Flow Measurement and Instrumentation 80 (2021) 102007

Contents lists available at ScienceDirect

Flow Measurement and Instrumentation

journal homepage: www.elsevier.com/locate/flowmeasinst

Discharge coefficient for vertical sluice gate under submerged condition using contraction and energy loss coefficients

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ARTICLE INFO

Keywords:
Contraction coefficient
Discharge coefficient
Energy loss coefficient
Multivariate adaptive regression spline
Open channel
Vertical sluice gate

ABSTRACT

A novel method is suggested for the determination of flow discharge in vertical sluice gates with considerably small bias. First, in order to derive an equation for the discharge coefficient, energy-momentum equations are implemented to define the physical realization of the phenomenon. Afterward, the discharge coefficient is presented in terms of contraction and energy loss coefficients. Subsequently, discharge coefficient, contraction, and energy loss coefficients were determined through an implicit optimization technique on the data. Data analysis illustrated that there is a meaningful power relationship between the contraction and energy loss coefficients. Thereafter, dimensional analysis is performed and an explicit best-fit regression equation is developed for defining the energy loss coefficient. The obtained equations for contraction and energy loss coefficients were then used in the computation of the discharge coefficient and determination of the flow discharge in the vertical sluice gate. The performance of the developed approach is validated against the selected benchmarks existing in the literature.

1. Introduction

The sluice gates are of great importance when controlling and measuring discharge is concerned in open channel hydraulics [1–5]. However, the computation bias of the discharge in the submerged flow conditions was reported to exceed up to 40% [6]. To this end, none of the previous studies motivated by the evaluation of the relationship between energy and momentum equations could obtain lower than 10% bias in the computation of the flow under vertical sluice gates [7–13].

In this respect, Cheng et al. [14] evaluated the steady-state flow properties under a sluice gate and over a spillway using the boundary integral equation approach. It was concluded that geometrical changes can alter the contraction coefficient. Rajaratnam and Humphries [15] conducted lab experiments on the immediate flow conditions at the upstream of a vertical sluice gate in a rectangular channel. Furthermore, several properties of the flow including surface eddy, bed's pressure defect, and also the velocity field in the jet were analyzed. Lin et al. [5] used experimental data to develop a theoretical equation in the calculation of the contraction coefficient based on the free flow and hydraulic jump. It was pointed out that, the contraction coefficient varies with the type of gate used in the analysis. In addition, it was shown that, for a certain approaching depth, flow in a radial gate is less likely to become submerged compared to the vertical gates with smaller contraction coefficients. Belaud et al. [12] proposed a theoretical framework based on momentum and energy conservation to calculate the contraction coefficient under submerged flow conditions. It was concluded that the size of the opening is important in the credibility of the contraction coefficients which can exceed 0.6 when the opening is large. Castro-Orgaz et al. [9] used energy-momentum equations to calibrate the velocity coefficient in sluice gates for agricultural use. It was found out that, the calibration of the contraction coefficient, indirectly improves the energy-momentum approach considering non-uniform velocity effects within the energy-momentum equations. Belaud et al. [16] studied the contraction and correction coefficient (energy coefficient) in sluice gates using energy-momentum equations. For this, experimental studies were conducted and simulated using a turbulence re-normalization group (RNG) k-ε model, motivated by Reynolds-Averaged Navier-Stokes equations. It was concluded that the numerical and the experimental methods confirm the variations in contraction coefficient (C_c) and it was suggested that corrections must be applied to the Coriolis and Boussinesq coefficients, head loss, and friction forces to enhance the predictability of contraction and discharge coefficients in energy-momentum

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https://doi.org/10.1016/j.flowmeasinst.2021.102007
Received 26 January 2021; Received in revised form 18 June 2021; Accepted 5 July 2021
Available online 10 July 2021
0955-5986/

equations. Cassan and Belaud [17] studied the flow condition under the sluice gates and reported that the contraction coefficient is increased with increasing the gate opening. Castro-Orgaz et al. [10] evaluated the energy-momentum equations in vertical sluice gates to predict a more applicable contraction coefficient instead of the typical 0.61 value. Although the results were satisfactory, it was shown that the estimated discharges have a 10% bias compared to the measured counterparts. Viero and Defina [18] studied the hysteresis plots associated with the supercritical flow conditions upstream of a vertical sluice gate. Flow configuration around the gate was classified by means of the Froude number at the downstream and upstream of the gate. It was concluded that the two regions exist which admit a dual solution. More recently, Bijankhan et al. [13] conducted a set of experiments by means of velocity profiles near the submerged sluice gate measured by Acoustic Doppler Velocimetry (ADV). It was concluded that the classical energy-momentum approach fails to determine the accurate flow rates, while the implementation of the energy correction factors and head loss in the analysis enhances the credibility of the analysis.

Most of the studies in the literature determined the discharge coefficient in vertical sluice gates by neglecting the contraction or energy loss coefficients parameters. The aim of this study is to develop an explicit equation to formulate the discharge coefficient using contraction and energy loss coefficients of vertical sluice gates when the flow condition is submerged. The need for such an extension is recognized when considering that the bias in calculation and/or estimation of discharge, reaches up to 10%. Respectively, the scopes and novel aspects of this study are as follows: (i) using semi-empirical models based on the relationship between contraction and energy loss coefficients in energy-momentum equations, (ii) developing an explicit model to interpret the discharge coefficient based on geometrical properties of the setup, and also (iii) applying multivariate adaptive regression spline (MARS) method in the development of those equations in practice which is comparable to the equations suggested by the previous studies. The subsequent chapters respectively detail the implementation of energy-momentum equations in obtaining discharge coefficient, evaluation of the experimental data including data analysis to understand the relationship between variables, application of dimensional analysis and Pi-Buckingham theorem, development of the equations in the determination of energy loss and contraction coefficients, application of MARS model, and discussion about the results based on the reference models taken from the literature.

2. Energy-momentum equations for flow under vertical sluice gates

2.1. Development of the theoretical aspect

The submerged hydraulic condition of a typical sluice gate is depicted in Fig. 1. Based on this illustration (i.e. Fig. 1), variables that are effective on the phenomenon are, q as the discharge per unit width, y_1 as the approaching depth in the upstream, y downstream depth immediately downstream of the gate; y_2 as the thickness at the vena contracta, y_3 as the depth in the downstream far from the gate, w as the height of the gate opening, and b as the width of the channel. Therefore, other variables such as C_c (contraction coefficient), k (energy loss coefficient), and Re_1 (approaching Reynolds number) are particularly obtained based on the data.

The energy equation between the upstream (cross-section 1, Fig. 1) and the vena contracta (section 2, Fig. 1) in the downstream of the submerged flow can be defined as

$$y_1+\frac{q^2}{2gy_1^2}=y+\frac{q^2}{2gy_2^2}+k\left(\frac{q^2}{2gy_2^2}\right) \quad (1)$$

or,

$$y_1-y=\frac{q^2}{2gy_2^2}\left(1+k-\frac{y_2^2}{y_1^2}\right) \quad (2)$$

In addition, the momentum equation between the sluice gate and the vena contracta at the downstream can be given as

$$y^2-y_3^2=\frac{2q^2}{g}\left(\frac{1}{y_3}-\frac{1}{y_2}\right) \quad (3)$$

Therefore, Equation (2) can be rewritten as

$$y=y_1-\frac{q^2}{2gy_2^2}\left(1+k-\frac{y_2^2}{y_1^2}\right) \quad (4)$$

or,

$$y^2=y_1^2+\left(\frac{q^2}{2gy_2^2}\right)^2\left(1+k-\frac{y_2^2}{y_1^2}\right)^2-2y_1\frac{q^2}{2gy_2^2}\left(1+k-\frac{y_2^2}{y_1^2}\right) \quad (5)$$

and by inserting y^2 from Equation (5) in Equation (3), it gives

$$y_1^2+\left(\frac{q^2}{2gy_2^2}\right)^2\left(1+k-\frac{y_2^2}{y_1^2}\right)^2-2y_1\frac{q^2}{2gy_2^2}\left(1+k-\frac{y_2^2}{y_1^2}\right)-y_3^2=\frac{4q^2}{2gy_2^2}\left(\frac{y_2^2}{y_3}-y_2\right) \quad (6)$$

or,

$$\left(1+k-\frac{y_2^2}{y_1^2}\right)^2\left(\frac{q^2}{2gy_2^2}\right)^2-\left[4\left(\frac{y_2^2}{y_3}-y_2\right)+2y_1\left(1+k-\frac{y_2^2}{y_1^2}\right)\right]\frac{q^2}{2gy_2^2}+y_1^2-y_3^2=0 \quad (7)$$

By solving the second order equation above, it can be concluded that

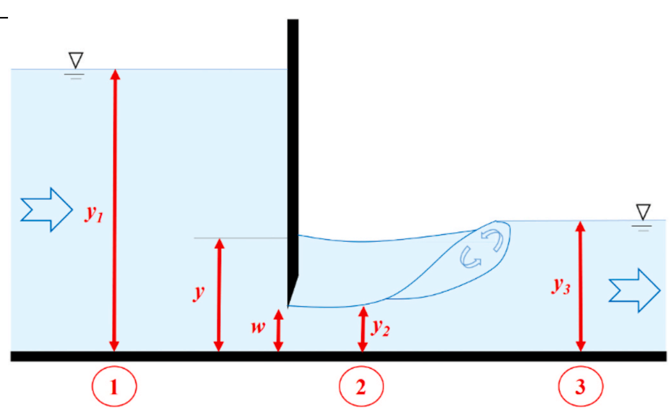

Fig. 1. Schematic of the vertical sluice gates under submerged flow condition.

$$\frac{q^2}{2gy_2^2} = \frac{\left[2\left(\frac{y_2^2}{y_3} - y_2\right) + y_1\left(1 + k - \frac{y_2^2}{y_1^2}\right)\right] \pm \sqrt{\left[2\left(\frac{y_2^2}{y_3} - y_2\right) + y_1\left(1 + k - \frac{y_2^2}{y_1^2}\right)\right]^2 - \left(1 + k - \frac{y_2^2}{y_1^2}\right)^2 (y_1^2 - y_3^2)}}{\left(1 + k - \frac{y_2^2}{y_1^2}\right)^2} \tag{8}$$

or,

$$\frac{q^2}{2gy_2^2} = \frac{y_1\left\{\left[2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right)\right] \pm \sqrt{\left[2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right)\right]^2 - \left(1 + k - \frac{y_2^2}{y_1^2}\right)^2 \left(1 - \frac{y_3^2}{y_1^2}\right)}\right\}}{\left(1 + k - \frac{y_2^2}{y_1^2}\right)^2} \tag{9}$$

Since $y_2 = C_c.w$ therefore,

$$q^2 = C_c^2 w^2 2gy_1 \frac{2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right) - \sqrt{\left[2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right)\right]^2 - \left(1 + k - \frac{y_2^2}{y_1^2}\right)^2 \left(1 - \frac{y_3^2}{y_1^2}\right)}}{\left(1 + k - \frac{y_2^2}{y_1^2}\right)^2} \tag{10}$$

or,

$$q = C_c \sqrt{\frac{2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right) - \sqrt{\left[2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right)\right]^2 - \left(1 + k - \frac{y_2^2}{y_1^2}\right)^2 \left(1 - \frac{y_3^2}{y_1^2}\right)}}{\left(1 + k - \frac{y_2^2}{y_1^2}\right)^2}}\, w\sqrt{2gy_1} \tag{11}$$

In this respect, the following equation can be written for both the free and submerged flow in the sluice gates [19] as

$$q = C_d w \sqrt{2gy_1} \tag{12}$$

By comparing Equation (11) and Equation (12), it can be concluded that

$$C_d = C_c \sqrt{\frac{2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right) - \sqrt{\left[2\left(\frac{y_2^2}{y_1y_3} - \frac{y_2}{y_1}\right) + \left(1 + k - \frac{y_2^2}{y_1^2}\right)\right]^2 - \left(1 + k - \frac{y_2^2}{y_1^2}\right)^2 \left(1 - \frac{y_3^2}{y_1^2}\right)}}{\left(1 + k - \frac{y_2^2}{y_1^2}\right)^2}} \tag{13}$$

or by replacing the $C_c w$ instead of the remaining y_2, it can be reformulated as

$$C_d = C_c \frac{\sqrt{2\left(C_c^2\frac{w^2}{y_1y_3} - C_c\frac{w}{y_1}\right) + 1 + k - C_c^2\left(\frac{w}{y_1}\right)^2 - \sqrt{\left[2\left(C_c^2\frac{w^2}{y_1y_3} - C_c\frac{w}{y_1}\right) + 1 + k - C_c^2\left(\frac{w}{y_1}\right)^2\right]^2 - \left[1 + k - C_c^2\left(\frac{w}{y_1}\right)^2\right]^2\left[1 - \left(\frac{y_3}{y_1}\right)^2\right]}}}{1 + k - C_c^2\left(\frac{w}{y_1}\right)^2} \tag{14}$$

The theory of the flow in vertical sluice gates relies on the energy and momentum equations in open channel flow. Through solving these equations, contraction (C_c) and energy loss (k) coefficients appear which are the parameters that cannot be measured directly during the experiments. As emphasized by Castro-Orgaz et al. [9], when the flow condition is submerged in vertical sluice gates at the vena contracta section, velocity profile is highly non-uniform. To this end, the discharge coefficient that consists of contraction (C_c) and energy loss (k) coefficients at its structure, represents the impacts of turbulence, non-uniform distribution of velocity, viscosity, and velocity head in the approach channel. Furthermore, the energy loss coefficient (k) can be linked to the eddy and jet characteristics in the shear layer [10]. Consequently, in order to establish a reliable approach for the determination of the discharge coefficient in a vertical sluice gate, contraction (C_c) and energy loss (k) coefficients must be defined [13].

It is seen from Equation (14) that, the effective parameters on the discharge coefficient are contraction coefficient (C_c), the ratio of the gate opening to the upstream depth (w/y_1), energy loss (k), the ratio of the gate opening to the downstream depth (w/y_3), and the ratio of the downstream to the upstream depths (y_3/y_1). It can be understood from Equation (14) that C_d is a function of y_1, y_3, w, and b as the characteristics of the flow, gate and channel; in addition to the presence of the C_c and k coefficients in the equation. The variables y_1, y_3, w and b can be measured directly; however, as mentioned before determination of the relationship between C_c and k is not an easy task to perform. In this respect, the C_d values can be determined, from Equation (12), and only by utilizing the experimental data. However, a challenging task comes from the determination of C_c and k in an implicit way when Equation (14) is used. Except for C_c and k in Equation (14), all other variables including C_d can be obtained directly from experimental measurements in an explicit way. Therefore, in the subsequent section, it is aimed to generate data for C_c and k through solving Equation (14), by utilizing the experimental data implicitly.

2.2. Generating data for contraction and energy loss coefficients

In order to generate data for C_c and k, values associated with the C_d should be obtained from Equation (12) initially. For this, two data sets were taken from the literature to determine the C_d values. The data used in this study were based on preliminary laboratory studies of Rajaratnam and Subramanya [20,21], and field data provided by Sepulveda et al. [8]. Accordingly, two data set with 55 experiment in total were used in the analysis. In addition, data sets related to the openings less than 6 cm were eliminated from the study, due to the scale effect [22].

Then, the C_c and k values at each data set were calculated using trial and error in Microsoft Excel's solver. For the sake of confidential modeling, each run was repeated several times to avoid misinterpretation and false detect. Afterward, the obtained C_c and k values were used in the remaining parts of the analysis. For this, Equation (14) was used and C_c and k values were obtained implicitly for each data experiment using generalized reduced gradient (GRC) nonlinear approach (with lower than 0.0001% bias). To come up with the best combination, the square error (*SE*) and root mean square percentage error (*RMSPE*) between observed and calculated C_d were minimized using,

$$SE = (x_i - \hat{x}_i)^2 \tag{15}$$

$$RMSPE = \frac{1}{\mu_x}\sqrt{\frac{\sum (x_i - \hat{x}_i)^2}{n}} \times 100 \tag{16}$$

where x_i, $\hat{x}_i$, μ_x, and n are observed values, estimated values (i.e. C_d), mean of the observed values, and number of values, respectively. Results showed that the calibrated C_c and k generates C_d values with bias range ± 0.04, the maximum *SE* of 2.48×10^{-3}, and the *RMSPE* of 5.21% which are reasonable and satisfactory. Alternatively, the C_c and k in the analysis can be obtained using velocity profiles [13], which may reveal bias higher than 10% as detailed previously.

The next step is to define a proper equation/realization for each of C_c and k. While the C_c and k were simplified, Equation (14) can be rearranged with less variable that can be more practical and parsimonious in application. Then, the most proper relationship was separately defined for C_c and k. In this stage, eliminating one of the C_c or k, would reduce the propagation of the bias through model development.

3. Data analysis and selection of effective parameters

In this stage, each of C_c and/or k should be expressed by means of other variables for the sake of parsimonious modeling such that

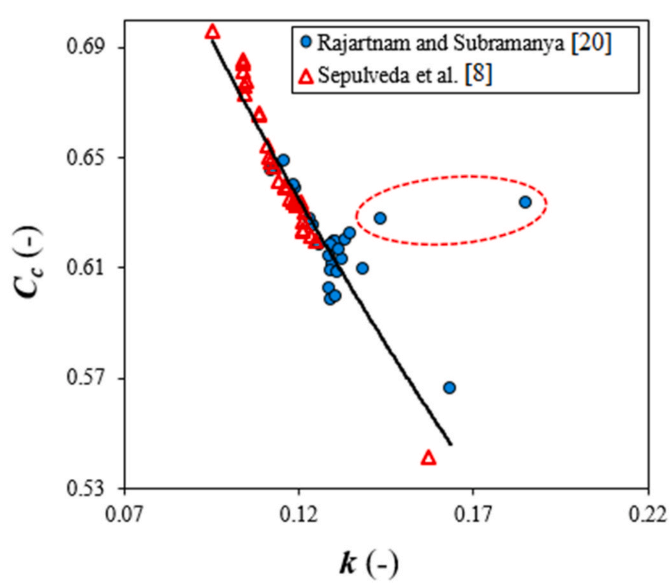

Fig. 2. Relationship between C_c and k.

$$C_c = 0.25k^{-0.43} \tag{17}$$

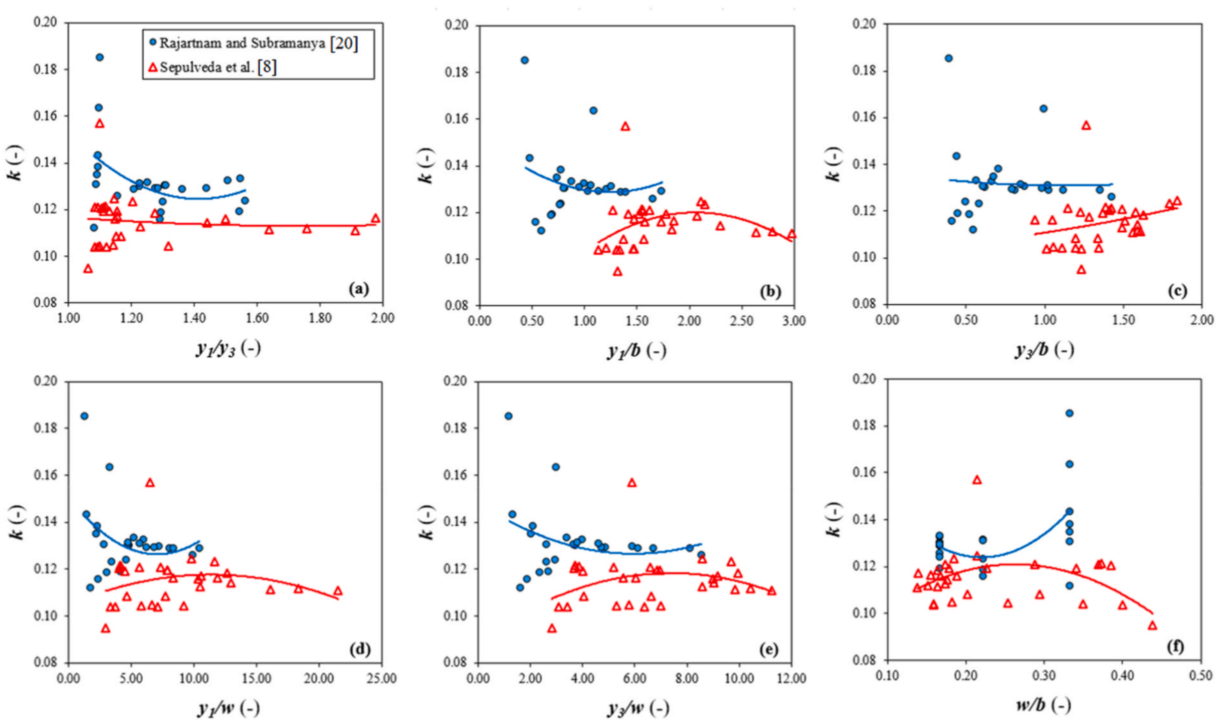

Fig. 3. Dimensionless variables used in definition of k. and eligable.tly ance criteria would be eligable to by rsimonious and owere bias ted.

Equation (14) can be solved without optimization in a strict forward way (i.e. explicit modeling). As detailed before, the main purpose is to predict C_c and k accurately. Therefore, it would be more practical either if C_c or k could be eliminated from the equation. The advantage of such simplification would be an equation which can be solved explicitly with one unknown variable to be predicted. In this respect, Fig. 2 depicts the nature of relationship between C_c and k. It was found that the C_c can easily be conceptualized using k and a proper curve fit. Thus, a power equation such as one given in Equation (16), can be effective, while eliminating the outliers (eliminated from Rajaratnam and Subramanya [20,21] data set) would prevent results from undesired deviations (given in red oval in Fig. 2). In this respect the determination coefficient (R^2) of the following equation is 0.92, which is reasonable and promising for application.

Accordingly, by substituting Equation (17) in Equation (14), the C_d can be expressed as

$$C_d = \frac{0.253\sqrt{2\left(\frac{(0.253w)^2}{k^{0.86}y_1y_3} - \frac{0.253w}{k^{0.43}y_1}\right) + 1 + k - \left(\frac{0.253w}{k^{0.43}y_1}\right)^2 - \sqrt{\left[2\left(\frac{(0.253w)^2}{k^{0.86}y_1y_3} - \frac{0.253w}{k^{0.43}y_1}\right) + 1 + k - \left(\frac{0.253w}{k^{0.43}y_1}\right)^2\right]^2 - \left[1 + k - \left(\frac{0.253w}{k^{0.43}y_1}\right)^2\right]^2\left[1 - \left(\frac{y_3}{y_1}\right)^2\right]}}}{k^{0.43}\left(1 + k - \left(\frac{0.253w}{k^{0.43}y_1}\right)^2\right)} \quad (18)$$

In this equation, the only unknown independent variable to be predicted is k which should be modelled in advance based on the initial values measured during the experiment. In Fig. 3, the relationship between dimensionless variables y_1/y_3 (Fig. 3a), y_1/b (Fig. 3b), y_3/b (Fig. 3c), y_1/w (Fig. 3d), y_3/w (Fig. 3e) and w/b (Fig. 3f) against k is given. These are the same variables used in Equations (14) and (18), therefore the estimation of k based on these variables would be more parsimonious. According to the mutual relationship between variables, it is still hard to distinguish the most effective variables. It is shown that the nature of the data provided by Rajaratnam and Subramanya [20,21] is different than those of Sepulveda et al. [8]. Hence, a model which satisfies the desired conditions in both of the data sets can be recognized as sufficient and eligible.

4. Developing an equation for k

As illustrated in Fig. 3, the relationship between k and other variables depicted a complicated behavior that is under the influence of large deviations, outliers, and sub-classes. Hence, a multi-stage decision model capable of incorporating classification technique in reduction of the model complexity and outliers would be favorable in predicting the best energy-loss coefficient, k.

4.1. Dimensional analysis

With respect to the information given in preceding sections, it is aimed to find relationships for calculation of k using b, y_1, y_3 and w. Therefore, the incorporation of dimensionless variables such as those introduced in Equation (14) or Equation (18) would be desirable. However, dimensional analysis is also designated to establish a functional relationship between k as dependent parameter and b, y_1, y_3, and w, similar to those of Equation (14) or Equation (18).

From the hydraulics point of view, a reliable method in finding a certain relationship between the numbers of variables is Pi-Buckingham

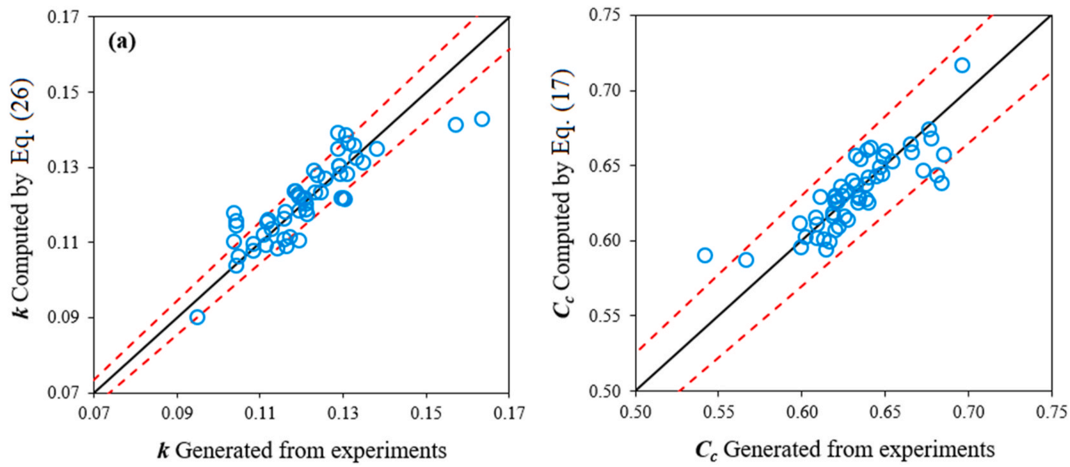

Fig. 4. Scatter diagram of the conducted modeling between (a) obtained data and Eq. (26) results in evaluation of k; and (b) obtained data and curve fitting in evaluation of C_c

Theorem. Having four variables as b, y_1, y_3, and w with length dimension [L] for calculation of a dimensionless variable of k, it is required to rearrange the equation to a dimensionless form. In this respect, by taking k as dependent variable and b, y_1, y_3, and w as independent variables, following expression can be written.

$$k = f(b, y_1, y_3, w) \tag{19}$$

Different combination of dimensionless parameters can be expressed in the analysis; whilst the highest similarity with those of Equation (14) and Equation (18) is preferred. Having four variables with length dimension, can be interpreted by one repeating variable, according to the Pi-Buckingham Theorem. Through a trial and error, it is found that while taking y_1 or y_3 as repeating variable better results can be achieved. The expressions based on y_1 or y_3 as repeating variables are given as

$$k = f\left(\frac{y_1}{w}, \frac{y_1}{b}, \frac{y_1}{y_3}\right) \tag{20}$$

$$k = f\left(\frac{y_3}{w}, \frac{y_3}{b}, \frac{y_3}{y_1}\right) \tag{21}$$

In order to enhance the capability of the developed equation in the calculation of k, Equations (20) and (21) were merged. Then, for the sake of better capturing the effect of channel width (b) and height of the gate opening (w), their ratio (w/b) was added to the final equation together with the Reynolds number at the gate opening in which fluid viscosity is incorporated ($Re_1 = w\sqrt{gy_1}/\nu$).

Consequently, the final expression can be considered as

$$k = f\left(\frac{y_3}{w}, \frac{y_3}{b}, \frac{y_3}{y_1}, \frac{y_1}{w}, \frac{y_1}{b}, \frac{y_1}{y_3}, \frac{w}{b}, Re_1\right) \tag{22}$$

which is satisfactorily structured based on the variables similar to those of Equation (14) and Equation (18).

4.2. Best fit equation for energy loss coefficient

In order to develop a best fit explicit equation considering the given parameters in Equation (22), multivariate adaptive regression spline (MARS) technique is implemented. It was due to the high potential of MARS in classification, outlier resistance, and flexibility that the method was preferred. MARS is a non-parametric regression method introduced by Friedman [23]. Since MARS is a computational robust, it provides explicit equation and widely used in engineering studies [24]. Accordingly, MARS has a useful feature in recognition of the impact of the independent variables of model output to construct a robust model. Additionally, it establishes a non-linear function between the dependent and independent variables align with a number of linear functions called the basis-functions. The input variables are segregated to consecutive splits to determine a linear relationship for every split or class, known as knots, which is a basis function to the approach. To this end, MARS model development task consists of two fundamental steps, a forward and a backward step. At the forward step, the most essential variables are determined, while in the backward step, fewer effective variables are eliminated to promote model accuracy and prevent overfitting [23–25].

Consequently, the final MARS model is generated through the combination of several linear functions, and a basis functions $B(x)$ where an input variable x has a functional relationship with model output considering the following two expressions as

$$B(x) = \max(0, x - c) \tag{23}$$

$$B(x) = max(0, c - x) \tag{24}$$

in which c is a threshold quantity, while the maximum number of basis-functions in the modeling should be determined by the user. Thereafter, the specific number of basis-functions are implemented in forward stage and then, less important variables are eliminated in the backward stage to simplify the final model. For the sake of keeping connection of the basis-functions, contiguous splines were intersected at knots. Afterward, the provided linear functions were merged to construct the final MARS model as,

$$f = \sum_{i=1}^{n} \beta + \alpha\phi_i(x) \tag{25}$$

in which n is the number of basis functions and $\phi_i(x)$ is the ith basis function. In the present study, the maximum number of basis-functions is set as 40 in which 19 of them were evaluated by the best MARS functions in the modeling.

MARS algorithm is applied on parameters given in Equation (22) where parameters at right hand side of the relationship were considered as model inputs and k as model output, the following relationship is proposed for the computation of k.

$$
\begin{aligned}
k = & -0.42-1.21\left(\max\left(0,\frac{w}{b}-0.33\right)\right)+0.64\left(\max\left(0,0.33-\frac{w}{b}\right)\right)+0.02\left(\max\left(0,\frac{y_1}{w}-1.77\right)\right)+2.06e^{-6}(\max(0,Re_1-133812))\\
& -0.06\left(\max\left(0,\frac{y_1}{b}-1.47\right)\right)+0.25\left(\max\left(0,1.47-\frac{y_1}{b}\right)\right)-1.50e^{-6}(\max(0,Re_1-229882))+0.92\left(\max\left(0,\frac{y_1}{y_3}-1.15\right)\right)\\
& -0.39\left(\max\left(0,1.15-\frac{y_1}{y_3}\right)\right)+0.04\left(\max\left(0,\frac{y_3}{w}-1.63\right)\right)-0.88\left(\max\left(0,\frac{y_1}{y_3}-1.10\right)\right)-0.07\left(\max\left(0,\frac{y_3}{b}-1.43\right)\right)\\
& +0.19\left(\max\left(0,1.43-\frac{y_3}{b}\right)\right)+1.22e^{-6}(\max(0,Re_1-246447))+0.06\left(\max\left(0,\frac{y_1}{b}-1.13\right)\right)-0.34\left(\max\left(0,\frac{y_1}{b}-1.37\right)\right)\\
& +0.20\left(\max\left(0,\frac{y_1}{b}-1.27\right)\right)-0.02\left(\max\left(0,\frac{y_1}{w}-6.20\right)\right)-0.34\left(\max\left(0,\frac{w}{b}-0.22\right)\right)
\end{aligned}
\tag{26}
$$

5. Computation of discharge

Equation (26) is used to calculate the k values and C_c by means of Equation (17). Having k and C_c in hand, the C_d values could be calculated using Equation (14) in an explicit way. Later, the unit discharge q (m^2/s), and the flow discharge, Q (m^3/s) were estimated to be evaluated against observed values. Comparison is made using scatter plot, *RMSPE* and determination coefficient (R^2) as given bellow.

$$
R^2=\left(\frac{Cov\left(X,\widehat{X}\right)}{\sigma_x\sigma_{\widehat{x}}}\right)^2 \tag{27}
$$

where σ_x and $\sigma_{\widehat{x}}$ are the standard deviation of observed and calculated values respectively. The associated determination coefficient and *RMSPE* are found as 0.74 and 5.18%, respectively, which is a sign for good model establishment (Fig. 4a). Since the estimated results are found to be satisfactory, they can be used in calculating the C_c based on Equation (17). It is given in Fig. 4b and quite dependable since it has a determination coefficient of 0.68 and *RMSPE* of 2.5%.

6. Comparison of suggested approach to some benchmarks

The method proposed in this study is compared to the methods suggested by Sepulveda et al. [8] based on a classical approach as well as the method proposed by Ferro [25]. The classical approach claims that the flow rate can accurately be obtained when the contraction coefficient as well as the hydraulic condition of the flow are known [1]. By considering the energy, mass and momentum equations, discharge of the flow can be calculated using the following equation.

$$
Q=C_d.b.w.\sqrt{2gy_1} \tag{28}
$$

which is similar to Equation (12), and is applicable for computation of the discharge per unit width of the channel (q). With respect to Equation (28), the sole value which should be estimated is the discharge coefficient (C_d), and is usually obtained separately for submerged flow as,

$$
C_d=C_c\sqrt{\frac{\left(\dfrac{1}{\frac{1}{\frac{b.C_c}{y_1}}-1}\right)^2+2\left(\frac{y_1}{y_3}-1\right)-\sqrt{\left(\left(\dfrac{1}{\frac{1}{\frac{b.C_c}{y_1}}-1}\right)^2+2\left(\frac{y_1}{y_3}-1\right)\right)^2-\left(\dfrac{1}{\left(\frac{b.C_c}{y_1}\right)^2}\right)^2\left(1-\frac{h_3^2}{h_1^2}\right)}}{\frac{y_1}{b.c_c}-\frac{b.C_c}{y_1}}} \tag{29}
$$

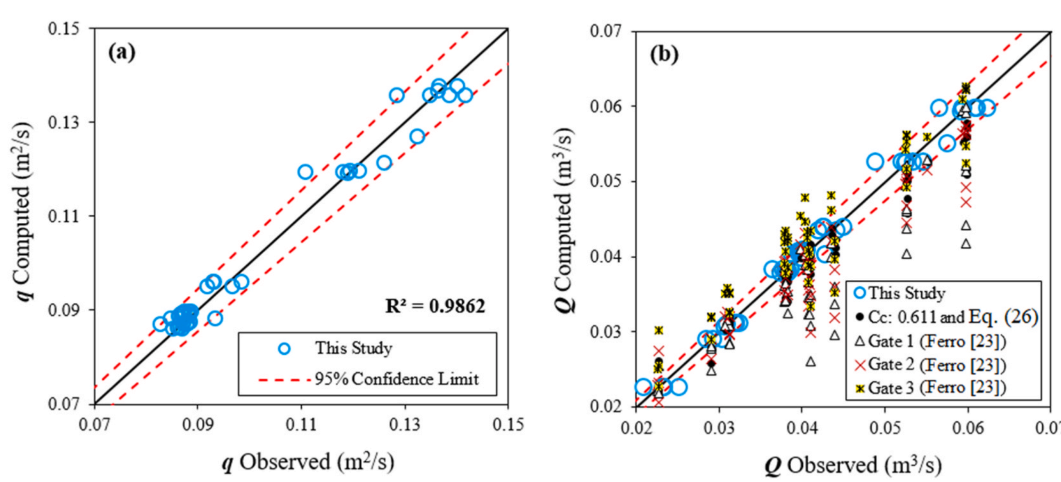

Fig. 5. Comparison of the method suggested in this study and selected benchmarks in calculating of discharge (Q) and discharge per unit width (q).

Table 1
Performance of the method suggested in this study and selected benchmarks in calculating the discharge (Q) using R^2, *RMSPE* and *MAPE*.

Method	R^2	*RMSPE* (%)	Equations and Details
This study	0.98	3.10	|k: Eq. (26) | C_c: Eq. (17) | C_d: Eq. (14)
	0.96	7.05	|k: Eq. (26) | C_c: 0.611| C_d: Eq. (14)
Ferro [25], Gate 1	0.80	15.26	|Eq. (30) |k'_0: 0.9176 |k'_1: 0.3489
Ferro [25], Gate 2	0.88	10.77	|Eq. (30) |k'_0: 0.9482 |k'_1: 0.34202
Ferro [25], Gate 3	0.89	9.16	|Eq. (30) |k'_0: 1.0097 |k'_1: 0.3154

However, the contraction coefficient can be effected by means of different factors such as the height of gate opening [12], shape of the lip, type of the gate and approaching water depth etc. [5]. This coefficient is usually calibrated based on the variations in geometry of the channel and the gate together with the flow condition [13]. On the other hand, empirical approaches for the vertical sluice gates in the submerged flow conditions indicates to the vital role of the C_d and the downstream depth [26]. Therefore, C_d was expressed as a function of depth and opening height and several constants.

Sepulveda et al. [8] suggested that the preliminary studies conducted by Ferro [27] and followed by Ansar [28], and Ferro [25] gives superior results compared to those suggested by Henry [29], Rajaratnam and Subramanya [21], and Swamee [30]. Accordingly, Ferro [25,27] used Pi-Theorem and the theory of self-similarity to introduce a new approach in calculating the discharge of a vertical sluice gate when the flow condition is submerged as,

$$Q = b\sqrt{g\left(wk_0'\left(\frac{y_1-y_3}{w}\right)^{k_1'}\right)^3} \tag{30}$$

while the k'_0 and k'_1 are constants to be predicted. In this respect, Sepulveda et al. [8] suggested that the k'_0 and k'_1 values, whilst the upstream water level (y_1) is much higher than the downstream water level (y_2), are respectively as high as 0.9176 and 0.3489 (Gate 1). Alternatively, the following values of k'_0 and k'_1; when there are small water level differences between upstream and downstream and larger gate openings can either be taken respectively as 0.9482 and 0.3202 (Gate 2); or as 1.0097 and 0.3154 (Gate 3). It was concluded that a typical combination of 1.0559 and 0.3344 for k'_0 and k'_1 in general can be selected for further application. Sepulveda et al. [8] also recommended C_d values of 0.609 (Gate 1), 0.678 (Gate 2), and 0.768 (Gate 3) for partially or totally submerged flow based on the analysis conducted by the Hydrologic Engineering Center's (HEC) River Analysis System (RAS) software (i.e. HEC-RAS). However, it was found that the method suggested by Ferro [25] is superior to those of HEC-RAS, classical methods [12], methods that uses velocity profile [13] or the approaches of which consider a constant, 0.611 for the contraction coefficient (C_c), in the analysis.

Thereby, the accuracy of the method proposed in this study was compared with the classical approach (Equations (28) and (29)), methods suggested by Ferro [25] as Equation (30), and a typical C_c: 0.611 value to get a transparent view on the subject. For this, the conceptualization of Equation (18) by means of Equation (17) and Equation (26) was used instead of Equation (14). In this respect, Fig. 5a, depicts the results of estimated q with R^2 and *RMSPE* of 0.98 and 3.12% respectively. This results, can be compared with the conclusion made by the Castro-Orgaz et al. [10] in modelling the discharge in vertical sluice gates with low precision. Table 1 also, summarize the performance of the selected methods in practice. Since the results obtained by the classical method (Equation (29)), also produce undefined conditions due to the negative values under the square root operator, the results of the classical method are neglected from Fig. 5 and Table 1. This defines the fundamental deficiency of the classical approach where for some cases, it gave undefined values. In this regard, for the sake of proposed method in the study, first the value of k is computed using Equation (26). Then C_c and C_d were respectively computed by Equation (17) and Equation (14). Afterward, a C_c equal to 0.611 is used together with the k obtained from Equation (26) to calculate C_d of Equation (14) and calculating q and Q to be compared with the results obtained by this study.

Based on the results of Fig. 5b and Table 1, the method suggested by this study has a superior performance in practice. Even, when the classical $C_c = 0.611$ approach is used, the combination of this coefficient with the k obtained using Equation (26), yields more confident results.

7. Validation of the model

The developed approach in this study for discharge computation of vertical sluice gate in submerged condition is validated using Bijankhan et al. [13] experimental data who performed experiments in channel having 7 m length and 1.179 m width. Similar to the obtained results in this study, it was concluded that the classical energy-momentum method is incapable of determining the flow rate for highly submerged flows. However, employing the interaction of the energy correction factors and head loss in the models would increase the accuracy of the head-discharge estimation. Performance of approach developed in this study is further evaluated on Bijankhan et al. [13] data and compared to the equation suggested by Ferro [25] and C_c: 0.611 instead of Eq. (26). For this, values of the k, C_c, C_d, and q are respectively calculated using Eq. (26), Eq. (17), Eq. (14), and Eq. (12). The equation suggested by Ferro [25] and C_c: 0.611 instead of Eq. (26) are used to illustrate the comparability of the suggested model. Results are given in Table 2, which confirm the results obtained in Table 1. Although the R^2 and *RMSPE* of the method which used C_c: 0.611 is better than the results of the suggested method (C_c calculated by Eq. (17)); a set of *RMSPE*: 1.24% and R^2: 0.98 is still indicate to promising results. The method suggested by Ferro [25] could not produce promising results. Therefore, it is concluded that the method suggested in this study is relatively robust and validated for further applications.

8. Discussion

The flowchart of the procedure used in the study is given in Fig. 6 to simplify the methodology used in determining the discharge of the vertical sluice gates under a submerged flow condition. According to the flowchart of the procedure in Fig. 6, and by considering the energy-momentum equations, C_d was determined as function of C_c and k. Then, the experimental data taken from the literature were used to generate C_c and k values through an optimization technique, implicitly.

Table 2
Validation of the method suggested in this study using data set of Bijankhan et al. [13].

Method	R^2	*RMSPE* (%)	Equations and Details
This study	0.98	1.24	|k: Eq. (26) | C_c: Eq. (17) | C_d: Eq. (14)
	0.99	0.21	|k: Eq. (26) | C_c: 0.611| C_d: Eq. (14)
Ferro [25], Gate 1	0.96	23.58	|Eq. (30) |k'_0: 0.9176 |k'_1: 0.3489
Ferro [25], Gate 2	0.97	19.61	|Eq. (30) |k'_0: 0.9482 |k'_1: 0.34202
Ferro [25], Gate 3	0.97	14.49	|Eq. (30) |k'_0: 1.0097 |k'_1: 0.3154

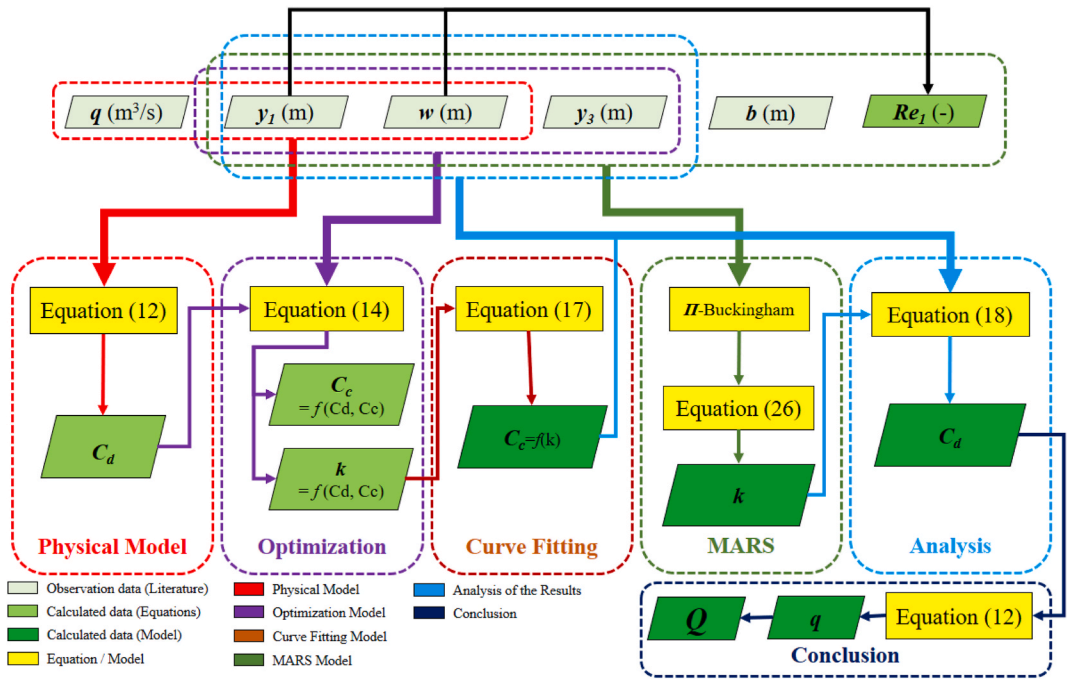

Fig. 6. The flowchart of the modelling/analyzing procedure in this study.

Afterward, a primary analysis is performed to investigate the relationship between C_c and k as given in Equation (17). Then, by considering the functional relationship between C_c and k, the energy-loss coefficient, was selected as a dependent variable to be calculated using y_1/y_3, y_1/b, y_3/b, y_1/w, y_3/w, w/b and Re_1. Thereby, by knowing the k and C_c, the values of C_d can easily be computed by means of Equation (14), and eventually be used in evaluation of the discharge (Q) of the vertical sluice gate when the flow condition is submerged.

Based on the previous studies, the proper definition of the discharge under the vertical sluice gates in a submerged flow conditions have to do with the discharge. In some cases, the classical approach of Equation (29), yield undefined values whilst the root square of a negative value is calculated. Accordingly, the method suggested by Sepúlveda et al. [8] based on the preliminary studies of Ferro [25,27], and Ansar [28] showed that classification of the coefficients is not always applicable. However, the method suggested in this study, suggests that the proper estimation of k and C_c in calculation of C_d, even with a predefined $C_c = 0.61$, and Equation (26) yields more dependable k values in practice. Hence, it can be concluded that the proposed method based on the MARS, curve fitting and energy-momentum equation is more applicable. In this regard, the MARS method can be applied as a global estimator for the similar conditions.

There is a significant uncertainty in the literature for determination of the exact relationship between the head-loss and contraction coefficients. As reported in Belaud et al. [12], for large gate openings, the head loss is small and the flow is largely submerged. On the other hand, as demonstrated by Fardjeli [31] and Belaud et al. [12] for large gate openings and large submergence, higher values of C_c are expected. These results tend to support the finding in Fig. 2. Belaud et al. [12] considered head-loss coefficient as a calibration coefficient and adopted $k = 1.05$. Habibzadeh et al. [32] also considered constant values for k and C_c variables where assuming $C_c = 0.611$ for both submerged and free flow conditions, they incorporated k values of 0.062 and 0.088 for free and submerged flow conditions, respectively. The exact and explicit relationship between C_c and k has not been reported in the literature yet and there is not any practical guidance in the literature that shows the complexity of the problem. The suggested relationship for k and C_c as Eq. (17) shows that taking k value of 0.088, it gives C_c value of 0.72 which is quite reasonable when compared to the results of Habibzadeh et al. [32]. On the other hand, as demonstrated in previous section, the novel approach presented in this study gives quite good results on Bijankhan et al. [13] data in the validation stage.

9. Conclusions

As a control structure, vertical sluice gates are widely used in regulation of flow and discharge measurement. Therefore, the accuracy of the calculated discharge is important in terms of environmental and operational regulations. Although there are several reported methods in determination of discharge and discharge coefficient in the sluice gate under free flow condition; lack of accuracy and serious bias were reported in the conducted studies for submerged flow condition. For this, two data sets from the literature were used in establishment of the link between geometric variables and contraction coefficient. A theoretical method based on the energy-momentum equations and MARS model is used to establish an equation for the calculation of k values which in return was used in computation of C_c, C_d, q, and Q consecutively.

Based on the conducted analysis it is concluded that

i. The methodology proposed in this study (Fig. 6) is effective and present superior results compared to those methods detailed in this study.
ii. Classical method produces undefined values when values under the square root are negative and therefore, cannot be used as global equation in practice.
iii. Results obtained by Ferro [25] yields higher bias in application. Since the definition of the flow conditions in each experiment is totally different, using a set of unchangeable coefficients would be troublesome in practice.
iv. Since the suggested method is based on simple properties of the flow in the channel, including depths and length, the only variable that alters the results is the energy loss coefficient which is technically based on the relationship between energy and momentum equation and is lumped into the discharge equation, C_d and there is need for further assistance in calibration of the contraction coefficient, C_c.

v. The application of MARS, is satisfactory that results in desirable reduction of bias in calculation of discharge of vertical sluice gates under submerged flow condition.

Credit author statement

Babak Vaheddoost: Conceptualization, Formal analysis, Methodology, Resources, Validation, Visualization, Writing - original draft, Writing - review & editing, Rasoul Ilkhanipour Zeynali: Conceptualization, Data curation, Investigation, Methodology, Supervision, Mir Jafar Sadegh Safari: Investigation, Methodology, Resources, Validation, Visualization, Writing - original draft, Writing - review & editing

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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