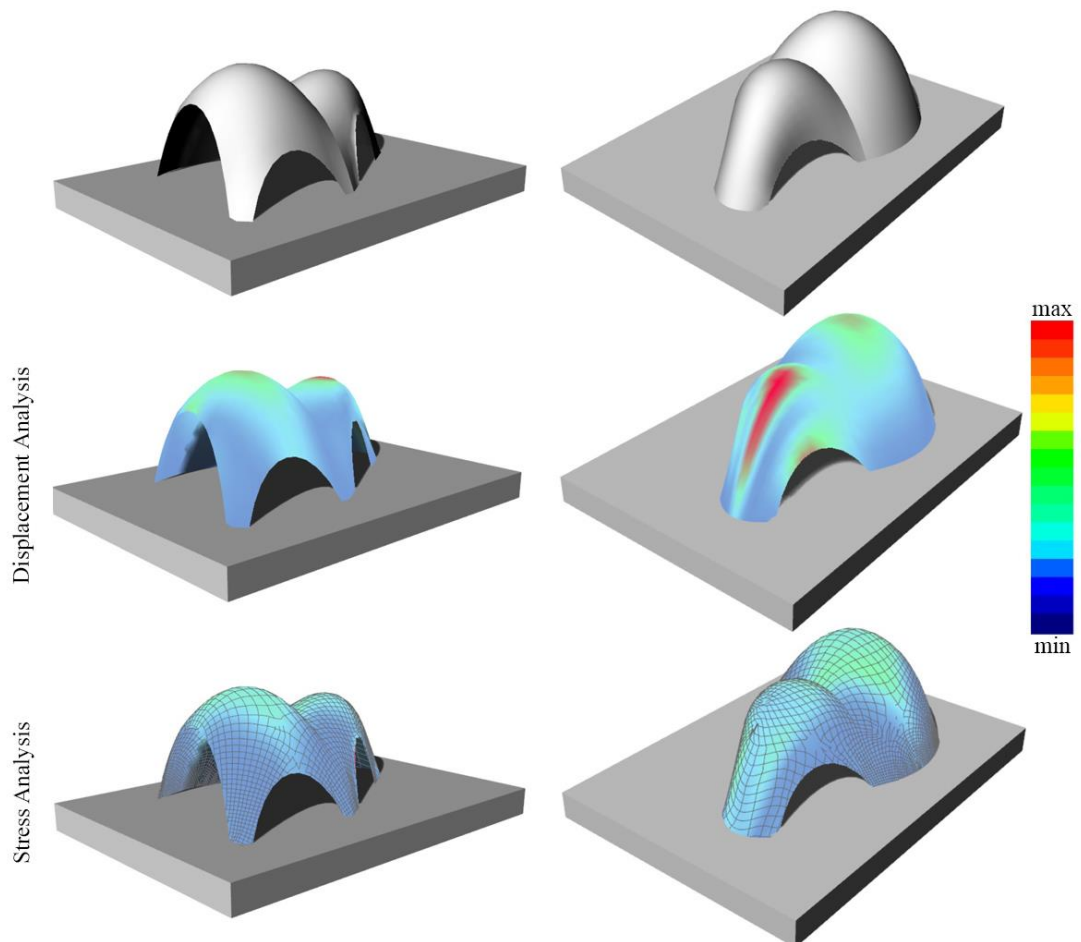


Figure 44. Coloured Visualizations of Displacement and Stress Analysis for Developed Generic Model

5.2. Research Contributions

Idea development is a key phase for designing. There are constraints and parameters for every design. If we can determine the constraints, criteria and parameters beforehand, we can easily control the process. Digital sketching depends on this very basic idea. It enables designers and architects to see how their design results depending on these determinations and to improve their intuitive attitude.

The application of digital sketch is important for the step of preliminary sketches of the design in the digital environment. This form and idea development method have been performed on this thesis, in accordance with designing shell structures with AAC blocks. For this study, by using digital sketching, the design not only has been considered in the context of form and geometry, but also been controlled in the context of gravity loading by the defined criteria. The design has been processed in relation with the tests and examinations made in chapter four and in this way, it is accompanied with choosing appropriate material and structural simulations. Thus, it is proved that digital sketching of this research is not an elementary sketch.

The proposed model of this study can be accepted as an advanced level digital sketch for designers and architects. The design is done based on particle spring system methods and analysed by finite element methods. Besides, the model is generated by defining criteria, reflecting the decisions on form, geometry and material in the frame of these definitions to the final design and, visualizing the results. The committed research is sufficient for designers at a certain level because complex calculation methods have been not used.

A mature model is developed by using digital tools as Kangaroo and the structural analysis is done by Millipede. Using these digital tools in the design process opens a way for designers to see the relationship between form and structures in an intuitive approach. Indeed, by this study, a digital design tool set, which has been improved for designers' intuitive attitude, is included. Thus, architects are able to use a simple level of structural analysis and structural design programs instead of using complex design methods in designing shell systems.

Rhinovault is another structural form finding software tool, which is used as a plug-in in Rhinoceros software, for compression-only structures which is based on TNA approach. And it is advanced by Rippmann, Lachauer and Block from ETH Zurich.

Rhinovault is described as a graphic statics-based form finding tool and because of being intuitive proceeded, it has a closed system. While progressing the model, it gives a chance to obtain three dimensional shape from preparing force and form diagrams, otherwise it is not possible to make changes on the geometry.

If we are to compare Rhinovault with the proposed digital tool set in this study; it can be said that the digital tool set is more favoured. The advantage of the digital tool set is that it is more convenient for controlling the geometry, by defining the topology and funicular open edges by hanging chain law based on the determined range. Thus, the authority on the geometry increases and it is more than in Rhinovault.

The working principle of the proposed tool set for digital sketching is based on Hooke's hanging chain law. With the definition of hanging chains as constraints before shape optimization, it is sure that the founded geometry after the shape optimization process will be in an appropriate shape in the structural context. Therefore, having adverse results from structural analysis is decreased.

5.3. Recommendations and Further Research

The thesis focuses on the appropriateness of AAC for a material oriented design of vault and shell systems. With the tests and examinations conducted on AAC, the material's structural behaviour and performance on shell systems have been figured out and analysed. And with the digital sketching tool set, the model has been generated.

With the use of combined digital tool set and defined design process while designing the model, a useful and beneficial development is presented to architects and designers for digital sketch qualified form finding practices. Thus designers' perspective and attitude would be developed.

Furthermore, beyond being a dividing wall based on its thermal conductivity, with this research, a new area of usage is introduced for AAC material. The material will be enhanced in this area. The shape of the blocks and laying pattern of these masonry elements will be studied. Thus, besides from existing conventions, AAC blocks can be able to be recasted and can be shaped in new forms freely.

Regarding to the improvements in digital sketching and computational methods in the design of shell structures, there will be a need for new materials and service for

the market of building systems. Thus, further improvements in materiality of the structures have to be studied.

In this study, only gravity loading and dead load of the shell structure have been studied. For further researches, lateral loads such as earthquake loads will be also integrated to the scope of the studies.

Finally, regarding the structure, this study has a potential for further studies aimed at making various cross optimizations based on;

- shape and plan geometry,
- height and length of spans;
- surface pattern,
- material thickness
- structural and economic objectives.

REFERENCES

- Addis, B. (2014). Physical Modelling and Form Finding. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge. 33-44p.
- Akgül, A. F. (2013). *Gazbeton ve Üretim Süreci*. Maden Mühendisliği Anabilim Dalı, Fen Bilimleri Enstitüsü, İstanbul Teknik Üniversitesi.
- Andolsun S. (2006). *A Study on Material Properties of Autoclaved Aerated Concrete and Its Complementary Wall Elements: Their Compatibility in Contemporary and Historical Wall Sections*. Graduate School of Natural and Applied Sciences, Middle East Technical University. 105p.
- Adriaenssens, S., Block, P., Veenendaal, D., Williams C. (2014). *Shell Structure for architecture: Form Finding and Optimization*. Routledge.
- Adriaenssens, S., Lachouer, L., Rippmann, M. (2014). Dynamic Relaxation: Design of a Strained Timber Gridshell. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge 89-102p.
- Asmaljee, Z. (2013). *Form-Finding of Thin Shell Structures*. Master Thesis. University of the Witwatersrand. Johannesburg.
- Bertin, T. B. (2013). *Evaluating the Use of Particle-Springs in the Conceptual Design of Grid Shell Structure*. Master Thesis. Department of Civil and Environmental Engineering, MIT.
- Bhooshan, S., Veenendaal, D., Block, P. (2014). Particle Spring System: Design of a Cantilevering Concrete Shell. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge. 103-114p.
- Block, P., DeJong, M., Ochsendorf J. (2006). *As Hangs the Flexible Line: Equilibrium of Masonry Arches*. Nexus Network Journal, 8(2).
- Block, P. (June 2009). *Thrust Network Analysis; Exploring Three-dimensional Equilibrium*. PHD Thesis. Building Technology, Department of Architecture, Massachusetts Institute of Technology.
- Block, P., Lachauer, L., Rippmann, M. (2010). *Validating Thrust Network Analysis using 3D-printed, structural models*. Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium, Shanghai.

- Block, P., Lachauer, L. (2014). *Three-dimensional funicular analysis of masonry vaults*. Mechanics Research Communications, 56, 53-60p.
- Block, P., Lachauer, L. (2014). *Three-dimensional (3D) equilibrium analysis of gothic masonry vaults*. International Journal of Architectural Heritage, 8(3), 312-335p.
- Block, P., Lachauer, L., Rippmann, M. (2014). Thrust Network Analysis: Design of a Cut-stone Masonry Vault. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge. 71-88.
- Block, P., Knippers, J., Mitra, N.J., Wang, W. (2015). *Advances in Architectural Geometry 2014*. Springer, 61-77p.
- Boothby, T.E. (2001). *Analysis of masonry arches and vaults*. Progress in Structural Engineering and Materials. 3, 246-256p.
- Bose, S., Rai, D.C. (2014). *Behavior of AAC Infilled RC Frame Under Lateral Loading*. Proceedings of the 10th National Conference in Earthquake Engineering, Earthquake Engineering Research Institute, Anchorage, Alaska.
- Burkhardt, B., Bächer M. (1978). *Multihalle Mannheim*. Institute for Lightweight Structures. University of Stuttgart.
- Chilton, J. (2000). *The Engineer's Contribution to Contemporary Architecture: Heinz Isler*. Thomas Telford Press. London.
- Çiçek, Y.E. (2002). *Pişmiş Toprak Tuğla, Bimsbeton, Gazbeton VE Perlit Yapı Malzemelerinin Fiziksel, Kimyasal, Mekanik Özelliklerinin Karşılaştırılması Olarak İncelenmesi*. Fen Bilimleri Enstitüsü, İstanbul Teknik Üniversitesi.
- Cooney, R.C., King, A.B. (1988). *Serviceability Criteria for Buildings*. Building Research Association of New Zealand. Branz Study Report SR14, Judgeford.
- Düzgün, H., & Polatoğlu, Ç. (2016). *Güncel Mimarlık Ortamında Kabuk-Bağlam İlişkisinin Sorgulanması*. Megaron, 11(1).
- Ferretti, D., Michelini, E., Rosati, G. (2015). *Cracking in autoclaved aerated concrete: Experimental investigation and XFEM modeling*. Cement and Concrete Research, 67, 156-167p.
- Fresl, K., Gidak, P., Vrancic R. (2013). *Generalized minimal nets in form finding of prestressed cable nets*. Gradevinar, 8.

- Galassi, S., Paradiso, M. (2014). *BrickWORK software-aided analysis of masonry structures*. IERI Procedia, 7, 62-70p.
- Garcia, J. R., (2011). *Numerical study of dynamic relaxation methods and contribution to the modelling of in atable lifejackets*. PHD Thesis, Mechanical engineering, University of Bretagne Sud.
- Gidak, P., Fresl, K. (January, 2012). *Programming the force density method*. In IASS-APCS 2012, From spatial structures to space structures.
- Hall, M. R., Lindsay R., Krayenhoff, M. (2012). *Modern Earth Buildings: Materials, Engineering, Constructions and Applications*, Elsevier.
- Heyman, J. (1995). *The Stone Skeleton; Structural Engineering of Masonry Structures*. Cambridge University Press.
- Huerta S. (2006). *Structural Design in the Work of Gaudi*. Architectural Science Review 49(4), 324-339p.
- Hüttner, M., Maca, J., Fajman, P. (2014). *Analysis of cable: membrane structures using the Dynamic Relaxation Method*. 9th International Conference on Structural Dynamics, EUROLYN.
- Ioannou, I., Hamilton, A., Hall, C. (2008). *Capillary absorption of water and n-decane by autoclaved aerated concrete*. Cement and Concrete Research, 38(6), 766-771p.
- Kaveh, A. (2013). *Computational Structural Analysis and Finite Element Methods*. Springer Science and Business Media, 432p.
- Kilian, A., Ochsendorf, J. (2005). *Particle-Spring Systems for Structural Form Finding*. Journal of The International Association for Shell and Spatial Structures. IASS. 46(147).
- Korkmaz K.A., Çarhođlu A.I., Orhon A.V., Nuhođlu A. (2014). *Farklı Yapısal Malzeme Özelliklerinin Yıđma Yapı Davranışına Etkisi*. Nevşehir Bilim ve Teknoloji Dergisi, 3(1), 69-78p.
- Kozak, Ş. (2010). *Gazbeton Üretiminde Uçucu Külün Hammadde Olarak Kullanılmasının Araştırılması*. Fen Bilimleri Enstitüsü, Afyon Kocatepe Üniversitesi.
- Kömürlü, R., Önel, H. (2007). *Usage of Aerated Concrete Construction Elements In Houses*. Megaron, 2(3), 145-158p.

- Lee, K.S., Han, S.E. (2011). *Advanced Shape Finding Algorithm of Force Density Method Based on FEM*. *Advanced Steel Construction*,7(4), 313-329p.
- Linkwitz, K. (2014). Force Density Method: Density of a Timber Shell. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge 59-70.
- Michalatos, P., Kaijima, S. (2014). Eigenshells: Structural Patterns and Modal Forms. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge. 195-210p.
- Michalatos, P., Kaijima, S. (2014). *Millipede Handbook*. Retrieved 20 December 2016,from http://www.sawapan.eu/sections/section88_Millipede/files/MillipedeMarch2014.pdf.
- Mostafa, N.Y. (2005). *Influence of air-cooled slag on physicochemical properties of autoclaved aerated concrete*. *Cement and Concrete Research*, 35, 1349-1357p.
- Lewis, W. J. (2003). *Tension Structures: Form and Behaviour*, Thomas Telford, 201p.
- Lourenço, P. B. (2008). *Structural masonry analysis: Recent developments and prospects*. In 14th international brick and block masonry conference. University of Newcastle, Australia.
- Narayanan, N., Ramamurthy K. (2000). *Structure and properties of aerated concrete: a review*. *Cement & Concrete Composites*, 22, 321-329p.
- Ochsendorf, J., & Block, P. (2014). Exploring shell forms. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge 7-14.
- Öztaş, V. (2009). *Yığma Yapıların Güçlendirilmesi ve Bir Yığma Örneği Güçlendirme Analizi*. Yüksek Lisans Tezi. Mimarlık Anabilim Dalı, Fenbilimleri Enstitüsü, İstanbul Teknik Üniversitesi.
- Pehlivanlı Z. (2010). *Gazbeton Malzemesinin Farklı Sıva Malzemeleriyle Birlikte Isıl Özelliklerinin Nem ve Sıcaklık Değişiminin İncelenmesi*. Doktora Tezi. Fen Bilimleri Enstitüsü, Kırklareli Üniversitesi. 130p.
- Pendergrast, R. A. (2010). *Thin Shell Structures Design Tool*. Master Thesis. Rensselaer Polytechnic Institute Troy, New York.
- Pedron, C. (2006). *An innovative tool for teaching structural analysis and design*. vdf Hochschulverlag, ETH Zürich.

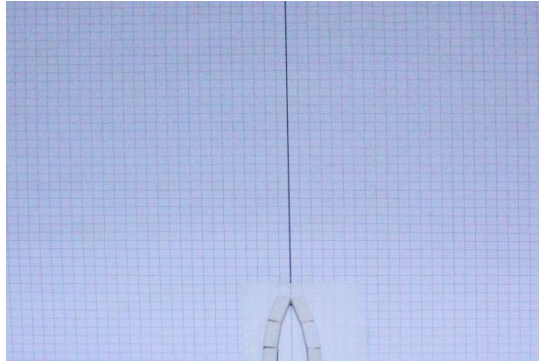
- Piker, D. *Using Kangaroo (Grasshopper Version)*. Manual. Retrieved 26.12.2016, from
https://docs.google.com/document/d/1XtW7r7tfC9duICi7XyI9wmPkGQUPIm_8sj7bqMvTXs/preview.
- Piker, D. (2013). *Kangaroo: Form Finding with Computational Physics*. Architectural Design, 83(2).
- Rippmann, M., Block, P. (2012). *Rethinking structural masonry: unreinforced, stone-cut shells*. Construction Materials, Proceedings of the Institution of Civil Engineers.
- Rippmann M., Lachauer L. and Block P. (2012). *Interactive Vault Design*. International Journal of Space Structures, 27(4).
- Rippmann, M., Block, P. (2013). *Funicular Shell Design Exploration*, ACADIA, Waterloo, Canada.
- Rippmann, M., Block, P. (2013). *Funicular Funnel Shells*. Design Modelling Symposium, Berlin.
- Saran, A. Z. (2014). *Yapılarda Gazbeton Paneller İle Alternatif Çözümler*. 7. Ulusal Çatı ve Cephe Sempozyumu. Yıldız Teknik Üniversitesi. İstanbul.
- Sezer, H. (2010). *Öğütülmüş Diyatomitin Gazbeton Üretiminde Kullanılmasının Araştırılması*. Fen Bilimleri Enstitüsü, Afyon Kocatepe Üniversitesi.
- Schenk, M. (2009). *On the shape of Cables, Arches, Vaults and Thin Shells*. University of Cambridge.
- Southern, R. (2011). *The force density method: A brief introduction*. The National Centre for Computer Animation, Bournemouth University, United Kingdom.
- Tomlow, J. (1989). *The Model: Antoni Gaudi's Hanging Model and Its Reconstruction-New Light on the Design of the Church of Colonia Guell*. Institute For Lightweight Structures. University of Stuttgart.
- Tzamtzis, A. D., & Asteris, P. G. (2003, June). *Finite Element Analysis of Masonry Structures Part I-Review of Previous Work*. In 9th North American masonry conference, 101-11p.
- Ünerdi, A. (2006). *Yüksek Sıcaklık Altında Gazbeton Kırıklı Betonların Dayanımlarının İncelenmesi*. İnşaat Mühendisliği Anabilim Dalı, Fen Bilimleri Enstitüsü, Osmangazi Üniversitesi.

- Van Mele, T., Block, P. (2011). *A novel form finding method for fabric formwork for concrete shells*. Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium, Shanghai. 52, 217-224p.
- Van Mele, T., Lachauer, L., Rippmann, M., & Block, P. (2012). *Geometry-based understanding of structures*. Journal of the International Association of Shell and Spatial Structures, 53(4), 285-295p.
- Veenendaal, D., & Block, P. (2012). *35 Computational form-finding of fabric formworks: an overview and discussion*. Institute of Technology in Architecture, Department of Architecture, ETH Zurich.
- Veenendaal, D., & Block, P. (2012). *An overview and comparison of structural form finding methods for general networks*. International Journal of Solids and Structures, 49(26), 3741-3753p.
- Veenendaal, D., & Block, P. (2014). Comparison of Form Finding Methods. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge. 115-130p.
- Vent I.A.E. (2011). *Prototype of a diagnostic decision support tool for structural damage in masonry*. PHD Thesis. Faculty of Architecture, Delft University of Technology. 154p.
- West, M. (2006). *Flexible fabric moulds for precast trusses*. BFT International, 72(10), 46-52p.
- Weller, M. W. (2011). *Form Finding, Force and Function: Mass-Spring Simulation for a Thin Shell Concrete Trolley Barn*. Master Thesis. Department of Architecture, University of Washington.
- Williams, C. (2014). What is A Shell?. *Shell Structures for Architecture: Form Finding and Optimization*. Routledge 21-32.
- Wittmann, F. (1992). *Advances in Autoclaved Aerated Concrete*. Proceeding of the 3rd Rilem International Symposium of Autoclaved Aerated Concrete, Switzerland.
- Yazıcı, A. P. D. G., Yazıcı, A. P. D. Y. E. (2012). *The use of finite element analysis applications in architectural education*. Int. J. New Trends Educ. Implications, 3(4), 148-155p.

APPENDIX 1 – Hanging Chain Test Results

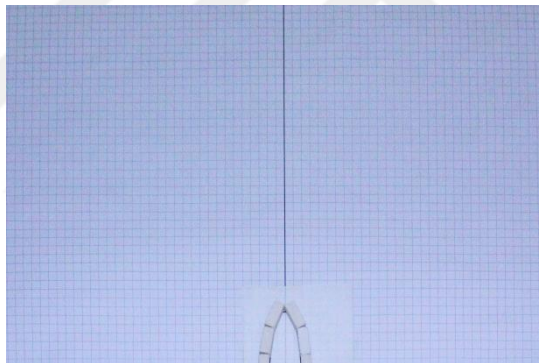
A. Funicular Arches with 6 Blocks

3cm Span Length



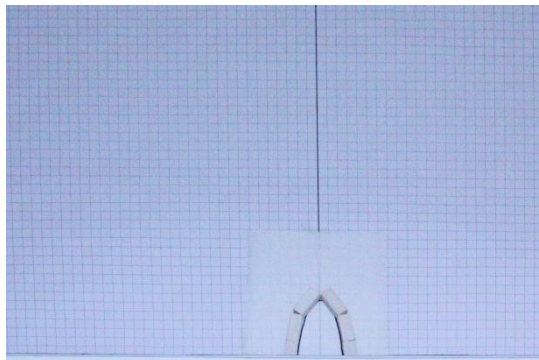
The photo refers to 3cm. span length arch. The arch is feasible however, it is seen that the shape of the arch is not funicular. It is more straight shape.

4cm Span Length



The photo refers to 4cm. span length arch. The arch is feasible and the shape is proper to be funicular. So, 4cm. is accepted as minimum span length.

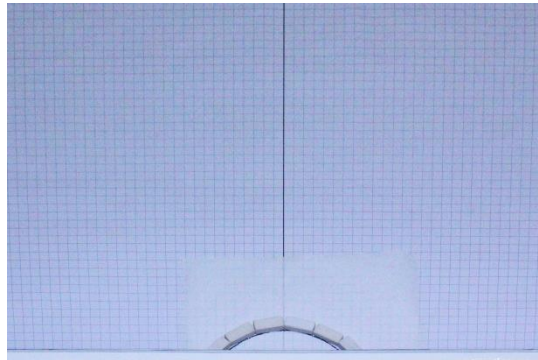
5cm Span Length



The photo refers to 5cm. span length arch. The arch is feasible and the shape is funicular.

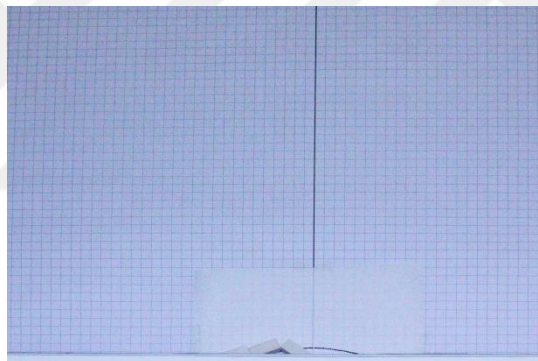
In the range between 5cm. to 15cm. span length the arches are modelled. It is seen that all modelled arches are stable and gives funicular shapes.

15cm Span Length



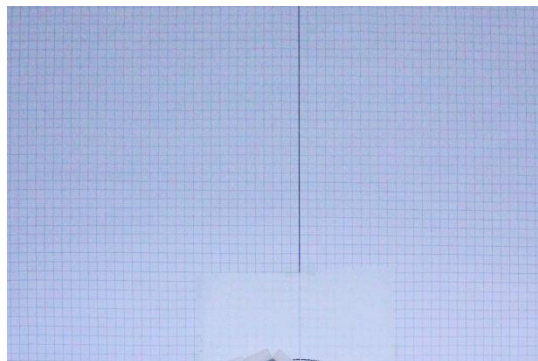
The photo refers to 15cm. span length arch. The arch is feasible and the shape is proper to be funicular. For the span length longer than 15cm, the arches are collapsed. So, 15cm. is accepted as maximum span length.

16cm Span Length



The photo refers to 16cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

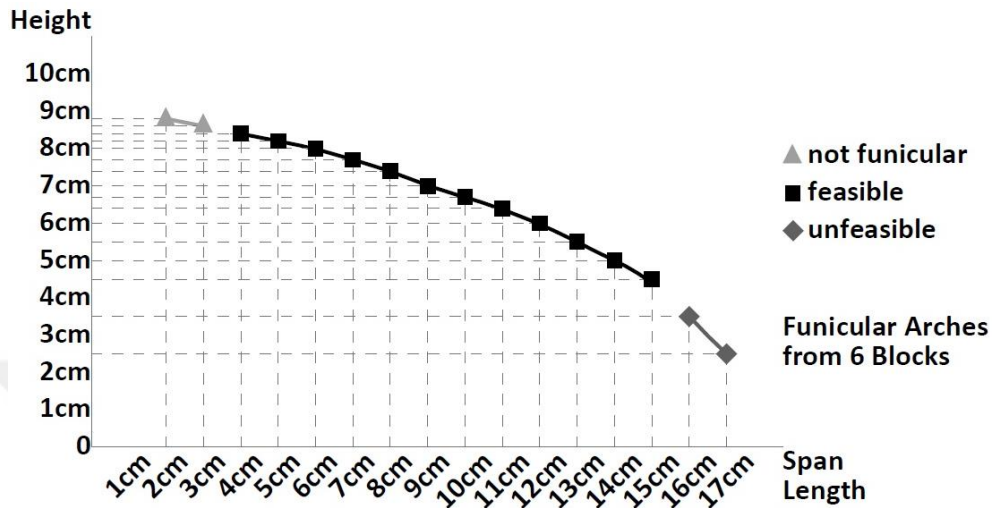
17cm Span Length



The photo refers to 17cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

Result

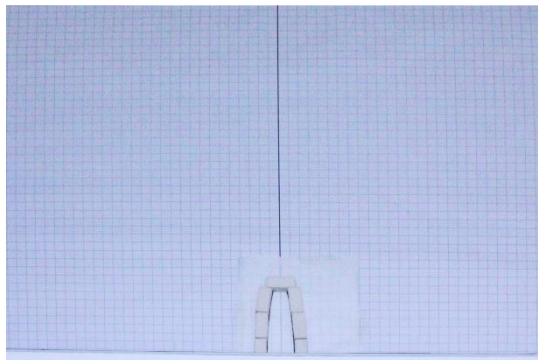
For 6 blocks and 18cm. arch length funicular arches, minimum span length is found 4cm. and maximum span length is found 15cm. The heights of these arches are changed between these ranges proportionally.



Span – Height Relation of Funicular Arches from 6 Block

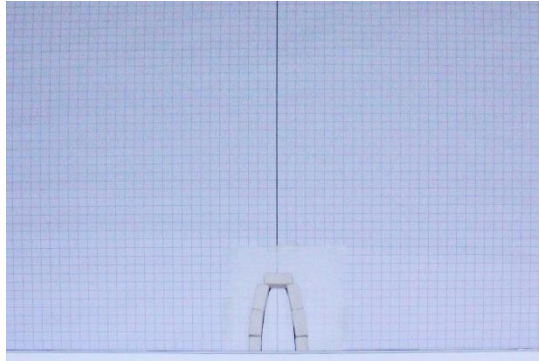
B. Funicular Arches with 7 Blocks

3cm Span Length



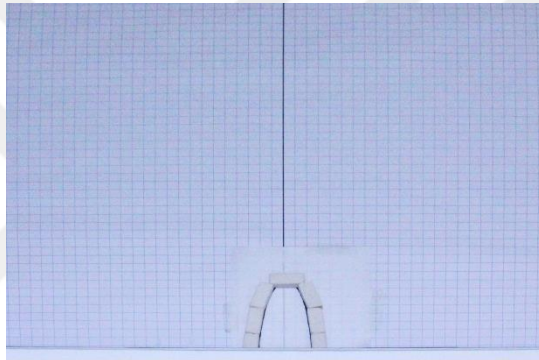
The photo refers to 3cm. span length arch. The arch is feasible however, it is seen that the shape of the arch is not funicular. It is more straight shape.

4cm Span Length



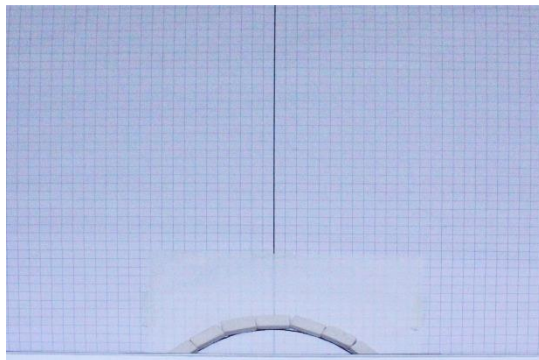
The photo refers to 4cm. span length arch. The arch is feasible and the shape is proper to be funicular. So, 4cm. is accepted as minimum span length.

5cm Span Length



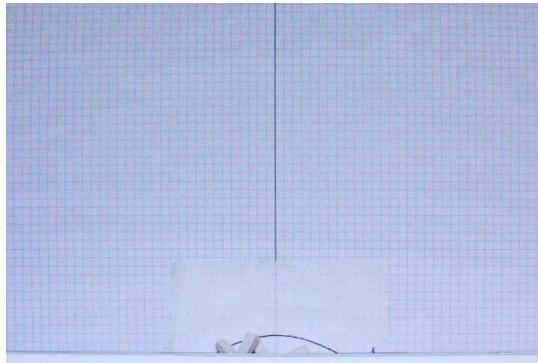
The photo refers to 5cm. span length arch. The arch is feasible and the shape is funicular.

18cm Span Length



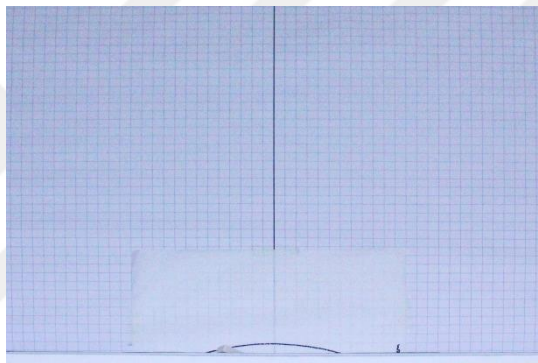
The photo refers to 18cm. span length arch. The arch is feasible and the shape is proper to be funicular. For the span length longer than 18cm, the arches are collapsed. So, 18cm. is accepted as maximum span length.

19cm Span Length



The photo refers to 19cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

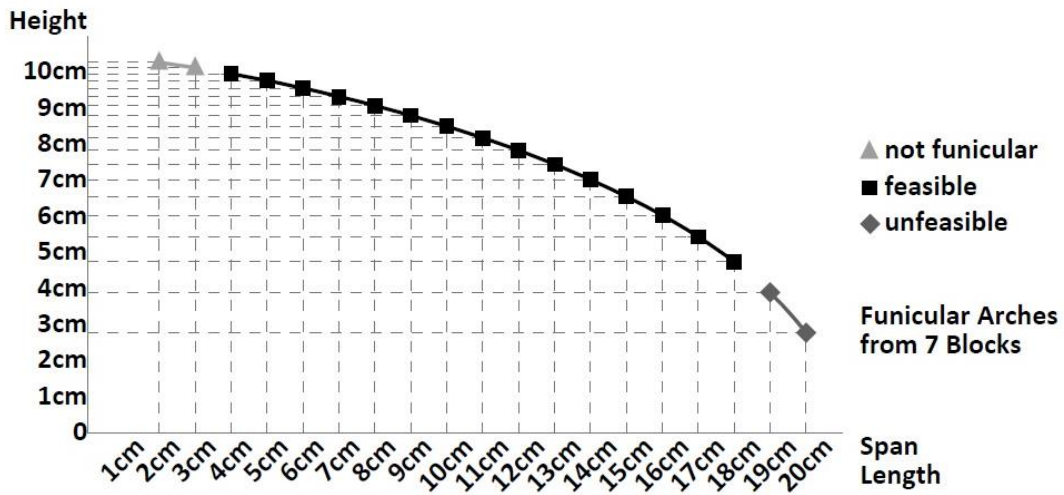
20cm Span Length



The photo refers to 20cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

Result

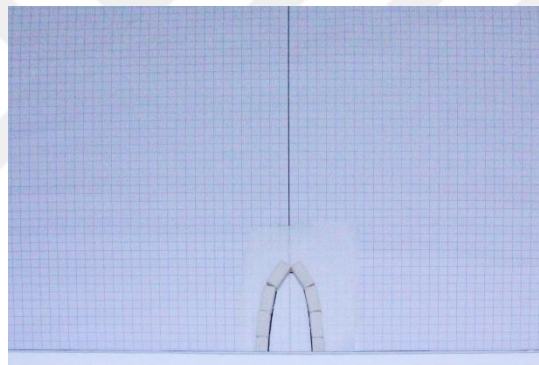
For 7 blocks and 21cm. arch length funicular arches, minimum span length is found 4cm. and maximum span length is found 18cm. The heights of these arches are changed between these ranges proportionally.



Span – Height Relation of Funicular Arches from 7 Block

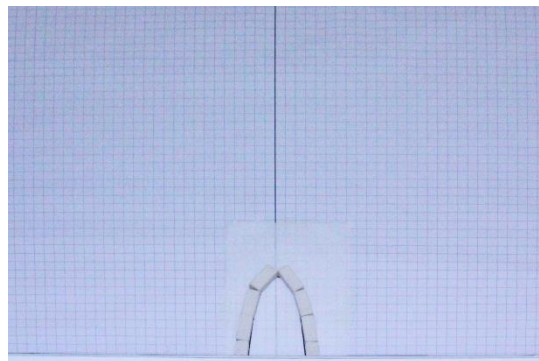
C. Funicular Arches with 8 Blocks

4cm Span Length



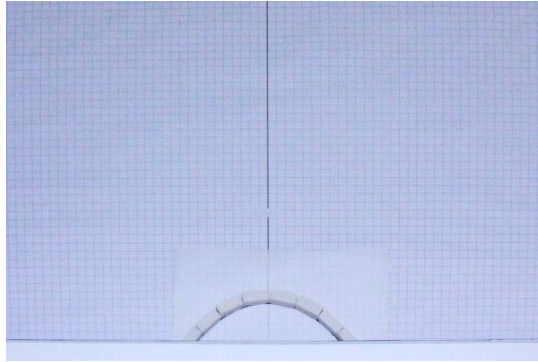
The photo refers to 4cm. span length arch. The arch is feasible however, it is seen that the shape of the arch is not funicular. It is more straight shape.

5cm Span Length



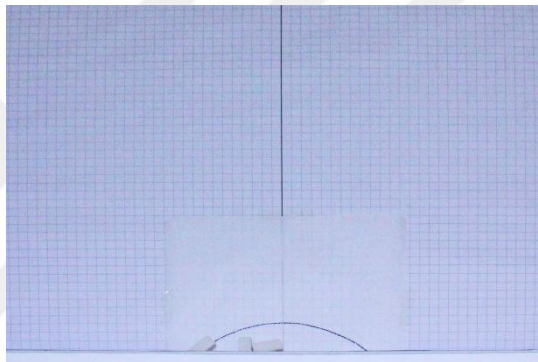
The photo refers to 5cm. span length arch. The arch is feasible and the shape is proper to be funicular. So, 5cm. is accepted as minimum span length.

19cm Span Length



The photo refers to 19cm. span length arch. The arch is feasible and the shape is proper to be funicular. For the span length longer than 19cm, the arches are collapsed. So, 19cm. is accepted as maximum span length.

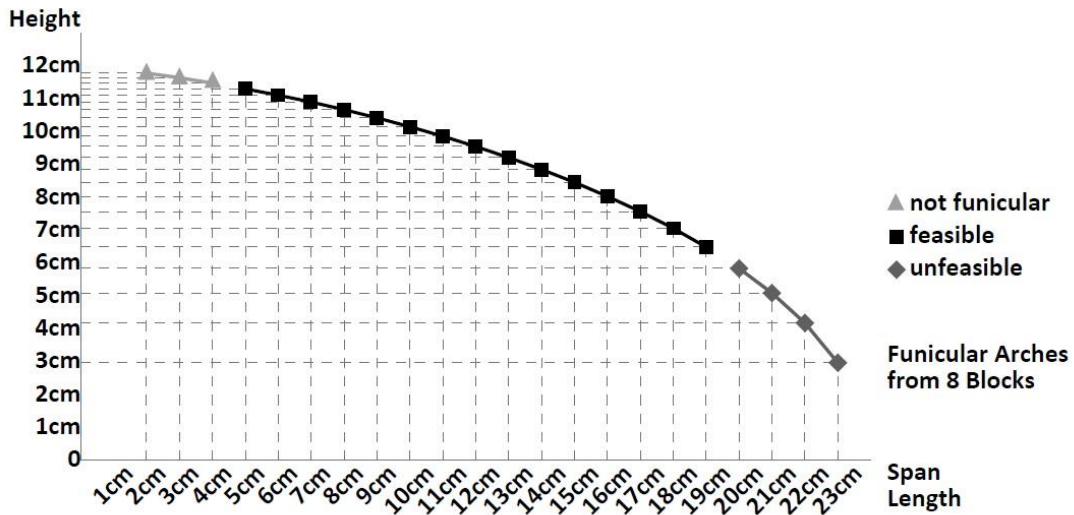
20cm Span Length



The photo refers to 20cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

Result

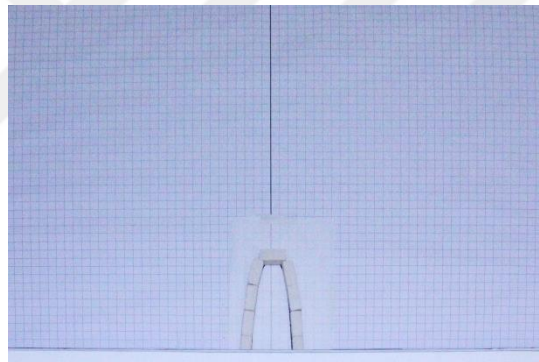
For 8 blocks and 24cm. arch length funicular arches, minimum span length is found 5cm. and maximum span length is found 19cm. The heights of these arches are changed between these ranges proportionally.



Span – Height Relation of Funicular Arches from 8 Block

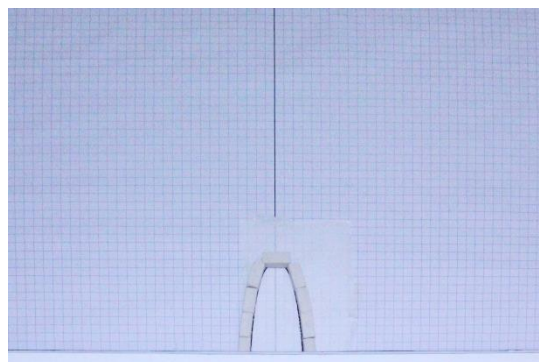
D. Funicular Arches with 9 Blocks

4cm Span Length



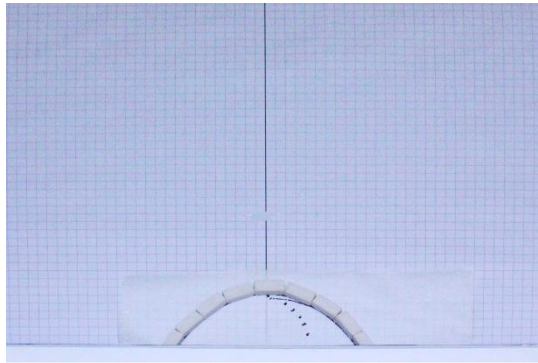
The photo refers to 4cm. span length arch. The arch is feasible however, it is seen that the shape of the arch is not funicular. It is more straight shape.

5cm Span Length



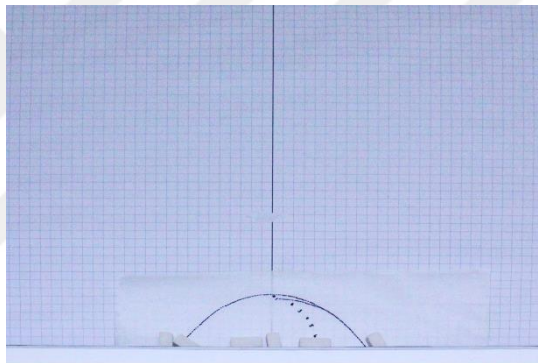
The photo refers to 5cm. span length arch. The arch is feasible and the shape is proper to be funicular. So, 5cm. is accepted as minimum span length.

20cm Span Length



The photo refers to 20cm. span length arch. The arch is feasible and the shape is proper to be funicular. For the span length longer than 20cm, the arches are collapsed. So, 20cm. is accepted as maximum span length.

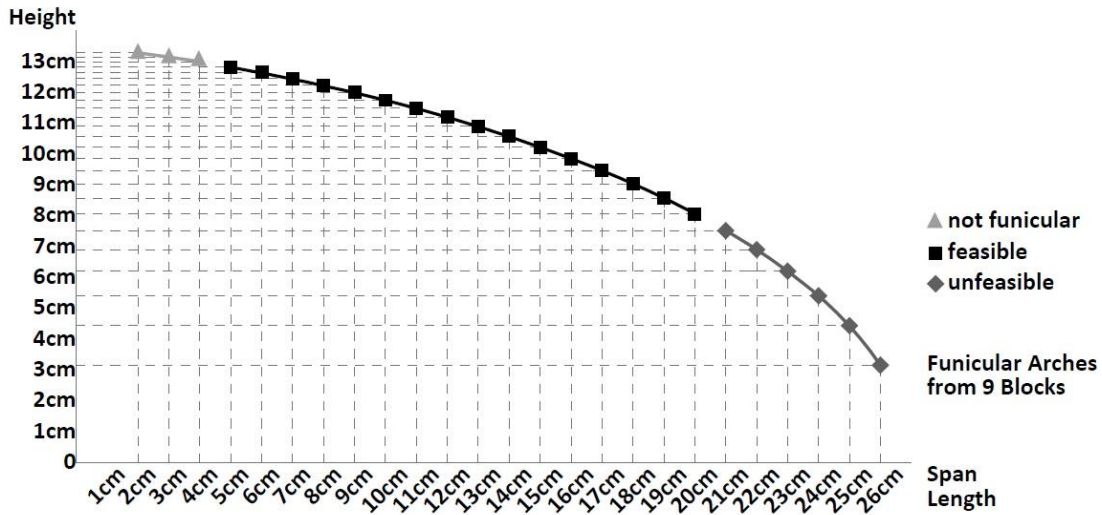
21cm Span Length



The photo refers to 21cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

Result

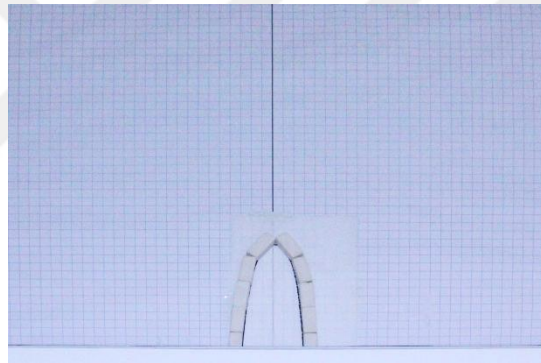
For 9 blocks and 27cm. arch length funicular arches, minimum span length is found 5cm. and maximum span length is found 20cm. The heights of these arches are changed between these ranges proportional.



Span – Height Relation of Funicular Arches from 9 Block

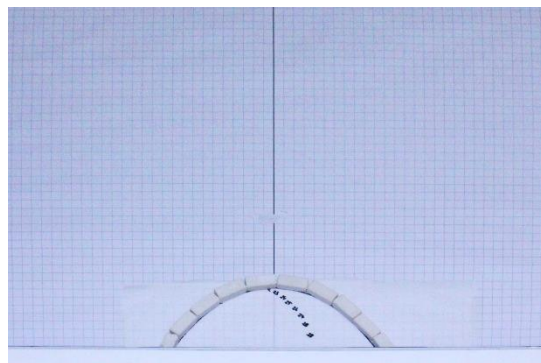
E. Funicular Arches with 10 Blocks

5cm Span Length



The photo refers to 5cm. span length arch. The arch is feasible and the shape is proper to be funicular. So, 5cm. is accepted as minimum span length.

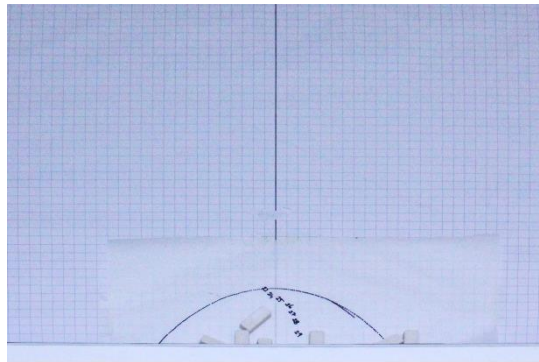
22cm Span Length



The photo refers to 22cm. span length arch. The arch is feasible and the shape is proper to be funicular. For the span length longer than 22cm, the arches are

collapsed. So, 22cm. is accepted as maximum span length.

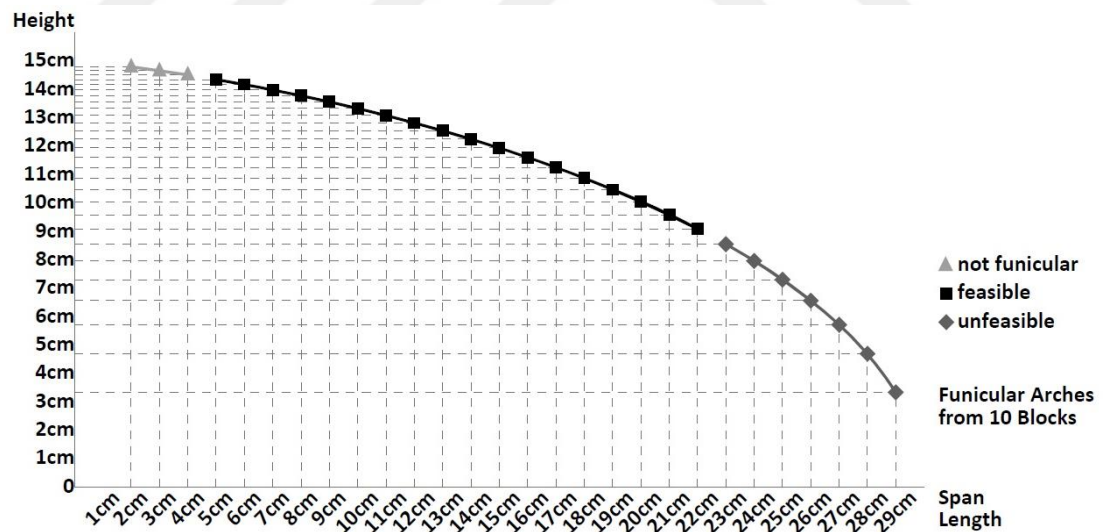
23cm Span Length



The photo refers to 23cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

Result

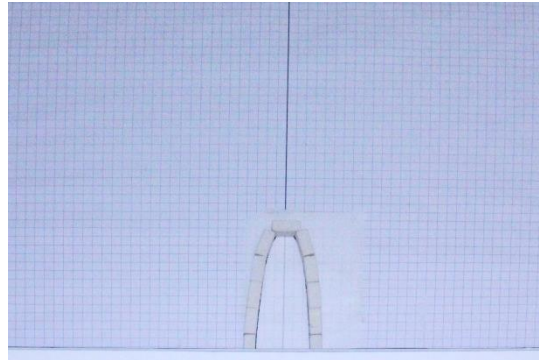
For 10 blocks and 30cm. arch length funicular arches, minimum span length is found 5cm. and maximum span length is found 22cm. The heights of these arches are changed between these ranges proportionally.



Span – Height Relation of Funicular Arches from 10 Block

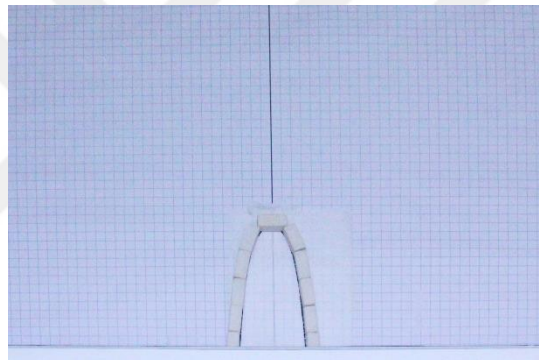
F. Funicular Arches with 11 Blocks

5cm Span Length



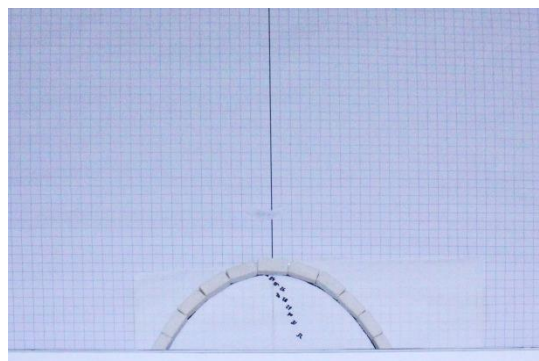
The photo refers to 5cm. span length arch. The arch is feasible however, it is seen that the shape of the arch is not funicular. It is more straight shape.

6cm Span Length



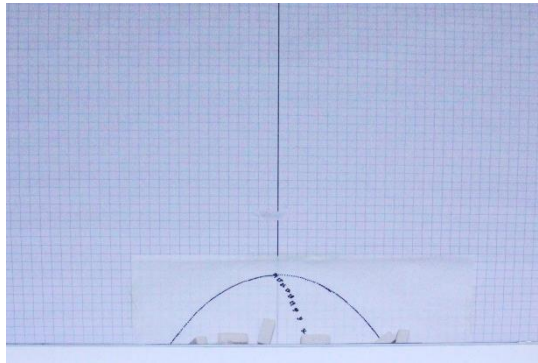
The photo refers to 6cm. span length arch. The arch is feasible and the shape is proper to be funicular. So, 6cm. is accepted as minimum span length.

23cm Span Length



The photo refers to 23cm. span length arch. The arch is feasible and the shape is proper to be funicular. For the span length longer than 23cm, the arches are collapsed. So, 23cm. is accepted as maximum span length.

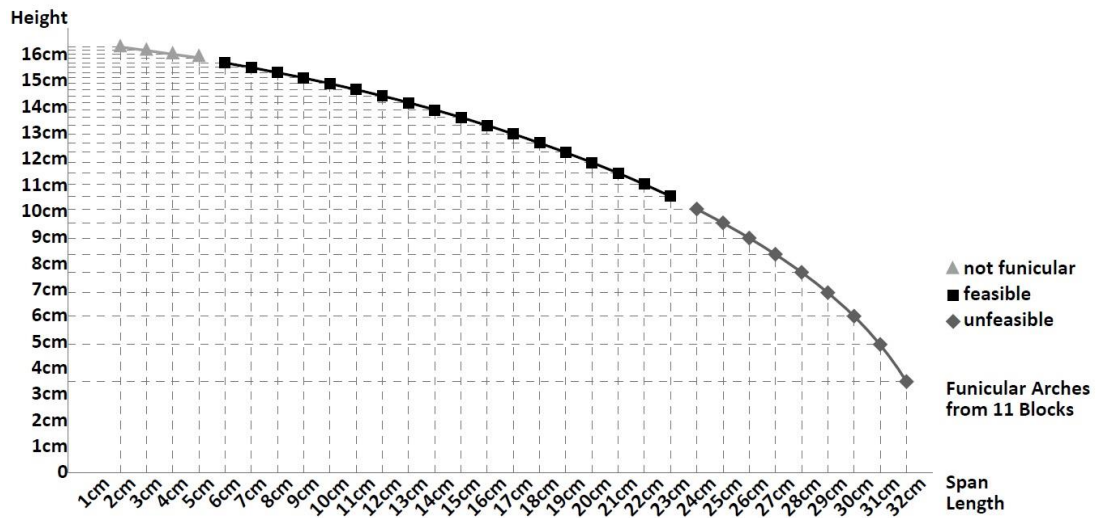
24cm Span Length



The photo refers to 24cm. span length arch. For that span length, the arch is collapsed and it is unfeasible.

Result

For 11 blocks and 33cm. arch length funicular arches, minimum span length is found 6cm. and maximum span length is found 23cm. The heights of these arches are changed between these ranges proportionally.



Span – Height Relation of Funicular Arches from 11 Block