



USING MIXTURE COPULA FOR THE CALCULATION
OF VALUE AT RISK: DOLAR-EURO PORTFOLIO

(MSc. Thesis)

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This study titled “Used mixture copula for calculating of value at risk: dolar-euro portfolio” and presented as master thesis. Thesis by Demet Çatal has been evaluated in compliance with the relevant provisions of Y.U Graduate Education and Training Regulation and Y.U Institute of Science Education and Training Direction and jury members written below have decided for the defence of this thesis and it has been declared by consensus / majority of votes that the candidate has succeeded in thesis defence examination dated 29/07/2013

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ÖZET

Riske Maruz Değer (RMD) hesaplamalarında portföyü oluşturan değerlerin bağımlılık yapılarının formu modelin performansı üzerinde büyük öneme sahiptir. Bu çalışmada yatırım araçlarının bağımlılık yapılarının bir çok formda modellenmesine olanak veren kopula çeşitleri, Dolar ve Euro portföylerinde incelenmiştir. İki yatırım aracının bağımlılık yapılarının negatif ve pozitif getiri bölgelerinde farklılık göstermesi nedeniyle bu bölgelere yönelik kopula karışımı önerilmiştir. Geleneksel metotlar, kopula fonksiyonları ve önerilen karışım kopulanın performansları geri dönük test ile ölçümlenmiştir. Türkiye’de döviz endeksleri üzerinde kopula alanında bir çalışma yapılmamış fakat hisse senetleri, bonolar ve hayat sigortası poliçeleri üzerinden modellemeler yapılmıştır.

Anahtar kelimeler: RMD, Kopula, Karışım Kopula, Geriye Dönük Test.

ABSTRACT

Dependence structure set up of financial assets has substantial importance in the performance of Value at Risk calculations. In this article, copula variations that allow modeling of rich dependency structures of financial assets, in particular of Dollar - Euro portfolios are examined. Due to significant dependency regime differences in positive and negative returns, adoption of regional copula mixture is proposed. Performances of traditional VaR calculation methods, copulas and mixture copulas are compared by back testing. Although copulas have been applied to stock returns, bonds and life insurance policies, until now there have been no copula studies related to exchange rates for Turkish markets.

Key Words: VaR, Copula, Mixture Copula, Back Testing.

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TEXT OF OATH

I declare and honestly confirm that my study titled “used mixture copula for calculating of value at risk: dolar-euro portfolio”, and presented as Master’s Thesis has been written without applying to any assistance inconsistent with scientific ethics and traditions and all sources I have benefited from are listed in bibliography and I have benefited from these sources by means of making references.

29/07/2013

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1. INTRODUCTION

Globalization in the recent years, availability of a number of new products in the financial markets with the technology and increase in the competition between the financial corporations have caused a remarkable increase in the risk factors. The increase of the interaction in the financial markets, the rapid change of the market conditions have caused extreme fluctuations of the prices. These changes and fluctuations makes the financial corporations all around the world open to risks and it makes the risk management gain importance. Today, many organizations want to know how big risk position it has against the financial developments. Perhaps, the best answer to this question can be given by means of “Value at Risk” method.

In financial mathematics and financial risk management, Value at Risk (VAR) is the estimation method of the maximum loss of a portfolio or asset under certain assumptions, that describes a certain confidence interval for a certain period. The methods used in VAR calculation are grouped under titles of parametric and non-parametric methods. Particularly, in the calculation of VAR for portfolios, the relations of the assets forming the portfolio is known to be an important factor in determining the overall risk. In cases where dependence structure of returns of assets with each other is linear, the standard statistical methods can be used in VAR calculation. In this thesis, copula method which gives opportunity to model the dependency structures of investment tools in many forms especially in situations when the financial markets has non-linear portfolios due to extreme fluctuations are analyzed. On the other hand, when the structure of dependence between asset returns is in a form other than the linearity, we do not have the opportunity to express that dependence with a single statistical parameter such as the correlation coefficient. Copulas are often used in modeling of dependence forms in many different areas recently (Nelsen, 2006). Dependence structures are extremely important in risk calculations and increasing rates of copula

applications may be found in financial literature (Cherubini, Luciano and Vecciato, 2004).

The word copula is a Latin word and it means “connection, relation”. It was used in mathematical and statistical meaning by Abe Sklar for the first time in 1959 while defining one-dimensional distribution functions in multidimensional form to mean “moving together”. The most important outcomes regarding the copulas occur during the development period of this theory continuing from 1958 to 1976. Sklar show that there is a relation between the copulas and the random variables in his article published in 1973. In short, even copula functions entered the literature in 1959, especially their statistical characteristics and applications are still being developed today.

Copula is defined as dependency function between the random variables. Copulas, with a more certain meaning, are the functions that relates the marginal distributions with their multivariable distributions. In fact, copula functions enable to separate the marginal distributions of the variables and identify the dependency structures clearly. In this way, single variable techniques can be used and a direct connection can be provided with the nonparametric dependency measures. Besides, the deficiencies of the linear correlation technique which is well-known and used up to now can be avoided.

Today, the relation between copula functions and finance is one of the most significant methodologies used to gather the variables studied in finance and the risk factors effective on the markets. As the data is gathered by using the statistical theory due to the indecisive, irregular and temporary attitudes in the financial market, this subject has gained importance for the people working in the application field of finance and the academicians and it has replaced some of the standard subjects used in the mathematical finance. None of the researchers today examines a statistical or financial problem regarding the financial markets without considering the problem of aberration. When the problems like pricing and risk measurement are one-dimensional, effective results can be obtained from the models founded out of the normality hypothesis but when the problem is multi-

dimensional, the models that are founded without using this hypothesis have many mistakes.

Copula use in VAR calculation depending on the characteristics of the related financial market can lead to the possibility of a non-return confusion in risk calculations, but the use of it providing high productivity can also be seen. Therefore, successful practices of copulas in the calculation of risk in some markets do not mean that they will show a similar success in other markets. For this reason, in literature market specific dependency structures formed according to the market qualities that requires the use of different forms of copula.

Two studies have been found regarding Turkey market in the literature. In Çifter and Özün's study (2007), the value at risk of the values is predicted with the methods of delta normal, EWMA, correlation with dynamic conditions and conditional symmetric joe-clayton copula with the daily interest rate of Turkish Lira and value at risk belonging to a portfolio weighed equally consisting of a daily US Dolar/TL exchange rate. In addition, modelling with copula of the interrelation with investment fund strategies and one application in Turkey are one of the studies held in this field. Avutman (2011) stopped at the investment funds in her thesis and by modelling with copula the funds formed up in two different strategies and two types belonging to Finansbank, she examined the copula structure in them. Although copulas have been applied to stock returns, bonds and life insurance policies, until now there have been no copula studies related to exchange rates for Turkish markets.

In literature, the values are produced by Monte-Carlo simulations from the dependency structures modeled with copula, these values are multiplied with weights determined in portfolio and VAR calculation is performed through the distribution of returns of the portfolio according to the account percentile (Ray, 2011). In summary, copulas are used to fine-tune the calculation of VAR through simulation.

In this thesis a model is developed with the parametric method used in the calculation of VAR enhanced with the copula. This model is operated with assets like Euro and the Dollar versus TL exchange rates that are most commonly used in the portfolios of the Turkish financial market and compared using the other methods of calculating VAR with Backtest methods. In the next section of this thesis, respectively, Value at Risk calculation methods without using copula, the backtest method, and general forms of copulas are briefly reviewed. The next section summarizes the use of the copula in VAR calculation with the simulation method. In the fourth chapter, the model developed in this thesis, using the mixture copula in VAR parametric calculation is described. The fifth section contains a comparison of different methods of VAR and final section concludes the thesis.

2. METHODOLOGY

2.1. Value At Risk

VAR analysis methods allows the direct calculation of the risk of a single investment instrument, as well as the portfolios consisting of more than one investment tool and is able to direct the investor's investment processes.

VAR is fundamentally used in banks with large trading portfolios where risk management is mandatory, pension funds, other financial institutions, regulatory bodies that are involved in monitoring and control activities of the industry , and non-financial institutions that are exposed to financial risk because of the financial instruments held by them (Jorion, 2000). The VAR usage areas in these institutions can be classified into three categories: passive use in terms of measuring and reporting of total risk, the defensive use where risk control is made by determination of position limits and consequently, through determining risk; finally, the active use in terms of risk management (Dowd, 1998).

We see the emergence and development of VAR techniques in financial risk management after the financial scandals in the early 1990s. The measurement of market risk by using VAR method in the field of finance of Turkey is set to the mandatory in new Banking Law which came into force in 1999, with the risk management and internal control systems.

In determining risk measurement methods; the return on the portfolio, returns of financial assets composing of portfolio and direct dependence are between most important factors. The biggest advantage of VAR is expressing the different positions in a single monetary value by taking into account the dependence usually expressed in correlations between risk factors.

The VAR values are meaningless without time zone and confidence levels. Because the investors trade with active portfolios, financial companies prefer a typical day time period, and institutional investors and non-financial corporations prefer long periods (Linsmeier J. & Pearson N.D., 1996). Because confidence level selection can be determined depending on the purpose while assessing capital requirements, managers who do not like taking risk can opt for a higher level of confidence.

Symbolically, the VAR calculation is expressed by the formula $VaR_{\alpha} = (\mu - Z_{\alpha} * \sigma) * W$. Here, α reflects the chosen confidence level, Z_{α} reflects corresponding to the value of α probability of the standard normal distribution, μ the portfolio average returns, σ risk of returns, and W reflects the value of the initial portfolio (Jorion, 2001).

Accordingly, when the initial portfolio value is 1 million, the annual return of the portfolio is 15%, the risk of the portfolio is 10% and 99% confidence level, 25-day VAR calculation gives the following result:

$$VaR_{99\%} = (15\% - 2.33 * 10\% * \sqrt{25/250}) * 1M = \text{₺ } 76,318$$

A year is assumed to be 250 trading days in the square root in the formula. According to this result, the likelihood of a loss lower than 7.63% is under one percent level. As it can be seen under the assumption of normality, the VAR calculation is very simple.

2.1.1. Value at risk approaches

In Value at Risk calculation, when parametric models are based on the statistical parameters of the risk distribution factors, the non-parametric models are divided into two as historical and simulation model (Ammann M. & Reich C., 2001). When price movements do not fit the normal distribution, methods based on simulation give healthy results because of the difficulty in VAR calculation methods and the unique possibility of expected changes for portfolios with different return distributions (Bolgün K. & Akcay M., 2005). In this section, the

three most common methods for calculating VAR, historical simulation, variance-covariance approach, and Monte Carlo simulation are summarized.

2.1.2. Historical simulation

Historical simulation is a simple, a theoretical approach that requires relatively few assumptions about the statistical distributions of the underlying market factors. In this approach, the VAR for a portfolio is estimated by creating a hypothetical time series of returns on that portfolio, obtained by running the portfolio through actual historical data and computing the changes that would have occurred in each period. Historical simulation approach estimates VAR and other risk measures using past observations of return and loss data. This approach is realized by tracking the history of price changes for all portfolio components and applying exactly the same movements to the underlyings for the target period. Historical simulation method is widely used, which can be explained by its easy implementation and fast calculations. Another advantage of this approach is its compliance to the regulators requirements.

Historical Simulation has obvious weaknesses. First, this method does not rest upon a probabilistic setup. Second, historical data may not be an adequate indication for the current economic agenda. Finally third, this method requires immense amount of data which is often not available.

2.1.3. Variance-covariance method

Variance-covariance method is an approach that has the benefit of simplicity but is limited by the difficulties associated with deriving probability distributions. In variance-covariance method of estimation of VAR, it assumes the returns (X) are normally distributed. It requires that we estimate first two moments i.e. mean μ and standard deviation σ which completely describe the normal distribution Z_α . Therefore, 99% $RMD_\alpha = (\mu - Z_\alpha * \sigma) * W$ i.e. $\mu - 2.33 * \sigma$ (where $\Pr((X - \mu) / \sigma < Z_\alpha) = .01$) (Jorion, 2001).

In the parametric methods used in VAR calculations, it is assumed that the financial asset income and portfolio risk have a normal distribution, and they are in a linear relation with the risk factors. In these methods, basic parameters consisting of standard deviations gathered from the past return series of the portfolio and the correlation are calculated to calculate variance and covariance matrix and the expected losses can be predicted by calculating the VAR of portfolio according to this. Different assumptions regarding the return distribution changes the VAR formula. That is to say, when return distribution is assumed log-normal with the same parameters of the sample above (15% average and 10% risk);

$$VaR_{99\%} = \left(15\% - \frac{10^2\%}{2} - 2.33 * 10\% * \sqrt{\frac{25}{250}} \right) * 1M = 71.318$$

a respectively lower VAR value is calculated. A similar logic works when a different probability model is assumed for return distribution. In this situation, the parameters of the assumed model are found with mostly likelihood estimation method.

2.1.4. Monte Carlo simulation

In modern financial risk management, one fundamental quantity of interest is value at risk (VAR). It denotes the amount by which a portfolio of financial assets might fall at the most with a given probability during a particular time horizon. For the sake of simplification we will only consider time horizons of one week length in this thesis. This covers portfolios that include assets that can be sold in one week. The discussions would have to be modified slightly if we were interested in longer time horizons.

Monte Carlo simulation method is used in VAR calculations of the complex portfolios including the financial assets having a nonlinear return structure like options (Morgan, 1996). Monte Carlo simulation is similar to the historical simulation; when historical simulation uses the real data to create

estimation portfolio return and loss, Monte Carlo simulation uses the unreal rational values by choosing a statistical distribution reflection the possible changes in the prices (Duman, 2000).

We present general information on how to compute the VAR of a portfolio using Monte Carlo simulation. The value of the portfolio at present time t will be denoted by V_t . Let us assume that V_t depends on n risk factors such as (relative) interest rates, foreign exchange (FX) rates, share prices, etc. Then a Monte Carlo computation of the VAR would consist of the following steps:

1. Choose the level of confidence $1 - \alpha$ to which the VAR refers.
2. Simulate the evolution of the risk factors from time t to time $t+1$ with appropriate marginal distributions and an appropriate joint distribution that describe the behaviour of the risk factors. The number m of these n -tuples has to be large enough (typically $m = O(1000)$) to obtain sufficient statistics in step 5.
3. Calculate the m different values of the portfolio at time $t + 1$ using the values of the simulated n -tuples of the risk factors. Let us denote these values by $P_{t+1,1}, P_{t+1,2}, \dots, P_{t+1,m}$.
4. Calculate the simulated returns and losses, i.e., the differences between the simulated future portfolio values and the present portfolio value, $\Delta P_i = P_{t+1,i} - P_t$ for $i = 1, \dots, m$.
5. Ignore the fraction of the α worst changes ΔP_i . The minimum of the remaining ΔP_i 's is then the VAR of the portfolio at time t . It will be denoted by $\text{VAR}(\alpha, t, t + 1)$. As soon as the time evolves from t to $t+1$, the real value of the (unchanged) portfolio changes from P_t to P_{t+1} . With this data at hand, one can backtest $\text{VAR}(\alpha, t, t+1)$ by comparing it with $\Delta P_i = P_{t+1,i} - P_t$.

The power of Monte Carlo simulation is due to not making normality estimation for the income. Even the estimations of the predictions are made by using the historical data, subjective data and other information can be easily

implemented to the system. On the other hand, wrong estimations regarding the pricing models and basic stochastic durations can cause to calculate VAR lower or higher than it really is.

2.2. Backtest Methods

To measure the performance of VAR techniques which is used as a tool for standard risk measurement by both financial institutions and non-financial institutions today, the backtest methods may be used. Therefore, the approval of the model is performed by to subject it to VARious tests and evaluation of results.

A backtest compares the VAR model values calculated with realized gains and loss values of portfolios and tests the accuracy of the VAR model. Then exception days which the value of the selected confidence level that VAR model proposed is higher than the daily loss are counted. Also, such tests are known as termless coverage tests (Jorion, 2001). In particular, to measure the performance of risk measurement models that they use, banks have to determine the deviation number by comparing portfolio returns and risk measurement models with value figures at estimated daily risk.

In this section, backtest methods used for VAR models; Pearson Chi-Square (χ^2) goodness of fit test is summarized.

2.2.1. Pearson (χ^2) goodness of fit test

Pearson's chi-square test was developed by the British statistician Karl Pearson in the 1900.

Chi-square goodness of fit test of VAR models are based on the differences between observed and expected frequencies of the data.

Consider a sample of independent observations of size n with unknown F (y, θ), θ is the parameter vector, y_1, y_2, \dots, y_n an example of sample, connected

in the form of an example of Labour was formed independent observations is assumed that $F(y)$, coincidence that the true but unknown cumulative distribution function and the origin of (Y) the theoretical cumulative normal distribution function to describe;

$$H_0: F(y, \theta) = F_0(y)$$

$$H_1: F(y, \theta) \neq F_0(y)$$

There is in the form of hypotheses to be tested.

If y_i observations, the sample σ and μ standard deviation and mean

$$x_i = \frac{y_i}{\sigma}$$

return the form, F_0 , the asymptotic $N(0,1)$ distributed. In this case, statistic test,

$$\chi^2 = \sum_{i=1}^n \frac{(n_i - np_{i0})^2}{np_{i0}}, \quad i = 1, 2, \dots, k$$

is obtained.

χ^2 is Pearson's cumulative test statistic and computed under the H_0 hypothesis of asymptotic value χ^2 chi-square distribution of degree of freedom (Dong and Giles, 2004). p_{0i} , i. the expected probability of observations per class, np_{i0} , expected frequencies, k , n the number of clusters in the data, and n_i , refers to the observed frequencies.

3. COPULA

Traditional representations of multivariate distributions assume that all random variables have the same marginal distribution. In this representation, the dependence structure between the variables are expressed by the correlation coefficient. This approach may fall short of the dependency structures of investment instruments. The purpose of the analysts making financial modeling is providing clearer results of VAR calculations by specifying a joint distribution with well-known functional forms. When these functional forms are connected with the help of copula, non-linear cross-dependencies between asset returns, thick tail, and even unusual effects can be modelled.

Because copula can capture dependency structures regardless of the forms of the marginals, it provides great flexibility for the covariance structure modeling, accordingly determining the dependency structures forming the portfolio in VAR calculations. As we will see, the main advantage of copula functions is that they enable us to tackle the specification problem of marginal univariate distributions separately from the specification of market co-movement and dependence. Using a copula to build multivariate distributions is a flexible and powerful technique, because it separates choice of dependence from choice of marginals on which no restrictions are placed.

There are many resources that can be used about the theoretical background and characteristics of copulas. For example, Nelsen (2006) described mathematical properties and derivations of copulas in general. In the same resource, simulation algorithms of multivariate distributions produced by using copula are also compiled. In the literature, Embrechts, Lindskog and McNeil (2003) provides example applications of general copula in the finance field. In addition, Cherubini, Luciano and Vecchiato (2004) examines the use of a copula for capital portfolio approach. They also commented about the time-varying correlation in the same study. Li (2000) arranged the correlation structure of financial assets with the Gaussian copula. Name of Gaussian copula stems from determining the variables of dependence structure as well as multivariate normal distribution with binary correlation parameter. However, Li(2000) didn't use the normal distribution for marginal distribution of assets. Gaussian and Frank copula

can not be used for modeling dependency structures of extreme values of assets; for this purpose Gumbel copula for the right tail and the Cook - Johnson copula for left tail are used. Cherubini and Luciano (2001) estimated VAR by using the Archimedean copula and the historical data constructed from the marginal distributions. Fortin and Kuzmics (2002) used a convex linear combination of copulas for VAR estimation of a portfolio of FSTE and DAX exchange indices. Cherubini and Luciano (2001) used Archimedean copula to estimate the marginal distributions and VAR using the historical empirical distribution, Meneguzzo and Vecchiato (2002) used copula for risk modeling of credit derivatives. McNeil, AJ, Frey, R., and Embrechts (2005) provides comprehensive information for theoretical concepts and modeling techniques of quantitative risk management. This book is a practical tool for financial risk analysts, actuaries, regulators, or students of quantitative finance.

The mixture copula is usually used as a linear combination of copula for different correlation parameters in the field of finance and economics. It is a simple and flexible model to summarize the dependence structure. Hu (2003) used the mixture copula approach to measure pre-determined components dependence in the financial markets. The mixture copula was constructed by combining Gaussian, Gumbel and Gaussian survival copulas. Based on the Gaussian copula, calculating the dependency is achieved by combining two other copulas and the left and right tail along with traditional demonstration. In this case, because it was constructed more than one copula, mixture copula has a more flexible structure than one-dimensional copula. In addition, the mixture copula is defined as the linear combination of the maximum copula, independent copula and the minimum copula with Spearman's rho coefficient (Ouyang et al., 2009).

3.1. Basic Features Of Copulas

In this section we summarise the basic definitions that are necessary to describe fundamentals of copulas. We then present important properties of copulas that are needed in financial applications. At this point we refer to the textbook by Nelsen (2006).

Definition (Copula): A n -dimensional copula is a multivariate distribution, C , with Standard uniform marginal distributions (Nelsen, 2006).

A (n -dimensional) copula is the joint cumulative density function of a pair of variables $(U; V)$ with marginal uniform distributions on $[0,1]$. Thus a copula is a function $C: [0, 1]^n \rightarrow [0, 1]$ satisfying the following three properties:

1. $\forall u \in [0, 1] C(1,1,1, \dots, u, 1, 1, 1) = u$,
2. $\forall u_i \in [0, 1], C(u_1, \dots, u_n) = 0$ if at least one of the u_i 's equal zero,
3. C is grounded and n -increasing, i.e., the C -volume of every box whose vertices lie in $[0, 1]^n$ is positive.

For the 2-dimensional case, these properties become;

$$C(u, 1) = u \text{ and } C(1, v) = v; \forall u, v \in [0, 1],$$

$$C(u, 0) = C(0, v) = 0; \forall u, v \in [0, 1],$$

$\forall u_1, u_2, v_1, v_2$ in $[0, 1]$ such that $u_1 < u_2$ and $v_1 < v_2$, we have:

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$$

Condition 1 provides the restriction for the support of the variables and the marginal uniform distribution. Conditions 2 and 3 correspond to the existence of a nonnegative “density” function (Wei Liu, 2006).

Copula is defined in the following theorem.

Theorem (Sklar): Let H be a joint distribution function with marginals F_1, F_2, \dots, F_n . Then there exists an n -copula C such that for all x, y in \bar{R} ,

$$H(F_1, F_2, \dots, F_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n)), \text{ for some copula } C.$$

If F_1, F_2, \dots, F_n are all continuous then C is unique; otherwise C is uniquely determined on $\text{Ran}F_1 * \dots * \text{Ran}F_n$, where $\text{Ran}F_i$ is the range of the marginal i . Conversely, if C is an n -copula and F_1, F_2, \dots, F_n are distribution functions, then the function H defined above is an n -

dimensional distribution function with marginals F_1, F_2, \dots, F_n (Malevergne, Y. and D. Sornette, *Extreme Financial Risks-2006*).

It is shown in Nelsen (2006) that H has margins F_1 and F_2 that are given by

$$F_1(x_1) = H(x_1, +\infty) \text{ and } F_2(x_2) = H(+\infty, x_2)$$

with Sklar's Theorem, the use of the name "copula" becomes obvious. For a proof we direct the reader to Nelsen(2006), the standard introductory text on the subject.

Example: Let H be a joint cumulative distribution function with margins F and G. Then, there exists a copula C such that $H(x, y) = C[F(x), G(y)]$; for any x, y .

For example Nelsen(2006) Farlie Gumbel Morgenstern(FGM) distribution;

$$F_{xy}(x, y) = F_x(x)F_y(y) \left\{ 1 - \alpha(1 - F_x(x))(1 - F_y(y)) \right\} \quad -1 < \alpha < 1$$

The copula of distribution clearly exists,

$$C(u, v) = u \cdot v \{ 1 + \alpha(1-u)(1-v) \} \quad -1 < \alpha < 1 \text{ and } u, v \in I^2$$

$$C(F_x(x), F_y(y)) = F_{xy}(x, y)$$

If x and y independent random variables then the FGM copula with the same independence copula for $\alpha=0$;

$$C(u, v) = u \cdot v \text{ such that } F_{xy}(x, y) = F_x(x)F_y(y) = C(F_x(x), F_y(y)).$$

Invariance theorem: If X_1, \dots, X_n has copula C, then $Y_1 = h_1(X_1), \dots, h_n(X_n)$; $Y_1 = h_1(X_1)$; has the same copula C, if Y_i is an increasing function of X_i . (Malevergne, Y. and D. Sornette, 2006).

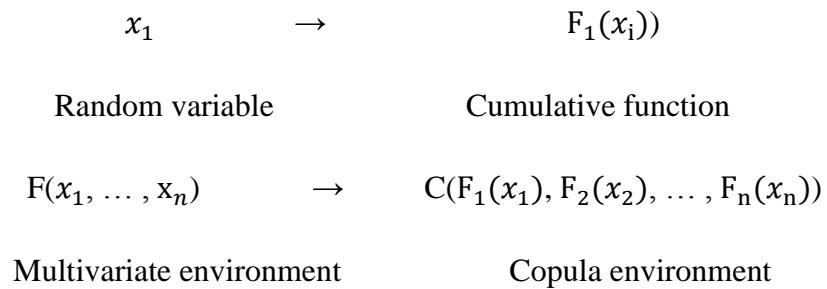
$$C(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) = C(h_1(F_1(X_1)), \dots, h_n(F_n(X_n)))$$

By the invariance theorem, it can be seen that the copula is not affected by non-linear transformations of the random variables.

Copulas are very useful models for representing multivariate distributions with arbitrary marginals. One can model the marginal distributions of a multivariate distribution and find a copula to capture the dependence between the marginals.

$$F(x_1, \dots, x_n) = P \{F(X_1 \leq x_1, \dots, X_n \leq x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n))\}$$

Once we find a copula for a multivariate distribution, we can interchange between the copula and the multivariate distribution environments. Thus working with copulas can be easier than working with multivariate distribution functions. For example if we want to simulate from a multivariate distribution, we can shift to the copula environment and simulate from the copula, then we find the corresponding random variates by transforming them back to the multivariate environment.



Due to the property that copulas are n-increasing, an upper and a lower bound can be found for any copulas. This situation explained by Frechet Bound limits.

Frechet bounds: We get $Max(u + v - 1, 0) \leq C(u, v) \leq Min(u, v)$, where the lower bound indicates the largest negative dependence and the upper bound represents the largest positive dependence(Malevergne, Y. and D. Sornette,2006).

The upper bound corresponds to the copula of two variables X and Y in a deterministic increasing (nonlinear) relationship. Similarly the lower bound provides the copula of two variables in a deterministic decreasing (nonlinear) relationship. Moreover, two variables corresponding to Frechet upper

(respectively lower) bound are said to be comonotonic (respectively countermonotonic).

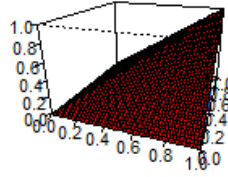
Notice that the correlation depends on the marginal distributions of the returns. The maximum value it can achieve can be computed by substituting the upper Fréchet bound in the formula:

$$\text{corr}_{\max}(X, Y) = \frac{1}{\sigma_X \sigma_Y} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\min(\phi(X), \phi(Y)) - \phi_X \phi_Y] dx dy$$

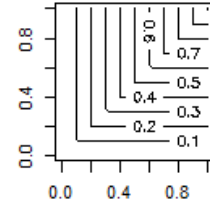
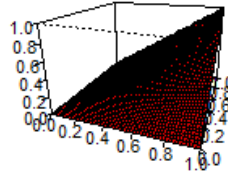
and the value corresponding to perfect negative correlation is obtained by substituting the lower bound

$$\text{corr}_{\min}(X, Y) = \frac{1}{\sigma_X \sigma_Y} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\max(\phi(X), \phi(Y) - 1) - \phi_X \phi_Y] dx dy$$

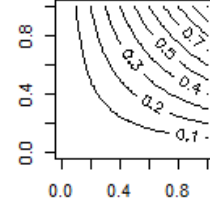
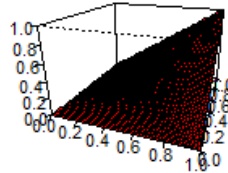
Of course everyone would expect these formulas to yield $\text{corr}_{\max} = 1$ and $\text{corr}_{\min} = -1$. The news is that this is not true in general. We may check this in the simple case of a variable z normally distributed and \mathbf{z}^2 which is obviously perfectly correlated with the first one, but has a chi-squared distribution.

Frechet-Hoeffding upper bound $W(u_1, u_2)$ (upper panel)

contour diagram upper bound

product copula $C(u_1, u_2)$ (middle panel)

contour diagram product copula

Frechet-Hoeffding lower bound $M(u_1, u_2)$ (lower panel)

contour diagram lower bound

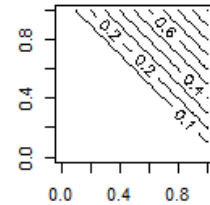


Figure 1: Frechet-Hoeffding upper bound $W(u_1, u_2)$ (upper panel), product copula $C(u_1, u_2)$ (middle panel), Frechet-Hoeffding lower bound $M(u_1, u_2)$ (lower panel).

Copulas have a very simple structure. These copulas are very important as they represent some characteristic features. Then we introduce families of copulas. A copula family is defined by a copula function that includes an additional free parameter.

4. FAMILIES OF COPULA

Copulas play an important role in the construction of multivariate density function and, as a consequence, having at one's disposal a variety of copulas can be very useful for building stochastic models having different properties that are sometimes indispensable in practice (e.g., heavy tails, asymmetries, etc.). Therefore, several investigations have been carried out concerning the construction of different families of copulas and their properties. Here, we present just a few of them, by focusing on the families that seem to be more popular in the literature (F. Durante and C. Sempi, 2009).

There are mainly two families of copulas used for financial applications: Elliptical Copulas and Archimedian Copulas. Elliptical copula was included Gaussian and student-t copula and Archimedian copulas was included Clayton, Gumbel, and Frank copulas are best known.

4.1. Elliptical Copula

The class of elliptical copula allow to model multivariate extreme events and forms of non-normal dependencies. Simulation from elliptical distributions is easy to perform. In this section we are going to present elliptical families of copulas. The most commonly used elliptical distributions are the multivariate normal and student-t distributions.

4.1.1. Normal (Gaussian) copula

The Gaussian (or normal) copula is the copula of the multivariate normal distribution. In fact, the random vector $X=(X_1, \dots, X_n)$ is multivariate normal iff: the univariate margins F_1, \dots, F_n are Gaussians; the dependence structure among the margins is described by a unique copula function C such that:

$$C_{\rho}^{Ga}(u_1, \dots, u_n) = \phi_{\rho}(F^{-1}(u_1), \dots, F^{-1}(u_n))$$

where ϕ_ρ is the standard multivariate normal density function with linear correlation matrix R and F^{-1} is the inverse of the standard univariate Gaussian density function.

Multivariate normal is commonly used in risk management applications to simulate the distribution of the n risk factors affecting the value of the trading book (market risk) or the distribution of the n systematic factors influencing the value of the credit worthiness index of a counterparty (credit risk).

If $n=2$, expression previous equation can be written as

$$C_\rho^{\text{Ga}}(u, v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right) dx dy$$

where ρ is simply the linear correlation coefficient between the two random variables .

The bivariate Gaussian copula does not have upper tail dependence if $\rho < 1$. Furthermore, since elliptical distributions are symmetric, the coefficient of upper and lower tail dependence are equal. Hence, Gaussian copulas do not even have lower tail dependence. (Malevergne, Y. and D. Sornette, 2006).

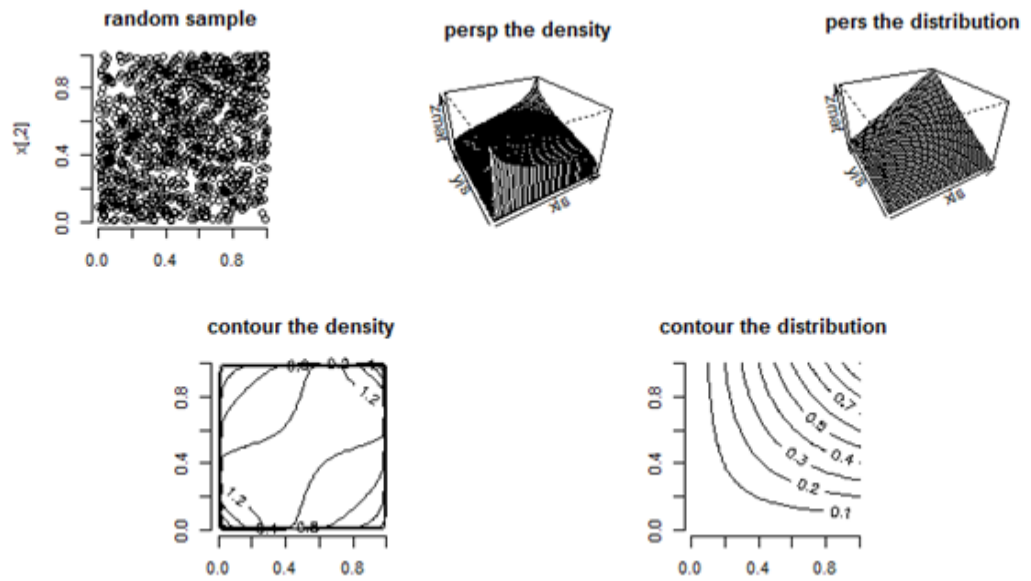


Figure 2: The graphs of normal copula

4.1.2. T copula

The copula of the multivariate t-Student distribution is the t-Student copula. Let X be a vector with an n-variate t-Student distribution with v degrees of freedom, mean vector μ (for $v > 1$) and covariance matrix $\frac{v}{v-2}\Sigma$. It can be represented in the following way:

$$X^d = \mu + \frac{\sqrt{v}}{\sqrt{X}}Z$$

where $\mu \in R^n$, $X \sim \chi_v^2$ and the random vector $Z \sim N_n(0, \Sigma)$ are independent.

The copula of vector X is the t-Student copula with v degrees of freedom. It can be analytically represented in the following way:

$$C_{\rho, v}^k(u_1, \dots, u_k) = t_{\rho, v}^n(t_v^{-1}(u_1), \dots, t_v^{-1}(u_k))$$

where $\rho_{ij} = \Sigma_{ij} / \sqrt{\Sigma_{ii}\Sigma_{jj}}$ for $i, j \in \{1, \dots, n\}$ where $t_{\rho, v}^k$ denotes the multivariate d.f. of the random vector $\sqrt{v}Y / \sqrt{X}$, where the random variable $X \sim \chi_v^2$ and the random vector Y are independent. t_v denotes the margins of $t_{\rho, v}^n$.

For $n=2$, the t-Student copula has the following analytic form:

$$= \int_{-\infty}^{t_v^{-1}(u_1)} \dots \int_{-\infty}^{t_v^{-1}(u_k)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \left\{ 1 + \frac{x^2 - 2\rho xy + y^2}{v(1-\rho^2)} \right\}^{-(v+2)/2} dx dy$$

where ρ is the linear correlation coefficient of the bivariate t-Student distribution with v degrees of freedom, if $v > 2$.

Unlike the Gaussian copula, it can be shown that the t-Student copula has upper tail dependence. As one would expect, such dependence is increasing in ρ and decreasing in v . Therefore, the t-Student copula is more suitable than the Gaussian copula to simulate events like stock market crashes or the joint default in most of the counterparties in a credit portfolio.

The description of the t-copula relies on two parameters: the correlation matrix ρ as for the normal copula, and in addition the number of degrees of freedom v . An accurate estimation of the parameter v is rather difficult and this can have an important impact on the estimated value of the shape matrix. As a consequence, the t-copula may be more difficult to calibrate than the normal copula (Malevergne, Y. and D. Sornette, 2006.)

4.2. Archimedean Copula

Every continuous, decreasing, convex function $\phi: [0, 1] \rightarrow [0, \infty)$ such that $\phi(1) = 0$ is a generator for an Archimedean copula. If furthermore $\phi(0) = +1$, then

the generator is called strict. Parametric generators give rise to families of Archimedean copulas.

An Archimedean copula can be written in the following form:

$$C(u_1, \dots, u_n) = \psi^{-1}[\psi(u_1) + \dots + \psi(u_n)]$$

For all $0 \leq u_1, \dots, u_n \leq 1$ and ψ is a function often called the generator, satisfying

- i) $\psi(1)=0$;
- ii) for all $t \in (0,1)$, $\psi'(t) < 0$ i.e ψ is decreasing;
- iii) for all $t \in (0,1)$, $\psi''(t) < 0$ i.e ψ is convex.

For Archimedean copulas, the complexity of the dependence structure between n variables, usually described by an n -dimensional function, is reduced and embedded into the function of a single variable, the generator ψ . (Malevergne, Y. and D. Sornette, 2006).

Every Archimedean copula behaves differently with respect to the tail dependence, a very important factor in dependence modelling. For these three copulas, their parameter ranges and the generators are given in Table;

name	generator $\psi(t)$	generator inverse $\psi^{-1}(t)$	parameter
Clayton	$(1+t)^{-1/\theta}$	$t^{-\theta} - 1$	$\theta \in (0, \infty)$
Frank	$-\frac{\log(1 - (1 - \exp(-\theta))\exp(-t))}{\theta}$	$-\log\left(\frac{\exp(-\theta t) - 1}{\exp(-\theta) - 1}\right)$	$\theta \in (0, \infty)$
Gumbel	$\exp(-t^{1/\theta})$	$(-\log(t))^\theta$	$\theta \in [1, \infty)$

Table 4 1 : Table with the most important generators

As it has been already mentioned there is no analytical expression for the Spearman's rho for Archimedean copula. On the other hand, Kendall's tau can be computed with this formula (Genest and MacKay, 1986).

$$\tau = 1 + 4 \int_0^1 \frac{\phi(t)}{\phi'(t)} dt$$

Different coefficients for the left and right tail dependence are desirable in many models, where complex dependencies occur. The Gumbel copula exhibits higher correlation in the right tail, whereas the Clayton copula shows tighter concentration of mass in the left tail. Quite often copulas can be flipped, i.e., instead of the original copula variable U we take $1-U$, which means that a left tail dependent copula becomes a right tail dependent copula. This works only if a given copula is asymmetric obviously. The Frank copula is the only bivariate Archimedean copula symmetric about the main diagonal and the antidiagonal of its domain, hence it has these coefficients equal (it is one of the copula implemented in Unicorn). It should be also noted that the Clayton and the Gumbel

copula in their standard forms realize only positive correlations. (Joe, 1997) and (Venter, 2002) provide a good overview of tail dependence for Archimedean copula.

Examples of bivariate Archimedean copulas are given in the following subsections::

4.2.1. Clayton copula

The generator function for the Clayton copula is given by $\psi(t) = t^{-\alpha} - 1/\alpha$ where $\alpha \in (0, \infty)$

$$C(u_1, u_2) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2)) = (u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-1/\alpha}$$

Extensions to the multivariate case are the following: Cook-Johnson copula

$$C(u_1, \dots, u_n) = \left(\sum_{j=1}^n u_j^{-\alpha} - n + 1 \right)^{-1/\alpha}$$

The Clayton copula has lower tail dependence (Nelsen, 2006).

4.2.2. Frank copula

The generator function for the Frank copula is given by $\psi(t) = -\ln \frac{e^{-\alpha t} - 1}{e^{-\alpha} - 1}$

where $\alpha \in \mathbb{R}$

$$C(u_1, u_2) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2)) = -\frac{1}{\alpha} \ln \left(1 + \frac{(e^{-\alpha u_1} - 1)(e^{-\alpha u_2} - 1)}{e^{-\alpha} - 1} \right)$$

Extensions to the multivariate case are the following: Frank copula

$$C(u_1, \dots, u_n) = \exp\{-[(-\ln u_1)^\alpha + (-\ln u_2)^\alpha + \dots + (-\ln u_n)^\alpha]^{-1/\alpha}\} \quad (\text{Nelsen, 2006}).$$

4.2.3. Gumbel copula

The generator function for the Gumbel copula is given by $\varphi(t) = (-\ln t)^\theta$ where $\theta \in (0, \infty)$

$$C(u_1, u_2) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2)) = \exp(-[(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{1/\theta})$$

Extensions to the multivariate case are the following: Gumbel-Hougaard copula

$$C(u_1, \dots, u_n) = \exp\{-[(-\ln u_1)^\alpha + (-\ln u_2)^\alpha + \dots + (-\ln u_n)^\alpha]^{-1/\alpha}\}$$

The Gumbel copula has upper tail dependence (Nelsen, 2006).

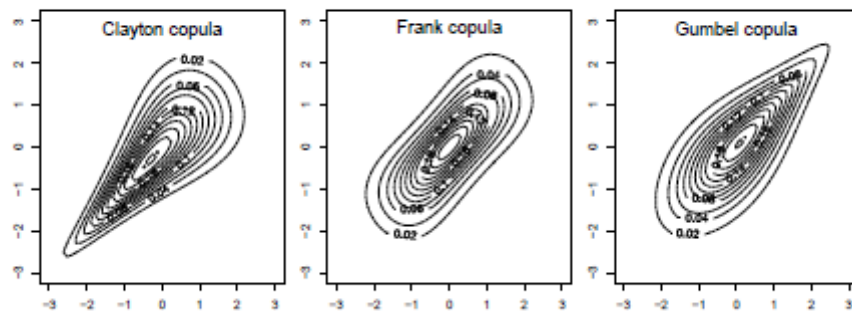


Figure 3: The Clayton and Gumbel and, Frank Copula are based on Normal margins.

4.3. Gaussian Mixture Copula

Copulas allow the overall assessment of dependency structures over distributions while preserving the characteristics of probability distributions. In the mixture copula, the unifying structure of the distributions is used to describe the dependence structure between asset returns. For example, in the mixture copula, dependency structure can be modeled using a standardized normal

distribution with two different distributions. So, while the united structure is formed, distributions should be even standard distribution [0,1].

In the following we will present our method which we call a mixture copula method. Different copulas can be combined to represent the distribution of a pair of variables. An example of mixtures of copulas is the mixture of Gaussian copula, obtained by combining two Gaussian copula. Since in this method a mixture distribution, or equivalently, a mixture copula is used to describe the dependence structure between asset returns, the mixture copula may mean the linear combination of two gaussian copula being the combination. So in this thesis, we extend his method to the case of two gaussian copula, and then we obtain the more generalized results.

Let the random vectors (U_1, V_1) and (U_2, V_2) be independent, and their joint distributions be Gaussian copulas C_1 and C_2 respectively. The random vector (U, V) is equal to (U_1, V_1) with probability p , and equal to (U_2, V_2) with probability $(1-p)$. Then the joint distribution of random vector (U, V) can be given by the Gaussian mixture copula $C_{1,2}$, which is expressed as

$$C_{1,2}(u, v) = pC_{\rho_1}(u, v) + (1 - p)C_{\rho_2}(u, v)$$

where ρ_1, ρ_2 can be Pearson linear correlation coefficient (Ouyang, Liao, Yang, 2009).

5. APPLICATION OF COPULA MODELLING

As mentioned earlier, copulas have been used in many different areas of study including financial markets. We will now use copulas to model the dependence of financial assets. We will use a Mixture Copula to model the dependence between two different financial assets indices.

5.1. Data

To model and find the best copula model for the weekly 201 existing data, VAR results have been found by applying eight different methods and the results have been backtested in order to compare the results of the tests that have been done. The closing values of the Euro and American Dollar have been downloaded from the Central Bank database. The closing values which have been downloaded in an Excel file in the CSV format have been organised according to their weekly closing values on Fridays. This has helped us to calculate the revenue that each marginal has provided in the Excel file. By subtracting the value of the first day from the second day and by dividing this result into the value of the first day, the return has been calculated.

$$\mathit{return} = \frac{P_1 - P_0}{P_0}$$

The formula above has been used to calculate the return. In this formula, “a” stands for the closing value of the dollar on the first day and “b” stands for the closing value of the dollar on the second day. After calculating the return on the Excel file, the returns have been transferred to software R to form the models. In order to develop the models, R, which is a software used for statistical purposes, has been used.

There are two marginal rates that are Dollar and Euro revenues. To import the data to the R software, the file needs to be in the CSV format. The data in the CSV file have been imported to the R software to be modelled. In order for the file to be read, with the “file” command, a name is to be given to the file in the read.csv command; if the names of the data is wanted to be seen then TRUE is

typed into the header command, if not, FALSE is to be typed and for the reagent used in the last csv file, a semi-colon is typed and the file is now imported into the programme. In order to import the file to the R software, the formula has been decided on as below:

```
read.csv(file = "dosyaad1.csv", header = TRUE, sep = ";")
```

The purpose of this thesis is to minimize the VAR value of the portfolio gained by two different assets. It has been already mentioned before that there are three different types of VAR that consist of a well-known parametric method which is variant and co-variant, a simulation which is not parametric but historical and the Monte Carlo method. Before the development of the copula models, these methods have been developed to gain the results. The methods used for the copula models are normal and t-student copulas that are in the elliptic copula category and the Clayton, Frank copula and the mixture copula models that are in the Archimedean copula category and that are thought to be the best copula models for the used data. The eight models that have been obtained and their VAR values have been backtested. The Pearson goodness of fit test has been used for backtesting.

5.1.1. Historical simulation

As known, Value at Risk (VAR) has been defined as “the loss estimated to occur in a period of time and on a confidence level”. So, if the trust value is chosen as α for the distribution of the returns and the losses in a certain period of time, the VAR occurs as $1 - \alpha$ at the end of this distribution. When the data belonging to the past are made ready in the software, the historical simulation method, which accounts to the information regarding the past, has been used.

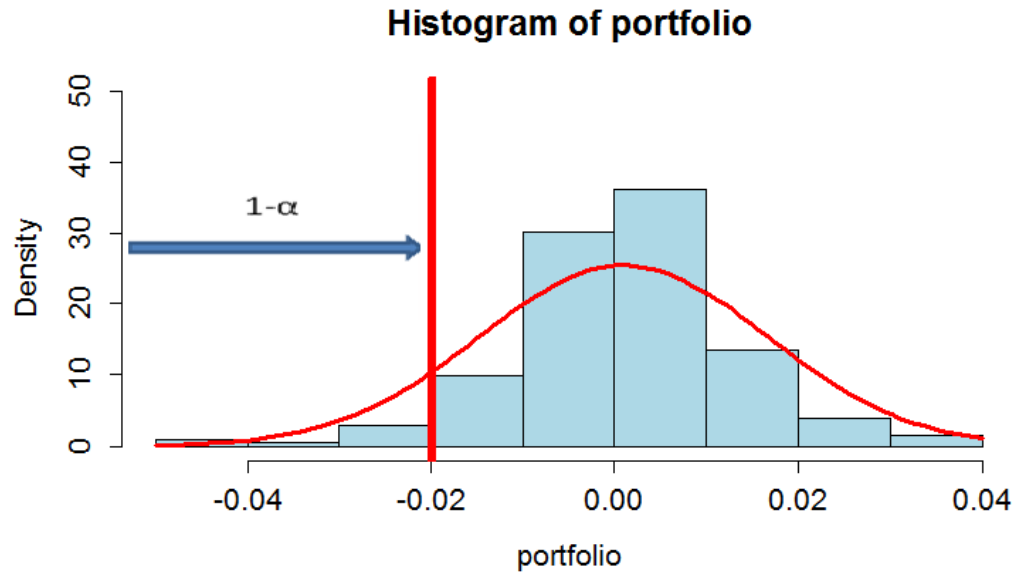


Figure 4: Histogram of the returns of the portfolio of (X1 and X2). VAR is represented with the vertical line (in red color).

This definition has also been mentioned before. As can be seen, if the asset is thought to be distributed normally, the VAR is given as a graph in Figure 1. Mathematically, VAR can be calculated with the formula mentioned below:

$$RMD_{\alpha} = \alpha * \sigma * W$$

Here, α stands for the confidence level, σ stands for the standard deviation of the portfolio value and R stands for the portfolio revenue.

To calculate the data on the R software, the data was imported to the software. The data consists of 201 observations for both currencies. In the figure 5, the correlation of the assets for 50 weeks have been demonstrated in a floating changeover graph in the regarding period. Except for the shocks in April 2005-2006 and November 2008-2009, the correlation between the two assets have decreased rapidly from 90% to 60%. Since January 2009, the correlation has been stuck in the band between 60% and 20% and does not show a distinctive tendency in this interim. The data used for the VAR calculation is the data belonging to the period between January 2009 and December 2012.

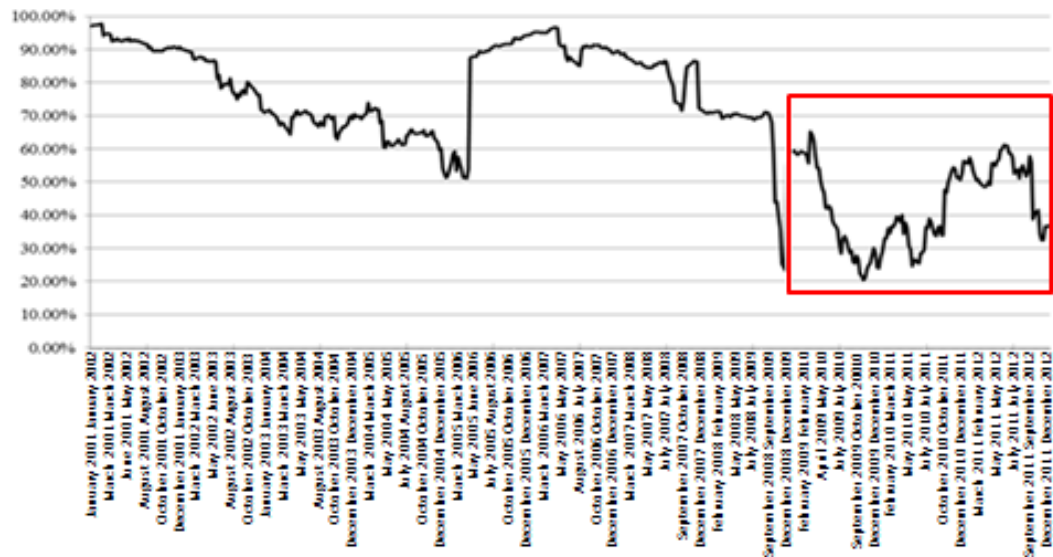


Figure 5: The graphic of the Dollar-Euro alteration correlation coefficient

After figuring out the changes in the correlations, the data has been started to be modified. If the marginals are to be X_1 and X_2 , X_1 would be designated for the dollar returns and X_2 would be designated for Euro returns and w would be designated to stand for the portfolio weight. To form the portfolio, a “ w ” would be taken from the first marginal and a “ $1-w$ ” would be taken from the second. The VAR of the portfolio has been calculated by the quantile command in the 5%, 10% and 1% shares. These percentages stand for the confidence level required to calculate the VAR values.

The VAR values have been calculated as below(R code):

```
X1=Dolar ; X2=Euro
portföy=w*X1+(1-w)*X2
port.VAR=quantile(portföy,probs= "quantile")
```

The distribution of the portfolio in the historical VAR context is as below(Figure 6):

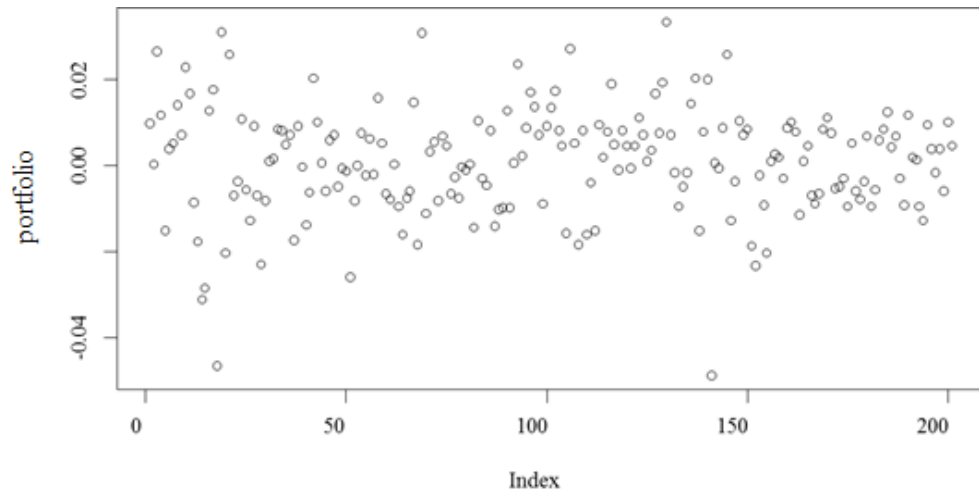


Figure 6: The distribution of the portfolio prepared for the historical data

5.1.2. Maximum likelihood estimation

In the second method, the maximum likelihood method has been used. This method lies on the basis that the description of a probability distribution function of a variable is dependent on another parameter. The probability distribution function show us how frequent the dependent variable relies on the other parameter of interest (Kay, 1993).

The probability distribution function of a random X can be considered as $f_x(\cdot, \theta)$, ($\theta \in R$) and the X_1, X_2, \dots, X_n can be considered to be a sample as an n unit taken from X 's distribution. The X_1, X_2, \dots, X_n are independent and are distributive as X . The shared probability distribution function of the X_1, X_2, \dots, X_n are

$$\begin{aligned} f_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n; \theta) \\ = f_{X_1}(x_1; \theta) f_{X_2}(x_2; \theta) \dots f_{X_n}(x_n; \theta), \quad (x_1, x_2, \dots, x_n) \in R \end{aligned}$$

For the observed x_1, x_2, \dots, x_n values of X_1, X_2, \dots, X_n , the function above is to be a function of θ .

$$L(\theta) = f_{X_1}(x_1; \theta) f_{X_2}(x_2; \theta) \dots f_{X_n}(x_n; \theta), \quad \theta \in R$$

function is named the likelihood function. The value that maximizes this function is $\hat{\theta}(x_1, x_2, \dots, x_n)$ and the $\hat{\theta}(X_1, X_2, \dots, X)$ statistic is named as the maximum likelihood estimation. Generally, in order to calculate the maximum likelihood estimation of θ , the function “ $\ln L(\theta)$ ” is maximized instead of “ $L(\theta)$ ”.

Let x be a random parameter and $f_x(\theta)$ to be a distributive function dependent on x in the θ parameter. If the x_1 and x_2 , x is to have two possible values to get, the calculation done for θ is to get one distributive values that depend on x_1 and x_2 . For each θ calculation, $f_{x_1}(\theta)$ and $f_{x_2}(\theta)$ will get different values. The x value estimated for a calculation will receive x value when the calculation is applied on probability distributive functions. For instance, the predicted x parameter for the θ calculation mentioned in the Figure 7 is x_2 because the distribution of the probability function is greater in the θ calculation.

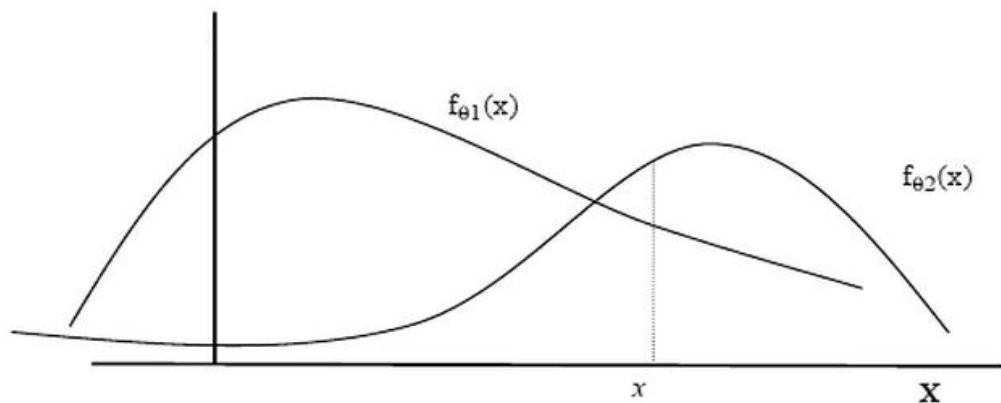


Figure 7: The x values receives for maximum likelihood estimation method and marginals

It has already been mentioned that the Dollar and Euro market indexes in hand belong to X_1 and X_2 as marginal values. Each margin's average is calculated by the “mean” command and their standard deviation is calculated by the “sd” command. In this method, in order to form the portfolio average and standard deviations have been used.

To produce the average p value in the first step;

$$w * m.X_1 + (1 - w) * m.X_2$$

w value is the weight for the average p value, $m.X_1$ is the first marginal income average and $m.X_2$ is the second marginal income average.

Deviation value is calculated in the second step.

$$w^2 * (s.X_1)^2 + (1 - w)^2 * (s.X_2)^2 + 2 * w * (1 - w) * cov(X_1, X_2)$$

w value is the weight for the average p value, $s.X_1$ is the first marginal income standard deviation, $s.X_2$ is the second marginal income standard deviation and $cov(X_1, X_2)$ states the two marginal income covariance value.

The “qnorm” command can directly calculate the value regarding the percentile. For instance, this can be done as `qnorm(0.05) = -1.644854`.

Calculated as the R value, the VAR value for maximum likelihood estimation is calculated by using the code below;

```
m.X1=mean(X1);m.X2=mean(X2);s.X1=sd(X1);s.X2=sd(X2)
mu.p=w*m.X1+(1-w)*m.X2
sigma.p<-w^2*s.X1^2+(1-w)^2*s.X2^2+2*w*(1-w)*cov(X1,X2)
VAR95<-mu.p+qnorm(VAR)*sqrt(sigma.p)
```

5.1.3. Hull-White copula

In the third method, the Hull-White copula model has been used. This method is an interesting one, suggested by Hull-White (1998) to model the multivariate distributions. This method shows that if the multivariate distributions do not distribute equally, the variance-covariance method can be used to equal the distributions and after this implementation the VAR values can be calculated by the usage of the Monte Carlo method.

Let it to be assumed that there are m numbered assets in the portfolio. An i numbered return-on-assets are to be e_i and the G_i function is to be the return-on-

assets distributions function of e_i , the turn to the normal can assumed as the e_i function in the G_i is equal to the f_i function.

$$f_i = N^{-1}[G_i(e_i)]$$

Here, N stands for the normal distributions function. While G_i is standing for the z^{th} percentile in the $[G_i(e_i)]$, f_i stands for the equal percentile in standard normal distribution. This way, the e_i that is counted as a return in the formula above, the normal standard normal is designated as f_i . The original return value can be found when the replacement is made in the equation:

$$e_i = G^{-1}[N(f_i)]$$

The first formula enables the returns to become standard normal and the second formula enables the standard normal data to be transformed into the original data. The G_i function can be turned into any form; for instance, there can be some heavy-tailed distributions or the function can be held equal to the empirical division function that result from the original data. Then, let the transformed data be in the f_i form. In this situation, the data divides as a normal multivariate and the average vector and the covariance matrix of the data can be estimated. As previously mentioned, Hull-White have offered to use the Monte Carlo method to simulate the return values of the normal standard distributions based on average and variance-covariance parameters. In return of the f_i values that have ben simulated by using the second equation, the irregular e_i values can be taken and the risk value can be predicted by the usage of the standard method. The nonparametric method that is based on the simulation seried can be held as an example for this case. This method is an uncomplicated one to use on a larger data series that consist of irregular variable distributions.

In this method, the maximum and minimum values have been decided on for the margins by assuming the margins are to be held on the X_1 ve X_2 variables .

$$\max.X1=\max(X1) ; \min.X1=\min(X1) ; \max.X2=\max(X2) ; \min.X2=\min(X2)$$

Since the margins are assumed to be modal able in the [0,1] area of the copula using the maximum and the minimum values, the data are standartized.

```
t.X1=(X1-min.X1)/(max.X1-min.X1);t.X2=(X2-
min.X2)/(max.X2-min.X2)
```

Later on, a mutual distributions function is decided on for each data. In this step, in order to prevent the maximum value to be equal to 1i the values equal to 1 have ben equalled to 0.999. If the maximum value is to be equalled to 1, in the latter steps the percentile functions returns infinite value.

```
e.X1=ecdf(t.X1)(t.X1);e.X1[e.X1==1]=0.999
e.X2=ecdf(t.X2)(t.X2);e.X2[e.X2==1]=0.999
```

The “ecdf” command that can be seen on the code helps to find the mutual distributions function. From the mutual distributions function, the percentile function can be found. The average of the percentile function is to be found. This process is done for each asset seperately.

```
f.X1=qnorm(e.X1);f.X2=qnorm(e.X2)
m.X1=mean(f.X1);m.X2=mean(f.X2)
```

The found results are the empirical results that have been f.X1 standartized. The “mean” command has been used to find the average values of the emprical results. The values are to be put in order by using the “cbind” command and the covariance is found by using the the “VAR” command.

```
X1.X2=cbind(f.X1,f.X2)
VC=VAR(X1.X2)
```

In order to form a copula after finding the covariance, a mutivariate distributions is made using the Monte Carlo method including the average and covariance forms.

```
f.X1.X2.dist=mvrnorm(MC,mu=c(m.X1,m.X2),Sigma=VC,empiri
cal=TRUE)
```

In this formula, if “mvrnorm” shows the multivariate distributions “mu” stands for average, “Sigma” stands for the covariance, “empirical” stands for empirical data, if it is not “TRUE”, the “FALSE” command is designated to get to the multivariate distributions. Then, with the help of multivariate distributions, two different probability functions are to be formed.

```
e.X1.X2.dist.1=pnorm(f.X1.X2.dist[,1],mean=m.X1,sd=sd(f.X1))*(max.X1-min.X1)+min.X1
```

```
e.X1.X2.dist.2=pnorm(f.X1.X2.dist[,2],mean=m.X2,sd=sd(f.X2))*(max.X2-min.X2)+min.X2
```

The formed two distributions functions are taken to form the portfolio by taking a w weight from the first function and a 1-w weight from the second function. For the portfolio formed, the VAR values have been found by using the “quantile” command, as previously mentioned.

```
port.X1.X2=w*e.X1.X2.dist.1+(1-w)*e.X1.X2.dist.2
port.VAR=quantile(port.X1.X2,probs=VAR)
```

Here, the “probs” command stands for the confidence level. The portfolio that has been formed by the data and the required equations have been showed in Figure 8.

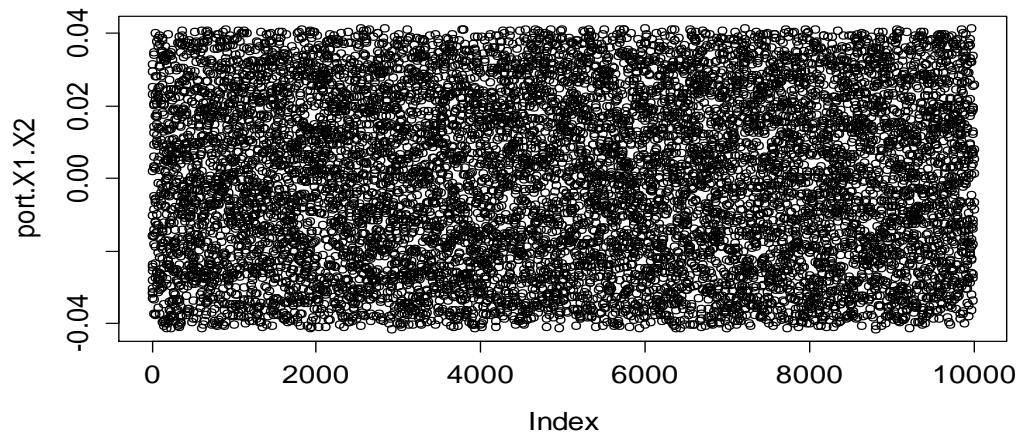


Figure 8: The distributions of the copula values that have been formed using the Hull-White method

5.1.4. Normal copula

The copulas, as mentioned previously, are studied under two titles that are the Elliptic copulas and the Archimedean copula family. The elliptic distributions are crucial distributions for the multivariate distributions because elliptic copulas have the features of the normal multivariate distributions. Moreover, multivariate distributions permit to model the extreme and irregular dependency forms. Elliptic copula is a simple form of copula that consists of elliptic distributions. To provide simulations from elliptic distributions is easy to do and the most commonly used elliptic distributions are normal and student t distributions. The “MASS” and “copula” packages that are found ready in the R software play an important role for the copulas to be modelled.

```
library("MASS")
library("copula")
```

In the code with the “library” command the packages are made ready to be used in the software. The function code found ready in the packages helps to enable the code to be transformed to an R format.

In the fourth method, the Gaussian (normal) copula method which is in the Elliptic copula has been used. This method is the simplest method to be applied on the copulas. To show the dependency form in the normal copula, the correlation parameter of the data-in-hand is used.

Correlation parameters show the direction and magnitude of the linear relationship between two random variables in statistics and probability theory. This coefficient varies between (-1) and (+1). Positive values show a direct linear relation, negative values show a reverse linear relation. If the correlation coefficient is 0, then there is no linear relation between the variables. Since the dependency between two assets are taken into account, it would be helpful during the modeling the correlation that shows deviancy from dependency. The correlation is obtained by dividing the variables 'covariance to the product of the variables' standard deviation. The correlation coefficient is calculated by:

$$\rho_{x,y} = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y}$$

“cov” is the covariance value, and σ is the standard deviation. The correlation can be found via coding by:

```
cor(X1, X2)
```

The correlation parameter is very important for a normal copula. The relations between two-variable distributions are expressed with the correlation parameter. In simplest cases, this parameter is independent from the value of the variable and the distribution changes with the changing correlation parameter.

A normal copula consists of multivariate Gaussian distributions. If ϕ is the standard normal distribution, for $\phi_{\rho,n}$ (where ρ is the correlation matrix) and the gaussian distribution is n-dimensional, the n-dimensional Gaussian Copula is expressed as:

$$C_{\rho}(u_1, \dots, u_n) = \phi_{\rho,n}(\phi^{-1}(u_1), \dots, \phi^{-1}(u_n))$$

The density function is described as:

$$c_{\rho}(u_1, \dots, u_n) = \frac{\partial C_{\rho}(u_1, \dots, u_n)}{\partial u_1 \dots \partial u_n}$$

As a result, the copula is expressed as:

$$c_{\rho}(u_1, \dots, u_n) = \frac{1}{2\pi\sqrt{\sigma_x\sigma_y(1-\rho^2)}} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x} - 2\rho_x \frac{(x-\mu_x)(y-\mu_y)}{\sqrt{\sigma_x\sigma_y}} + \frac{(y-\mu_y)^2}{\sigma_y}\right]\right)$$

The correlation parameter is ρ , the mean is μ and the standard deviation is σ . As it can be seen above, in order to calculate the copula values, the mean and standard deviation values are needed. While calculating, the d parameter that is constructed for the correlation coefficient, the d parameter is named as the copula generator. For a normal copula, the d parameter is calculated by:

$$d = \rho(X, Y) = \sin(\pi/2\rho)$$

d parameter can be found in the R program as following, with the encoded mean and standard deviations:

```
d<-sin(pi/2*a)
X1.mu<-mean(X1)
X1.sd<-sd(X1)
X2.mu<-mean(X2)
X2.sd<-sd(X2)
```

After the required mean, standard deviation and d parameter is calculated, the “copula” package in the R program is defined as above.

The copula value is named “mycop” and within the “normalcopula” command, the “dispstr” command is defined, using the d parameter. “dispstr” command is used to characterize the elliptic copula by identifying the symmetric positive matrix type. The copula is identified with the following command:

```
mycop<-normalCopula(param=d, dispstr="un")
```

After the copula is defined, the multi-variable distribution is named “myMvdc”, and under this command, with the “margins” command the marginal values are constructed normally, and with the “paramMargins” command the

means and standard deviations of the data is listed to construct a multivariate distribution. The following command is used to construct the multivariate distribution:

```
myMvdc <- mvdc(mycop, margins=c("norm", "norm"),
paramMargins=list(list(mean=X1.mu, sd=X1.sd),
list(mean=X2.mu, sd=X2.sd)))
```

After the multivariate distribution is constructed, a general copula is constructed with the Monte Carlo method.

```
mc.X1.X2=rmvdc(myMvdc, MC)
```

In this command, the desired amount of data will be assigned for Monte Carlo. For example: When MC=10000 is assigned, 10000 data can be constructed for the copula.

Afterwards, similar to the previous three methods, with the calculated results, a portfolio is constructed by taking a weight of w from the first asset value, and $1-w$ weight is taken from the second asset value, and after the portfolio is constructed, VAR value is found by the “quantile” command, as seen below:

```
por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
por.VAR<-quantile(por.X1.X2, probs=VAR)
```

The “probs” command determines the confidence level. The distribution of the portfolio after being constructed with the data is shown below(Figure 9):

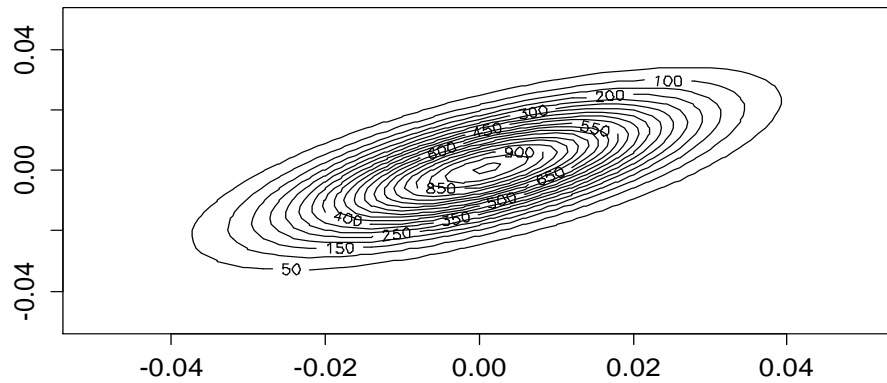


Figure 9: The dependency model of the values constructed by normal copula

5.1.5. T copula

The fifth method is the student t copula method which is in the elliptic copula class. T-copula is constructed just like the normal copula. The difference between these methods is the use of degrees of freedom in t-copula. The degree of freedom gives the numerical freedom in the change of the numbers of values while exact calculations.

In adapting the statistical model to the data, the error and residual vectors are generally shorter than the components of the general vector. The residual or error vector having a smaller dimension is called the degree of freedom of the error.

To put it simply, if the x values in the, X_1, \dots, X_n and μ random Variables with expected values:

$$\bar{X}_n = \frac{X_1 + \dots + X_n}{n}$$

After calculating the sample mean, the magnitude of $X_i - \bar{X}_n$ increases the residual of error predictions in $X_i - \mu$.

Contrary to the error term, the sum of the residuals must be 0. So they are restricted to be in the $n - 1$ dimensional space. If $n - 1$ residuals are known, the last one can be found as well. Therefore, the degree of freedom for the error term is $n - 1$.

As with the normal copula, in order to construct the t-copula the correlation parameter of the present data, the d parameter (the copula generator), the mean and standard deviations of the data are required. For coding, the required values are calculated with the same relations as in normal copula. Afterwards, the t-copula is named as “myCop”, the d parameter is coded as “param”, since there are 2 data the “dim” command is 2, to characterize the distribution the exp value is coded in “dispstr”, 4 for the degree of freedom, and to modify the degrees of freedom, the “df.fixed” command is assigned TRUE for fixed degrees of freedom and FALSE is for variable degrees of freedom.

```
myCop<-tCopula(param=d, dim = 2, dispstr = "ex", df =
4, df.fixed = FALSE)
```

After the t-copula is defined, it is named as “myMvdc”, and under the “mvdc” command the copula name, under the “margins” command the marginal values are defined normally, and with the “paramMargins” command the means and standard deviations of the data are listed to construct the multivariate distribution:

```
myMvdc=mvdc(myCop, margins=c("norm", "norm"),
paramMargins=list(list(mean=X1.mu, sd=X1.sd),
list(mean=X2.mu, sd=X2.sd)))
```

After the multivariate distribution is defined the Monte Carlo method is used to construct a general copula.

```
mc.X1.X2=rmvdc(myMvdc, MC)
```

Afterwards, similar to the previous methods, with the calculated results, a portfolio is constructed by taking a weight of w from the first asset value, and $1-w$ weight is taken from the second asset value, and after the portfolio is constructed, VAR value is found by the “quantile” command, as seen below:

```
por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
por.VAR<-quantile(por.X1.X2,probs=VAR)
```

The “probs” command determines the confidence level. The distribution of the portfolio after being constructed with the data is shown below (Figure 10):

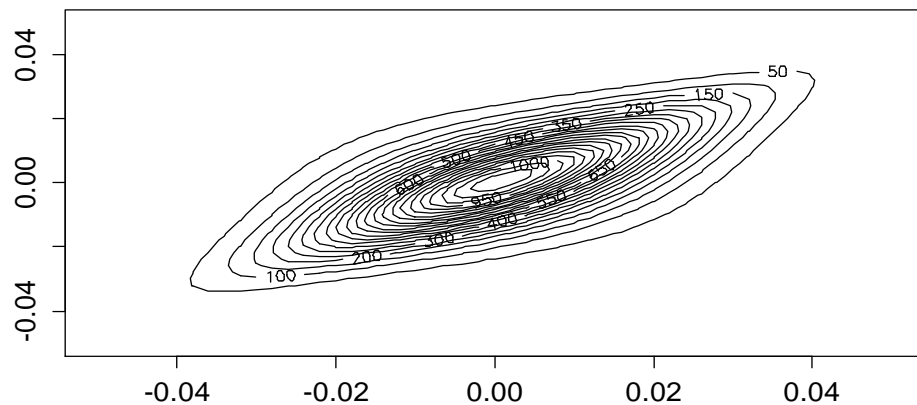


Figure 10: Dependency model of the values constructed by Student-t copula

5.1.6. Clayton copula

The Archimedean copula was named first by Ling in 1965, but before that, its validity was accepted by Sklar ve Schweizer at t norms. Many of the parametric copulas are Archimedean. The ease to construct them and the high presence of the related copula families gives many application areas to Archimedean copulas. Unlike other copulas, the Archimedean copula has not been derived from the multivariate distribution functions that use the Sklar theorem (Embrechts, 2001).

If $\phi = [0,1] \rightarrow [0, \infty]$ is defined and $\phi(1) = 0$ is a definite continuous decreasing function, and if ϕ^{-1} is the reverse of ϕ , The function of $C = [0,1]^2 \rightarrow [0,1]$ is:

$$C(u,v) = \phi^{-1}(\phi(u), \phi(v))$$

In order for C to become a copula ϕ must be convex. The copula above is defined as Archimedean Copula. ϕ function is defined as the copula generator. If $\phi(0) = \infty$, then ϕ is called the continuous generator. In this case,

$\phi[-1] = \phi^{-1}$ and $C(u, v) = \phi^{-1}(\phi(u) + \phi(v))$ are named as continuous Archimedean copula. (Nelsen, (2006;p.110))

The most frequently used Archimedean copulas are Clayton copula and Frank copula. In this thesis, the Clayton copula was used as the sixth method. Within the Clayton copula the d parameter is used, as with elliptical copula. For a data set of two variables, the d parameter is found by the following relation with the help of the correlation parameter:

$$d = \frac{2 * corr}{(1 - corr)}$$

“corr”, shows the correlation value. As a code:

```
a<-cor(X1,X2,,method="kendall")
d=2*a/(1-a)
```

Here, the correlation parameter was calculated as the kendall tau parameter. The reason is that the structure of the data was not assumed as a normal distribution. The normal correlation that was obtained does not fulfill the appropriate parameter, but the kendall tau parameter fulfills the desired structure. Similar to the elliptical copula, in the Clayton copula the mean and standart deviations are calculated for each asset.

```
X1.mu<-mean(X1)
X1.sd<-sd(X1)
```



```
X2.mu<-mean(X2)
X2.sd<-sd(X2)
```

Clayton copula was constructed by naming the command “myCop.clayton”. Within the “archmCopula” command, “clayton” command was assigned to “family” command, the value 2 was assigned to the “dim” command- as there were 2 data- and to the “param” command the d parameter for the Clayton copula was assigned.

```
myCop.clayton=archmCopula(family = "clayton", dim = 2,
param = d)
```

After the Clayton copula was defined, it was named as “myMvdc1”. Within the “mvdc” command, “myCop.clayton” was assigned in the “copula” command. Since the values in the “margin” command was assumed to have a normal distribution, the “norm” expression was assigned to each asset, and in the “paramMargins” command, the means and standard deviations of the assets are listed to construct a multivariate distribution.

```
myMvdc1=mvdc(copula = myCop.clayton, margins=c("norm",
"norm"), paramMargins=list(list(mean=X1.mu, sd=X1.sd),
list(mean=X2.mu, sd=X2.sd)))
```

After the multivariate distribution is defined, as in the elliptical copula, a general copula is constructed with the Monte Carlo method.

```
mc.X1.X2=rmvdc(myMvdc1, MC)
```

As in elliptical copula, a data amount will be assigned for the Monte Carlo. Afterwards, similar to the previous four methods, with the calculated results, a portfolio is constructed by taking a weight of w from the first asset value, and 1-w weight is taken from the second asset value, and after the portfolio is constructed, VAR value is found by the “quantile” command, as seen below:

```
por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
por.VAR<-quantile(por.X1.X2, probs=VAR)
```

The “probs” command determines the confidence level.

In Clayton copula, which is an asymmetrical archimedean copula, the dependency of the positive tail dependency is higher than that of the negative tail dependency. The distribution of the portfolio after being constructed with the data is shown below (Figure 11):

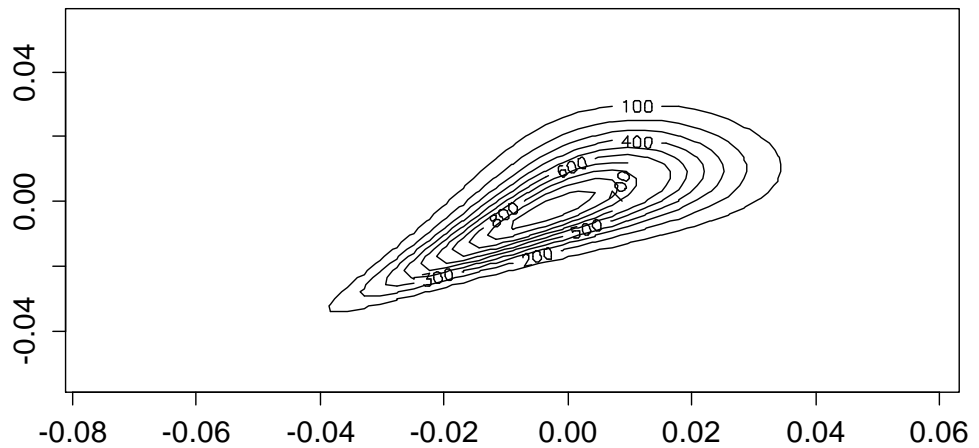


Figure 11: Dependency model of the values constructed by Clayton copula

5.1.7. Frank copula

Frank copulas are absolute Archimedean copulas. In this thesis, Frank copulas were used as the seventh copula. Frank copula has a unique d parameter of its own. For a dataset consisting of two variables, the d parameter, as in the Clayton copula, is calculated with the help of the correlation parameter.

$$d = 1 - \frac{\text{corr}}{4}$$

“corr”, expresses the correlation value. As a code:

```
a<-cor(X1,X2,,method="kendall")
d=2*a/(1-a)
```

As seen above, the correlation parameter, as in Clayton copula, was obtained with kendall tau parameter. Once again, for the Frank copula, the means and standard deviations of the assets are calculated.

```
X1.mu<-mean(X1)
X1.sd<-sd(X1)
X2.mu<-mean(X2)
X2.sd<-sd(X2)
```

Frank copula is named as “myCop.frank”. Within the “archmCopula” command, “frank” command was assigned to “family” command, the value 2 was assigned to the “dim” command-as there were 2 data- and to the “param” command the d parameter for the Frank copula was assigned.

```
myCop.frank <- archmCopula(family = "frank", dim = 2,
param = d)
```

After the Frank copula was defined, it was named as “myMvdc5”. Within the “mvdc” command, “myCop.frank” was assigned in the “copula” command. Since the values in the “margin” command was assumed to have a normal distribution, the “norm” expression was assigned to each asset, and in the “paramMargins” command, the means and standard deviations of the assets are listed to construct a multivariate distribution.

```
myMvdc5 <- mvdc(copula = myCop.frank, margins =
c("norm", "norm"),

paramMargins=list(list(mean=X1.mu, sd=X1.sd),

list(mean=X2.mu, sd=X2.sd)))
```

After the multivariate distribution was defined, a general copula was constructed with Monte Carlo, similar to the Clayton copula.

```
mc.X1.X2=rmvdc(myMvdc5, MC)
```

As in elliptical copula, a data amount will be assigned for the Monte Carlo. Afterwards, similar to the previous four methods, with the calculated results, a portfolio is constructed by taking a weight of w from the first asset value, and $1-w$ weight is taken from the second asset value, and after the portfolio is constructed, VAR value is found by the “quantile” command, as seen below:

```
por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
por.VAR<-quantile(por.X1.X2,probs=VAR)
```

The “probs” command determines the confidence level.

Frank copula, which is an Archimedean copula, is the only Archimedean copula that verifies the $C(u, v) = \hat{C}(u, v)$ for radial symmetry (Alhan, 2008).

If radial symmetry, for an X random variable, has a continuous F distribution, the symmetry can also be shown as follows:

$$F(a + x) = \bar{F}(a - x)$$

F is not absolutely continuous; the relation above is only applicable in the points of continuity.

Let X and Y be random variables, H be joint distribution function, and F and G be continuous random variables with marginal distribution functions. Let point (a, b) be any point. If and only if for all (x, y) points

$$H(a + x, b + y) = H(a - x, b - y)$$

(X, Y) is *radially symmetric* around point (a, b) .

The term *radial* means that the points $(a + x, b + y)$ and $(a - x, b - y)$ are on the opposite sides of point (a, b) (Nelsen, 2006).

The distribution of the portfolio after being constructed with the data is shown below (Figure 12):

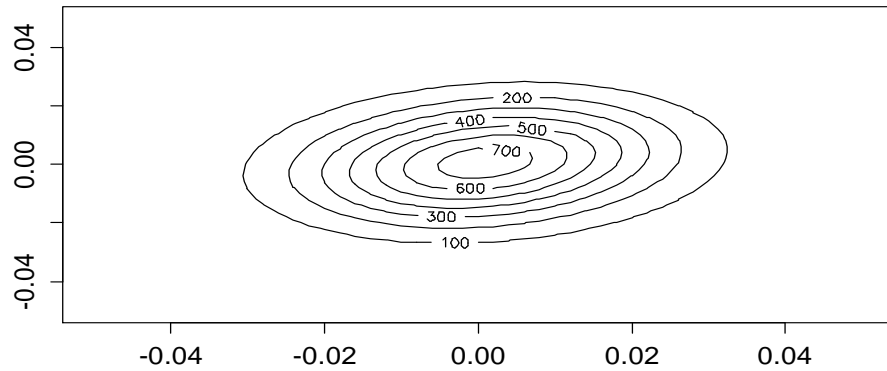


Figure 12: The dependency model of the values constructed by Frank copula

5.1.8. Mixture copula

The data used in this thesis has both negative and positive correlations, as seen on the double peaks of the figure below. For the mixture copula, which is the last method, the normal copula method was used, as it was the most appropriate copula model for the available data. The mixture copula method was constructed with the help of the “copula” package, with two normal copulas.

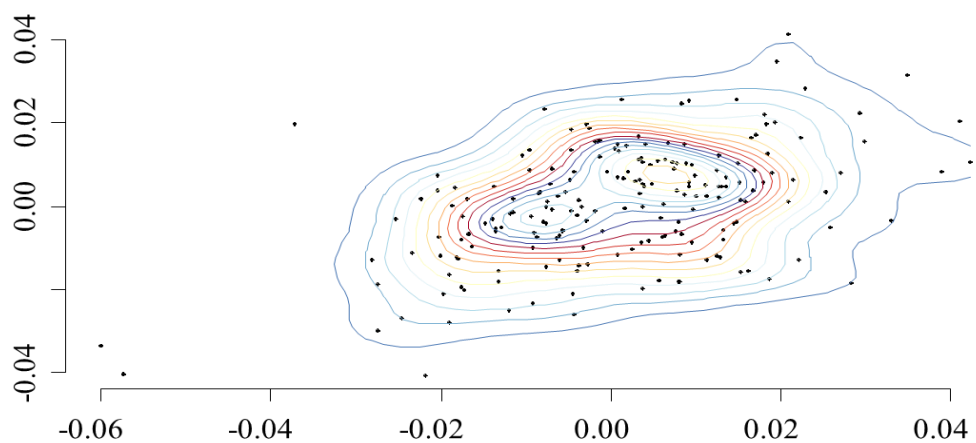


Figure 13: A distribution graph of US Dollar and Euro returns

In order to apply the mixture copula method and to find the positive and negative correlation values, the data has to be split. For this reason, the breakpoint was calculated. In the data that was divided, the kendall tau coefficient was calculated for the positive side. The reason for using the kendall tau as the correlation coefficient is to show the distribution as a normal distribution.

```
cordiff.finder=function(X1,X2,w){
  rr=0;aaa=0
  for (i in 5:(length(X1)-5)){
    a=cor(w*X1[1:i],(1-w)*X2[1:i],method="kendall")
    b=cor(w*X1[(i+1):length(X1)],(1-
w)*X2[(i+1):length(X1)],method="kendall")
    if (abs(a-b)>rr) {rr=abs(a-b)
                      aaa=i}
  }
  return(aaa)
}
```

As seen in the code, the breakpoint was named as “cordiff.finder” and was turned into a function for a direct calculation with the data. First, in order to calculate within the function, the asset value were defined as X_1, X_2 and the weight value to be taken from them were defined as w . In order to find the positive and negative correlation values that are greater than or less than zero, the “rr” value was equalized to zero. This will enable *for* loop that was used to equalize the “aaa” to zero and to assign calculated values to this variable. As seen above, the correlation parameters for a and b values in the *for* loop was calculated as the kendall tau correlation coefficient. In a variable, every asset began with the first 5 values, the assets were calculated with *for* loop until 5 values remained. For the b value, the calculation began with the first 6 values, and it was complete once the 201st value was scanned and calculated, as in value a. To prevent the case of the a and b correlation coefficients being negative, they were assigned to the “aaa” variable if the absolute values were greater than zero. The “aaa” variable which was found with *for* loop was taken as the breakpoint.


```

#MC 1
mydraws1=rcopula(copula=myCop1,n=MC/2)
qq1=qnorm(mydraws1)
mydraws1[,1]=qq1[,1]*sd(X[,1])+mean(X[,1])
mydraws1[,2]=qq1[,2]*sd(X[,2])+mean(X[,2])
}
#MC1+MC2
if(length(mydraws1)==1){mydraws=mydraws2}
if(length(mydraws2)==1){mydraws=mydraws1}

if(length(mydraws1)>1&length(mydraws2)>1){mydraws=rbind
(mydraws1,mydraws2)}
my.port=w*mydraws[,1]+(1-w)*mydraws[,2]
my.VAR=quantile(my.port,VAR)
}

```

The first copula is constructed by assigning the values in the X variable which are greater than zero. The most important parameter that determines the dependency structure is the correlation, and at the positive side, the Kendall Tau correlation was calculated. As discussed before, the reason to use the Kendall Tau parameter is to make the distribution appear as normal. In order to construct a normal copula, the d parameter, which is the generator, was calculated based on the data change, and the copula was restructured. After calculating the d parameter, the normal copula is defined- as in the fourth method- and a multivariate distribution was constructed. Since “mydraws1” variable was defined before, a general copula that was constructed in the previous step was used with the normal copula by assigning 5000 data. The “mydraws1” variable is the first normal copula values that were constructed from 5000 data.

In the normal copula that was found, the percentage function values were calculated with the “qnorm” function. The first value taken from the percentage was multiplied with the standard deviation and added to the mean and was

assigned to “mydraws1[,1]” variable, and the second data was assigned with the similar method to the variable “mydraws1[,2]” to construct the first copula.

The second copula is constructed by assigning the values in the Y variable which are greater than zero. As with the first copula, the second copula was calculated with the Kendall Tau correlation parameter. The d parameter was recalculated based on the change in variables .

As in the first copula, after the d parameter is calculated, the normal copula is defined, and a multivariate distribution was constructed for the normal copula. Since the “mydraws2” variable was predefined like “mydraws1” variable, the general copula that will be used with the normal copula constructed in the previous step, and 5000 data will constructed to be assigned to this variable. The “mydraws2” variable is the second normal copula values constructed from 5000 data.

In the normal copula that was found, the percentage function values were calculated with the “qnorm” function. The first value taken from the percentage was multiplied with the standard deviation and added to the mean and was assigned to “mydraws2[,1]” variable, and the second data was assigned with the similar method to the variable “mydraw21[,2]” to construct the first copula.

The two copulas that were found were cross-examined in 3 conditions to construct a portfolio. The mixture copula was named “mydraws” and the conditions for “mydraws1” and “mydraws2” copula conditions were determined. In the first condition, the first copula value of the “mydraws1” variable length was equalized to 1 and “mydraws” variable was equalized to the “mydraws2” copula values. In the second condition, the first copula value of the “mydraws2” variable length was equalized to 1 and “mydraws” variable was equalized to the “mydraws1” copula values. In the third condition, if the length of the copula values of the first and second copulas were greater than 1, the copula values of “mydraws” are taken as the “mydraws1” and “mydraws2” copula values. After

the conditions were determined, the copula models that were designed separately, and were combined as follows:

$$C_{1,2}(u, v) = wC_{\rho_1}(u, v) + (1 - w) C_{\rho_2}(u, v)$$

Here the $C_{\rho_1}(u, v)$ value is the copula values of “mydraws1” copula values, $C_{\rho_2}(u, v)$ value is the copula values of “mydraws2” copula values. As with the other methods, the VAR values are found with the “quantile” command.

Distribution of data and the portfolio constructed from the necessary calculations is illustrated with the drawing given below.

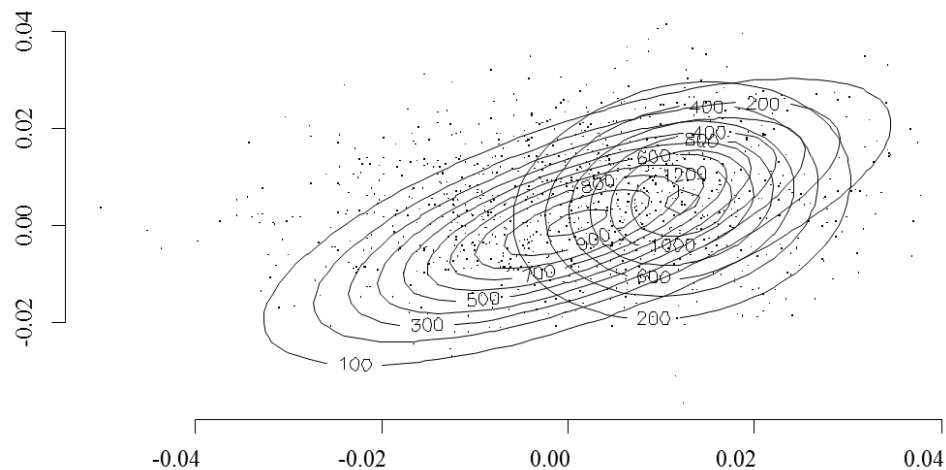


Figure 14: The aggregate model of the values constructed with mixture copula

After the models were defined separately, they were converted into a function in order to be used in the backtests easily. The next step was coded for the backtest.

```
backk <- function (alfa,ww,portw){
set.seed(1)
  DATA=aa[,2:3]
  WS=ww;dd=alfa
  BT.HW=0;BT.Hist=0;BT.normalcopula=0;BT.tcopula=0
```

```

BT.MLE=0;BT.mixture=0;BT.frankcopula=0;BT.claytoncopula
=0
  for (i in 1:(dim(DATA)[1]-WS)){
    X1.X2=DATA[i:(i+WS-1),]
    BT.X1.X2=DATA[(i+WS),]
    BT=portw*BT.X1.X2[1]+(1-
portw)*BT.X1.X2[2]#apply(BT.X1.X2,1,mean)
HW.VAR=HWTransCop(X1=X1.X2[,1],X2=X1.X2[,2],VAR=dd,w=po
rtw)
Hist.VAR=Historical(X1=X1.X2[,1],X2=X1.X2[,2],VAR=dd,w=
portw)
MLE.VAR=MLE(X1=X1.X2[,1],X2=X1.X2[,2],VAR=dd,w=portw)
mixture.cop.VAR=mixture.cop(X1=X1.X2[,1],X2=X1.X2[,2],V
AR=dd,w=portw)
NormalCopula.VAR=normalcopula(X1=X1.X2[,1],X2=X1.X2[,2]
,VAR=dd,w=portw)

tcopula.VAR=tcopula(X1=X1.X2[,1],X2=X1.X2[,2],VAR=dd,w=
portw)

frankcopula.VAR=frankcopula(X1=X1.X2[,1],X2=X1.X2[,2],V
AR=dd,w=portw)

claytoncopula.VAR=claytoncopula(X1=X1.X2[,1],X2=X1.X2[,
2],VAR=dd,w=portw)
  BT.HW.Temp=(BT<HW.VAR)
  BT.Hist.Temp=(BT<Hist.VAR)
  BT.MLE.Temp=(BT<MLE.VAR)
  BT.mixture.cop.Temp=(BT<mixture.cop.VAR)
  BT.normalcopula.Temp=(BT<NormalCopula.VAR)

```

```

BT.tcopula.Temp=(BT<tcopula.VAR)
BT.frankcopula.Temp=(BT<frankcopula.VAR)
BT.claytoncopula.Temp=(BT<claytoncopula.VAR)
BT.HW=c(BT.HW,BT.HW.Temp)
BT.Hist=c(BT.Hist,BT.Hist.Temp)
BT.MLE=c(BT.MLE,BT.MLE.Temp)
BT.mixture=c(BT.mixture,BT.mixture.cop.Temp)

BT.normalcopula=c(BT.normalcopula,BT.normalcopula.Temp)
BT.tcopula=c(BT.tcopula,BT.tcopula.Temp)

BT.frankcopula=c(BT.frankcopula,BT.frankcopula.Temp)

BT.claytoncopula=c(BT.claytoncopula,BT.claytoncopula.Temp)
}
a1=sum(BT.HW)/(length(BT.HW)-1)
a2=sum(BT.Hist)/(length(BT.Hist)-1)
a3=sum(BT.MLE)/(length(BT.MLE)-1)
a4=sum(BT.mixture)/(length(BT.mixture)-1)
a5=sum(BT.normalcopula)/(length(BT.normalcopula)-1)
a6=sum(BT.tcopula)/(length(BT.tcopula)-1)
a7=sum(BT.frankcopula)/(length(BT.frankcopula)-1)
a8=sum(BT.claytoncopula)/(length(BT.claytoncopula)-1)
outp=c(a1,a2,a3,a4,a5,a6,a7,a8)
return(outp)
}

```

As a backtest, 25, 50 and 100 pieces of data is scrolled over 201 pieces of data to calculate VAR values. In this section it is written in a function again to be used in the next step and it is named as “back”. Alfa, ww and portw variables are used in a function and alfa is used for the confidence level, ww for the window to

slide, portw for determining the weight of the item. Then, the data to be used are stabilized with the `set.seed(1)` command. After the data is stabilized, the available data is transferred to the program again and the the variables to be used for the test are equalized with zero.

A for loop is constructed to slide the windows on the data, data to be used for VAR calculation, marginal income forming the portfolio exposed to the test and portfolio income expose to the test is calculated, VAR and the months lower than VAR value are calculated and assigned to a variable. The values found are summed up and the length of data is divided to minus one to get the necessary VAR values.

Lastly, 0.01,0.05,0.1percentile and VAR values found with windows of 25,50 and 0.25,0.5,0.75 weight is calculated with for loop and it is kept in the variable named `tempp` and the results are gathered.

```

alfa=c(0.01,0.05,0.1)
ww=c(25,50,100)
portw=c(0.25,0.5,0.75)
outtt=c(0,0,0,0,0,0,0,0)
for (i in 1:3){
  for (j in 1:3){
    for (k in 1:3){
      tempp=backk(alfa[k],ww[j],portw[i])
      outtt=rbind(outtt,tempp)
    } } }

```

VAR values found are given below.

Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
0,034091	0,034091	0,017045	0,017045	0,017045	0,017045	0,017045	0,017045
0,039773	0,079545	0,0625	0,0625	0,045455	0,056818	0,056818	0,056818
0,073864	0,125	0,102273	0,096591	0,096591	0,096591	0,125	0,102273
0,013245	0,013245	0,006623	0,013245	0,006623	0,006623	0,006623	0,006623
0,019868	0,059603	0,046358	0,05298	0,033113	0,039735	0,066225	0,046358
0,02649	0,112583	0,092715	0,092715	0,07947	0,07947	0,112583	0,086093
0,017045	0,039773	0,017045	0,011364	0,011364	0,011364	0,017045	0,005682
0,039773	0,079545	0,039773	0,045455	0,039773	0,039773	0,0625	0,039773
0,073864	0,113636	0,085227	0,090909	0,079545	0,085227	0,113636	0,085227
0,006623	0,013245	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623
0,019868	0,046358	0,039735	0,059603	0,013245	0,013245	0,05298	0,019868
0,039735	0,092715	0,086093	0,092715	0,066225	0,072848	0,10596	0,07947
0,017045	0,022727	0,005682	0,005682	0,005682	0,005682	0,011364	0,005682
0,034091	0,068182	0,022727	0,022727	0,017045	0,017045	0,034091	0,022727
0,0625	0,107955	0,073864	0,073864	0,068182	0,068182	0,096591	0,073864
0,006623	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623
0,013245	0,05298	0,013245	0,02649	0,006623	0,013245	0,033113	0,013245
0,039735	0,092715	0,072848	0,072848	0,066225	0,066225	0,07947	0,066225

Table 5 1 : VAR values of the models constructed from portfolio values.

6. CONCLUSION

In this study, the market risks of a hypothetical portfolio constructed by using VAR analysis methods which is a current approach of risk management of the financial items in Turkey, backtests which are thought as an inseparable part of VAR and copula model are calculated on the 90%, 95 % and 99 % confidence level belonging to 2009-2012 term on basis of 201 weeks.

The reason to use copula besides VAR methods is that they reveal the dependency structures between the random variables in statistics and they connect the distribution functions with multiple variables to their marginal distribution functions with a single variable. Additionally, copulas include normality hypothesis and they have a structure that can be used when independency between the variable can be obtained. Copula shows that marginal actions of index income can be modeled separately by presentation. In this study, a new algorithm design is made by using mixture copulas for financial risk valuation. The valuation of VAR in portfolio is related to simulation problem. A new simulation algorithm is presented while calculating financial risk in Turkey market based on copula theory and dependency.

In this thesis, it is found that total risk is less if there are risks that eliminate each other by considering the correlation between the VAR methods and the risk factors. Copula models give better results than VAR methods in terms of their characteristics. Normal, t-student, Gumbel, Frank copulas used in finance are examined and as mixture copula is thought to be more suitable the specification of the available data, this method is examined to be modeled as well. VAR values are found with these eight methods and VAR values found are compared by means of backtest.

6.1. Pearson Goodness Of Fit Test

Chi-square goodness of fit test takes the differences between data frequency expected and inspected for the statistical models as basis. VAR model calculates

the probability of inspections to accrue under a certain limit. When model predictions change in time, it may show differences between the off-limit values rate observed and the expected rate. Chi-square goodness of fit test tests the statistical significance of this difference. The data window name to be tested is put into groups given to use the Chi-square significance test for VAR models. It should be supposed that the data is divided in k number of groups and there are n numbers of inspections in each group. According to VAR model the rate of off-limit inspection is p_i^0 , so that the expected off-limit inspection number is np_i^0 . If off-limit inspection number is shown as n_i within each group,

$$\chi^2 = \sum_{i=1}^k \frac{(n_i - np_{oi})^2}{np_{oi}}, i=1,2, \dots, k$$

Chi-square test statistics is for the differences of the expected frequency by inspections. This test statistics is distributed asymptotically with a independency level (Dong and Giles, 2004).

After the available data and probabilities are calculated, the calculations are done as follow for Pearson goodness of fit test:

- 1) The inspections of the data and the expected values are calculated.
- 2) The inspected situation is subtracted from the expected situation and it is squared.
- 3) The value found by squaring is divided the value of the expected value.
- 4) Same case, according to number of observations, by applying one by one, the results obtained are collected.
- 5) The summed up values are taken as χ^2 distribution. Independency level is calculated for χ^2 distribution.
- 6) It independency level data number is n, n is mentioned as -1. As we have two pieces of data in hand, independency level is taken as 2-1=1.
- 7) The result found determines whether to refuse or not the assumed hypothesis for the available data.

As an example, exceptional and unexceptional conditions are considered for our data. This means we have exceptional and unexceptional conditions to apply the test. If the probability rate is taken as 1% for the exceptional conditions, it is 99% for the unexceptional conditions. For the exceptional condition, the

conditions inspected from the data 23, expected conditions 556 and for unexceptional conditions the conditions inspected from the data is 5.79 and unexpected conditions are 573.21, it is found as follows;

$$\frac{(23-5,79)^2}{5,79} + \frac{(556-573,21)^2}{573,21} = 51,67113$$

With 1% 51,67113 χ^2 statistics is obtained. Hypothesis for supposed sample is refused with this number.

VAR values are modeled in R program and it is obtained from VAR methods and they are obtained over copula models. The values obtained are adjusted on an excel file for their weight of portfolio, necessary windows and safety level. Portfolios in the weights of 25%, 50% and 75 %, windows sliding over the whole data, 25 and 50 pieces of data and their security levels of 90%, 95% and 99% are taken. The results obtained from R program are transferred to an excel file on R program to get VAR values. As it is seen, 18 results are obtained for each method. These results are the ones of models produced with data on R program and they are shown in the table below.

portfolio	window	alfa	Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
0,25	25	0,01	0,034091	0,034091	0,017045	0,017045	0,017045	0,017045	0,017045	0,017045
0,25	25	0,05	0,039773	0,079545	0,0625	0,0625	0,045455	0,056818	0,056818	0,056818
0,25	25	0,1	0,073864	0,125	0,102273	0,096591	0,096591	0,096591	0,125	0,102273
0,25	50	0,01	0,013245	0,013245	0,006623	0,013245	0,006623	0,006623	0,006623	0,006623
0,25	50	0,05	0,019868	0,059603	0,046358	0,05298	0,033113	0,039735	0,066225	0,046358
0,25	50	0,1	0,02649	0,112583	0,092715	0,092715	0,07947	0,07947	0,112583	0,086093
0,5	25	0,01	0,017045	0,039773	0,017045	0,011364	0,011364	0,011364	0,017045	0,005682
0,5	25	0,05	0,039773	0,079545	0,039773	0,045455	0,039773	0,039773	0,0625	0,039773
0,5	25	0,1	0,073864	0,113636	0,085227	0,090909	0,079545	0,085227	0,113636	0,085227
0,5	50	0,01	0,006623	0,013245	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623
0,5	50	0,05	0,019868	0,046358	0,039735	0,059603	0,013245	0,013245	0,05298	0,019868
0,5	50	0,1	0,039735	0,092715	0,086093	0,092715	0,066225	0,072848	0,10596	0,07947
0,75	25	0,01	0,017045	0,022727	0,005682	0,005682	0,005682	0,005682	0,011364	0,005682
0,75	25	0,05	0,034091	0,068182	0,022727	0,022727	0,017045	0,017045	0,034091	0,022727
0,75	25	0,1	0,0625	0,107955	0,073864	0,073864	0,068182	0,068182	0,096591	0,073864
0,75	50	0,01	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623
0,75	50	0,05	0,013245	0,05298	0,013245	0,02649	0,006623	0,013245	0,033113	0,013245
0,75	50	0,1	0,039735	0,092715	0,072848	0,072848	0,066225	0,066225	0,07947	0,066225

Table 6 1 : Result of R for all methods

After the results found in R program are transferred to an excel file as above, the number of exceptional days are found by using the results. Exceptional day number is found by subtracting the windows to slide from the whole data and multiplying with the obtained result on R program. To give an example;

For Hull-White method, we have 201 pieces of data. The number 176 is found by subtracting the 76 pieces of windows number from 201 pieces of data. The exceptional day number is found to be 6 by multiplying the number 176 with the number 0,034091 found with the Hull-White method on R program.

Exceptional day number= (whole data number – whole data minus the number of windows to slide)* VAR value found for Hull-White

Exceptional day number= $(201-25) * 0,034091=6$. Each value is calculated separately and the results found are shown below.

Hull-White	Historical RMD	MLE RMD	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
6	6	3	3	3	3	3	3
7	14	11	11	8	10	10	10
13	22	18	17	17	17	22	18
2	2	1	2	1	1	1	1
3	9	7	8	5	6	10	7
4	17	14	14	12	12	17	13
3	7	3	2	2	2	3	1
7	14	7	8	7	7	11	7
13	20	15	16	14	15	20	15
1	2	1	1	1	1	1	1
3	7	6	9	2	2	8	3
6	14	13	14	10	11	16	12
3	4	1	1	1	1	2	1
6	12	4	4	3	3	6	4
11	19	13	13	12	12	17	13
1	1	1	1	1	1	1	1
2	8	2	4	1	2	5	2
6	14	11	11	10	10	12	10

Table 6 2 :Exceptional date numbers for all methods

Exceptional date numbers are calculated and then expected and unexpected day numbers are detected for Pearson Chi-square goodness of fit that is used for backtest. As it known, Pearson Chi-square goodness of fit test test the accuracy of the hypothesis determined for the cumulative normal distribution function. After the expected and unexpected values are found, total expected value is found. To calculate for Hull-White;

- 1) Expected number of days is subtracted from the exceptional day number and it is squared to be divided to the number of expected number of days.
- 2) Exceptional day number is subtracted from the total expected day number and the result found is subtracted from the unexpected day number to be squared and it is divided unexpected exceptional day number.

3) The results of the section found are summed up to find Pearson Chi-square goodness of fit values.

$$\frac{(\text{Exceptional day number} - \text{expected day number})^2}{\text{expected day number}} +$$

$$\frac{(\text{Unexpected day number} - (\text{total expected day number} - \text{exceptional day number}))^2}{\text{Unexpected day number}}$$

$$\text{Hence; } \frac{(6-1,76)^2}{1,76} + \frac{(174,24-(176-6))^2}{174,24} = 10,31772268$$

The results are found as below by doing these calculations for other methods.

EXPECTED	Expect Non	Sum	Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
1,76	174,24	176	10,31772268	10,31772	0,882461	0,882461	0,882461	0,882461	0,882461	0,882461
8,8	167,2	176	0,387559809	3,23445	0,578947	0,578947	0,076555	0,172249	0,172249	0,172249
17,6	158,4	176	1,335858586	1,222222	0,010101	0,022727	0,022727	0,022727	1,222222	0,010101
1,51	149,49	151	0,16061275	0,160613	0,173992	0,160613	0,173992	0,173992	0,173992	0,173992
7,55	143,45	151	2,886371558	0,293133	0,042175	0,028233	0,906588	0,33496	0,836877	0,042175
15,1	135,9	151	9,066225166	0,265636	0,089036	0,089036	0,707138	0,707138	0,265636	0,324503
1,76	174,24	176	0,882460973	15,75849	0,882461	0,033058	0,033058	0,033058	0,882461	0,331497
8,8	167,2	176	0,387559809	3,23445	0,38756	0,076555	0,38756	0,38756	0,578947	0,38756
17,6	158,4	176	1,335858586	0,363636	0,426768	0,161616	0,818182	0,426768	0,363636	0,426768
1,51	149,49	151	0,173991571	0,160613	0,173992	0,173992	0,173992	0,173992	0,173992	0,173992
7,55	143,45	151	2,886371558	0,042175	0,33496	0,293133	4,294528	4,294528	0,028233	2,886372
15,1	135,9	151	6,093451067	0,089036	0,324503	0,089036	1,913907	1,236939	0,059603	0,707138
1,76	174,24	176	0,882460973	2,879706	0,331497	0,331497	0,331497	0,331497	0,033058	0,331497
8,8	167,2	176	0,937799043	1,22488	2,755981	2,755981	4,023923	4,023923	0,937799	2,755981
17,6	158,4	176	2,75	0,123737	1,335859	1,335859	1,979798	1,979798	0,022727	1,335859
1,51	149,49	151	0,173991571	0,173992	0,173992	0,173992	0,173992	0,173992	0,173992	0,173992
7,55	143,45	151	4,29452771	0,028233	4,294528	1,757058	5,981527	4,294528	0,906588	4,294528
15,1	135,9	151	6,093451067	0,089036	1,236939	1,236939	1,913907	1,913907	0,707138	1,913907

Table 6 3 : Result of all methods

The results found are distributed as χ^2 according to Pearson Chi-square goodness of fit test. For this reason, the values in the column above for this method are summed up. The first line in the table given below gives the total Pearson Chi-square goodness of fit values.

In the second line, right tailed probabilities of chi-square distribution are calculated for total Pearson Chi-square goodness of fit values. We need independency level to calculate χ^2 distribution. The calculated Pearson Chi-square goodness of fit values are 27 pieces as seen in the table above. For this reason, independency level is taken as 27.

- 1) Total Pearson Chi-square goodness of fit values are found.
- 2) Chi-square distribution probabilities are calculated by giving the value in the amount of the independency level to the independency level to use Chi-square distribution for Pearson Chi-square goodness of fit test.

	Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
Total chi-square value (18)	83,3692	44,60595	23,67952	16,07785	38,02961	34,15212	11,52603	26,79763
P Value	1,15E-07	0,017887	0,648028	0,951447	0,077395	0,16167	0,995904	0,474754

Table 6 4 : Goodness of Fit chi-square(1) values

The results found here include the whole data i.e. all of the portfolios constructed. As it is seen, the results that are found to be successful in the test belong to Mixture Copula and Frank Copula. According to these results, we can say that Frank Copula may give a better result than mixture copula due to its radial symmetry feature.

The results found according to the weights of the portfolios are shown in the table below. In the beginning three portfolios were constructed according to their weights. The calculations are made for the first 25% portfolio, second 50% portfolio and third 75% portfolio. As we mentioned above, as the independency levels per portfolio are 6 values of Chi-square goodness of fit, it is taken as 6. As we have applied to the whole data;

- 1) Pearson Chi-square goodness of fit total values are found for 25% portfolio.
- 2) Pearson Chi-square goodness of fit total values are found 25% portfolio.

The results are found by applying this algorithm to 50% and 75% portfolios.

portfolio		Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
25%	Total-Chi Square Values (6)	33,311303	16,4756	2,262482	2,951857	4,702743	3,580639	4,722854	2,320536
	P Value	0,0001179	0,05759	0,986605	0,966173	0,859413	0,936788	0,857768	0,985327
50%	Total-Chi Square Values (6)	22,396584	21,34356	5,255605	1,058397	11,40964	10,34126	2,088593	7,638687
	P Value	0,0077036	0,011209	0,811482	0,999291	0,248671	0,323571	0,989988	0,570927
75%	Total-Chi Square Values (6)	27,66131	6,786795	16,16143	12,0676	21,91722	20,23022	4,714584	16,8384
	P Value	0,0010864	0,659306	0,063586	0,209525	0,009145	0,016544	0,858445	0,051308

Table 6 5 : Goodness of Fit Chi-Square(1) values for all methods of P values

The results show that mixture copula is successful in 50% portfolio. If we look at the other portfolios, clayton copula is successful in 25% portfolio and frank copula is successful in 75% portfolio.

As a result, VAR calculations can be made with VAR methods and copula models. After the dependency structures are obtained, VAR values are obtained and when they are compared with the results of VAR methods, the results gathered from the dependency structures and the copula methods from the backtest give better results as seen in the table above.

In the risk management, the copulas including the possibility of true and flexible modeling of the dependency structure between the financial assets estimates the VAR with left-right tail dependency for the extreme values. The findings show that copula is a very effective method to model the dependency structure between the individual risks for risk measurement. On the other hand, mixture copula method is found to be better than the other methods. This thesis contributes to the decision period and estimations of the (risk) managers to calculate for a certain term. As VAR method has some restrictions and it cannot

provide a complete risk management by itself, copula is used as a new tool for risk measurement and a new method is developed to replace the points which VAR method lacks.

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APPENDICES

APPENDIX I

Common code;

```

library("MASS")
aa<-read.csv(file ="usdeuro.csv",header= TRUE,sep=";")
Historical <-function (X1,X2,VAR=0.05,w) {
  X1.X2=w*X1+(1-w) *X2
  port.VAR=quantile(X1.X2,probs=VAR)
}

MLE <- function (X1,X2,VAR=0.05,w) {
  m.X1=mean(X1);m.X2=mean(X2);s.X1=sd(X1);s.X2=sd(X2)
  mu.p=w*m.X1+(1-w) *m.X2
  sigma.p<-w^2*s.X1^2+((1-w)^2)*s.X2^2+2*w*(1-
w)*cov(X1,X2)
  var95<-mu.p+qnorm(VAR)*sqrt(sigma.p)
}

HWTransCop <-function (X1,X2,MC=10000,VAR=0.05,w) {
max.X1=max(X1);min.X1=min(X1);max.X2=max(X2);min.X2=min
(X2)
  t.X1=(X1-min.X1)/(max.X1-min.X1);t.X2=(X2-
min.X2)/(max.X2-min.X2)
  e.X1=ecdf(t.X1)(t.X1);e.X1[e.X1==1]=0.999
  e.X2=ecdf(t.X2)(t.X2);e.X2[e.X2==1]=0.999
  f.X1=qnorm(e.X1);f.X2=qnorm(e.X2)
  m.X1=mean(f.X1);m.X2=mean(f.X2)
  X1.X2=cbind(f.X1,f.X2)
}

```

```

VC=var(X1.X2)

f.X1.X2.dist=mvrnorm(MC,mu=c(m.X1,m.X2),Sigma=VC,empirical=TRUE)

e.X1.X2.dist.1=pnorm(f.X1.X2.dist[,1],mean=m.X1,sd=sd(f.X1))*(max.X1-min.X1)+min.X1

e.X1.X2.dist.2=pnorm(f.X1.X2.dist[,2],mean=m.X2,sd=sd(f.X2))*(max.X2-min.X2)+min.X2

port.X1.X2=w*e.X1.X2.dist.1+(1-w)*e.X1.X2.dist.2
port.VAR=quantile(port.X1.X2,probs=VAR)
}

library("copula")
normalcopula<-function(X1,X2,MC=10000,VAR=0.05,w){
  a<-cor(X1,X2)
  d<-sin(pi/2*a)
  X1.mu<-mean(X1)
  X1.sd<-sd(X1)
  X2.mu<-mean(X2)
  X2.sd<-sd(X2)
  mycop<-normalCopula(param=d,dispstr="un")

  myMvdc=mvdc(mycop,margins=c("norm","norm"),paramMargins=
list(list(mean=X1.mu,sd=X1.sd),
list(mean=X2.mu,sd=X2.sd)))
  mc.X1.X2=rmvdc(myMvdc,MC)
  por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
  por.VAR<-quantile(por.X1.X2,probs=VAR)
}

```

```

tCOPULA<-function(X1,X2,MC=10000,VAR=0.05,w){
  a<-cor(X1,X2)
  d<-sin(pi/2*a)
  X1.mu<-mean(X1)
  X1.sd<-sd(X1)
  X2.mu<-mean(X2)
  X2.sd<-sd(X2)
  myCOP<-tCOPULA(param=d, dim = 2, dispstr = "ex", df =
4, df.fixed = FALSE)
  myMvdc=mvdc(myCOP, margins=c("norm", "norm"),

paramMargins=list(list(mean=X1.mu, sd=X1.sd),

list(mean=X2.mu, sd=X2.sd)))
  mc.X1.X2=rmvdc(myMvdc,MC)
  por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
  por.VAR<-quantile(por.X1.X2,probs=VAR)
}
value1<-contour(myMvdc, dmvc, xlim = c(-0.05, 0.05),
ylim = c(-0.05, 0.05),
nlevel=20)

claytonCOPULA<-function(X1,X2,MC=10000,VAR=0.05,w){
  a<-cor(X1,X2,,method="kendall")
  d=2*a/(1-a)
  X1.mu<-mean(X1)
  X1.sd<-sd(X1)
  X2.mu<-mean(X2)
  X2.sd<-sd(X2)

```

```

myCop.clayton=archmCopula(family = "clayton", dim =
2, param = d)
myMvdc1=mvdc(copula = myCop.clayton,
margins=c("norm", "norm"),

paramMargins=list(list(mean=X1.mu, sd=X1.sd),

list(mean=X2.mu, sd=X2.sd)))
mc.X1.X2=rmvdc(myMvdc1, MC)
por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
por.VAR<-quantile(por.X1.X2, probs=VAR)
}

value1<-contour(myMvdc1, dmvdc, xlim = c((min(USD)-
sigma), (max(USD)+sigma)),
ylim = c((min(EURO)-sigmab),
(max(EURO)+sigmab)))

frankcopula<-function(X1,X2,MC=10000,VAR=0.05,w) {
a<-cor(X1,X2,method="kendall")
d<-1-a/4
X1.mu<-mean(X1)
X1.sd<-sd(X1)
X2.mu<-mean(X2)
X2.sd<-sd(X2)
myCop.frank <- archmCopula(family = "frank", dim = 2,
param = d)
myMvdc5 <- mvdc(copula = myCop.frank, margins =
c("norm", "norm"),

paramMargins=list(list(mean=X1.mu, sd=X1.sd),

```

```

list(mean=X2.mu, sd=X2.sd))
  mc.X1.X2=rmvdc(myMvdc5, MC)
  por.X1.X2=w*mc.X1.X2[,1]+(1-w)*mc.X1.X2[,2]
  por.VAR<-quantile(por.X1.X2, probs=VAR)
}

mixture.cop=function(X1,X2,MC=10000,VAR=0.05,w){
  X=cbind(X1,X2)
  Y=X
  cutter.i=cordiff.finder(X1,X2,w)
  X=X[1:cutter.i,]
  Y=Y[(cutter.i+1):length(X1),]
  MCX=(length(X[,1])/length(X1))*MC
  MCY=10000-MCX
  mydraws1=NA;mydraws2=NA
  if (length(X>0)){
    a= cor(X[,1],X[,2],method="kendall")
    d<-sin(pi/2*a)

m1=mean(X[,1]);m2=mean(X[,2]);s1=sd(X[,1]);s2=sd(X[,2])
  myCop1<-normalCopula(param=d, dim = 2, dispstr =
"ex")
  myMvdcx1=mvdc(myCop1, margins=c("norm", "norm"),
                paramMargins = list(list(mean = m1, sd
=s1 ),
                                     list(mean =m2, sd
=s2 )))

  mydraws1=rcopula(copula=myCop1,n=MC/2)

```

```

qq1=qnorm(mydraws1)
mydraws1[,1]=qq1[,1]*sd(X[,1])+mean(X[,1])
mydraws1[,2]=qq1[,2]*sd(X[,2])+mean(X[,2])
}

if(length(Y>0)){
  a= cor(Y[,1],Y[,2],method="kendall")
  d<-sin(pi/2*a)
  myCop2<-normalCopula(param=d, dim = 2, dispstr =
"ex")
  myMvdcx2=mvdc(myCop2, margins=c("norm", "norm"),
  paramMargins= list(list(mean = mean(Y[,1]),sd
= sd(Y[,1])),list(mean =mean(Y[,2]),sd=sd(Y[,2]))))
  #MC 2
  mydraws2=rcopula(copula=myCop2,n=MC/2)
  qq2=qnorm(mydraws2)
  mydraws2[,1]=qq2[,1]*sd(Y[,1])+mean(Y[,1])
  mydraws2[,2]=qq2[,2]*sd(Y[,2])+mean(Y[,2])
}
if(length(mydraws1)==1){mydraws=mydraws2}
if(length(mydraws2)==1){mydraws=mydraws1}

if(length(mydraws1)>1&length(mydraws2)>1){mydraws=rbind
(mydraws1,mydraws2)}
my.port=w*mydraws[,1]+(1-w)*mydraws[,2]
my.RMD=quantile(my.port,VAR)
}

cordiff.finder=function(X1,X2,w){
  rr=0;aaa=0

```



```

for (i in 5:(length(X1)-5)){
  a=cor(w*X1[1:i],(1-w)*X2[1:i],method="kendall")
  b=cor(w*X1[(i+1):length(X1)],(1-
w)*X2[(i+1):length(X1)],method="kendall")
  if (abs(a-b)>rr) {rr=abs(a-b)
                    aaa=i}
}
return(aaa)
}
backk <- function (alfa,ww,portw){
set.seed(1)
DATA=aa[,2:3] #[-(1:400),]
WS=ww;dd=alfa
BT.HW=0;BT.Hist=0;BT.normalcopula=0;BT.tcopula=0

BT.MLE=0;BT.mixture=0;BT.frankcopula=0;BT.claytoncopula
=0
  for (i in 1:(dim(DATA)[1]-WS)){
    X1.X2=DATA[i:(i+WS-1),] # Data used in the
calculation of VaR
    BT.X1.X2=DATA[(i+WS),] # Tested in the portfolio
marginal return
    BT=portw*BT.X1.X2[1]+(1-
portw)*BT.X1.X2[2]#apply(BT.X1.X2,1,mean) # Tested in
the portfolio return

HW.VaR=HWTransCop(X1=X1.X2[,1],X2=X1.X2[,2],VAR=dd,w=po
rtw) #Hull-White VAR

Hist.VaR=Historical(X1=X1.X2[,1],X2=X1.X2[,2],VAR=dd,w=
portw) #Historical VAR

```

```

MLE.VaR=MLE (X1=X1.X2 [, 1] ,X2=X1.X2 [, 2] ,VAR=dd,w=portw)
#MLE VAR

mixture.cop.VaR=mixture.cop (X1=X1.X2 [, 1] ,X2=X1.X2 [, 2] ,V
AR=dd,w=portw) #Mixture Copula VAR

NormalCopula.VaR=normalcopula (X1=X1.X2 [, 1] ,X2=X1.X2 [, 2]
,VAR=dd,w=portw) #Normal Copula VAR

tcopula.VaR=tcopula (X1=X1.X2 [, 1] ,X2=X1.X2 [, 2] ,VAR=dd,w=
portw) #T Copula VAR

frankcopula.VaR=frankcopula (X1=X1.X2 [, 1] ,X2=X1.X2 [, 2] ,V
AR=dd,w=portw) #GUMBEL VAR

claytoncopula.VaR=claytoncopula (X1=X1.X2 [, 1] ,X2=X1.X2 [,
2] ,VAR=dd,w=portw)
    BT.HW.Temp=(BT<HW.VaR) #Hul-White VAR's lower weeks
    BT.Hist.Temp=(BT<Hist.VaR) #Tarihsel VAR's lower
weeks
    BT.MLE.Temp=(BT<MLE.VaR)
    BT.mixture.cop.Temp=(BT<mixture.cop.VaR)
    BT.normalcopula.Temp=(BT<NormalCopula.VaR) #Normal
Copula VAR's lower weeks
    BT.tcopula.Temp=(BT<tcopula.VaR) #t copula VAR's
lower weeks
    BT.frankcopula.Temp=(BT<frankcopula.VaR) #Gumbel
VAR's lower weeks
    BT.claytoncopula.Temp=(BT<claytoncopula.VaR)
    BT.HW=c (BT.HW, BT.HW.Temp)

```

```

BT.Hist=c (BT.Hist,BT.Hist.Temp)
BT.MLE=c (BT.MLE,BT.MLE.Temp)
BT.mixture=c (BT.mixture,BT.mixture.cop.Temp)

BT.normalcopula=c (BT.normalcopula,BT.normalcopula.Temp)
BT.tcopula=c (BT.tcopula,BT.tcopula.Temp)

BT.frankcopula=c (BT.frankcopula,BT.frankcopula.Temp)

BT.claytoncopula=c (BT.claytoncopula,BT.claytoncopula.Temp)
}

a1=sum (BT.HW) / (length (BT.HW) -1)
a2=sum (BT.Hist) / (length (BT.Hist) -1)
a3=sum (BT.MLE) / (length (BT.MLE) -1)
a4=sum (BT.mixture) / (length (BT.mixture) -1)
a5=sum (BT.normalcopula) / (length (BT.normalcopula) -1)
a6=sum (BT.tcopula) / (length (BT.tcopula) -1)
a7=sum (BT.frankcopula) / (length (BT.frankcopula) -1)
a8=sum (BT.claytoncopula) / (length (BT.claytoncopula) -1)

outp=c (a1, a2, a3, a4, a5, a6, a7, a8)
return (outp)
}

Finding p values for VAR;

alfa=c (0.01,0.05,0.1)
ww=c (25, 50, 100)
portw=c (0.25,0.5,0.75)
outtt=c (0,0,0,0,0,0,0,0)
for (i in 1:3){

```

```
for (j in 1:3){  
  for (k in 1:3){  
    temp=backk(alfa[k],ww[j],portw[i])  
    outtt=rbind(outtt,temp)  
  }  
}  
}  
write.csv(outtt,file="C:/Users/Altan  
Semih/Documents/out2.csv")
```

APPENDIX II

ADDITIONAL NUMERICAL RESULTS

portfolio	window	alfa	Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
0,25	25	0,01	0,034091	0,034091	0,017045	0,017045	0,017045	0,017045	0,017045	0,017045
0,25	25	0,05	0,039773	0,079545	0,0625	0,0625	0,045455	0,056818	0,056818	0,056818
0,25	25	0,1	0,073864	0,125	0,102273	0,096591	0,096591	0,096591	0,125	0,102273
0,25	50	0,01	0,013245	0,013245	0,006623	0,013245	0,006623	0,006623	0,006623	0,006623
0,25	50	0,05	0,019868	0,059603	0,046358	0,05298	0,033113	0,039735	0,066225	0,046358
0,25	50	0,1	0,02649	0,112583	0,092715	0,092715	0,07947	0,07947	0,112583	0,086093
0,25	100	0,01	0,009901	0,019802	0,009901	0,009901	0,009901	0,009901	0,019802	0,009901
0,25	100	0,05	0,019802	0,049505	0,049505	0,069307	0,029703	0,039604	0,059406	0,039604
0,25	100	0,1	0,019802	0,09901	0,079208	0,118812	0,069307	0,069307	0,09901	0,079208
0,5	25	0,01	0,017045	0,039773	0,017045	0,011364	0,011364	0,011364	0,017045	0,005682
0,5	25	0,05	0,039773	0,079545	0,039773	0,045455	0,039773	0,039773	0,0625	0,039773
0,5	25	0,1	0,073864	0,113636	0,085227	0,090909	0,079545	0,085227	0,113636	0,085227
0,5	50	0,01	0,006623	0,013245	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623
0,5	50	0,05	0,019868	0,046358	0,039735	0,059603	0,013245	0,013245	0,05298	0,019868
0,5	50	0,1	0,039735	0,092715	0,086093	0,092715	0,066225	0,072848	0,10596	0,07947
0,5	100	0,01	0,009901	0,019802	0,009901	0,009901	0,009901	0,009901	0,009901	0,009901
0,5	100	0,05	0,009901	0,039604	0,029703	0,039604	0,019802	0,019802	0,049505	0,029703
0,5	100	0,1	0,019802	0,079208	0,059406	0,09901	0,059406	0,059406	0,09901	0,059406
0,75	25	0,01	0,017045	0,022727	0,005682	0,005682	0,005682	0,005682	0,011364	0,005682
0,75	25	0,05	0,034091	0,068182	0,022727	0,022727	0,017045	0,017045	0,034091	0,022727
0,75	25	0,1	0,0625	0,107955	0,073864	0,073864	0,068182	0,068182	0,096591	0,073864
0,75	50	0,01	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623	0,006623
0,75	50	0,05	0,013245	0,05298	0,013245	0,02649	0,006623	0,013245	0,033113	0,013245
0,75	50	0,1	0,039735	0,092715	0,072848	0,072848	0,066225	0,066225	0,07947	0,066225
0,75	100	0,01	0,009901	0,019802	0,009901	0,009901	0,009901	0,009901	0,009901	0,009901
0,75	100	0,05	0,009901	0,039604	0,019802	0,009901	0,009901	0,009901	0,029703	0,019802
0,75	100	0,1	0,009901	0,069307	0,039604	0,069307	0,039604	0,039604	0,069307	0,039604

Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
6	6	3	3	3	3	3	3
7	14	11	11	8	10	10	10
13	22	18	17	17	17	22	18
2	2	1	2	1	1	1	1
3	9	7	8	5	6	10	7
4	17	14	14	12	12	17	13
1	2	1	1	1	1	2	1
2	5	5	7	3	4	6	4
2	10	8	12	7	7	10	8
3	7	3	2	2	2	3	1
7	14	7	8	7	7	11	7
13	20	15	16	14	15	20	15
1	2	1	1	1	1	1	1
3	7	6	9	2	2	8	3
6	14	13	14	10	11	16	12
1	2	1	1	1	1	1	1
1	4	3	4	2	2	5	3
2	8	6	10	6	6	10	6
3	4	1	1	1	1	2	1
6	12	4	4	3	3	6	4
11	19	13	13	12	12	17	13
1	1	1	1	1	1	1	1
2	8	2	4	1	2	5	2
6	14	11	11	10	10	12	10
1	2	1	1	1	1	1	1
1	4	2	1	1	1	3	2
1	7	4	7	4	4	7	4

Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
6	6	3	3	3	3	3	3
7	14	11	11	8	10	10	10
13	22	18	17	17	17	22	18
2	2	1	2	1	1	1	1
3	9	7	8	5	6	10	7
4	17	14	14	12	12	17	13
1	2	1	1	1	1	2	1
2	5	5	7	3	4	6	4
2	10	8	12	7	7	10	8
3	7	3	2	2	2	3	1
7	14	7	8	7	7	11	7
13	20	15	16	14	15	20	15
1	2	1	1	1	1	1	1
3	7	6	9	2	2	8	3
6	14	13	14	10	11	16	12
1	2	1	1	1	1	1	1
1	4	3	4	2	2	5	3
2	8	6	10	6	6	10	6
3	4	1	1	1	1	2	1
6	12	4	4	3	3	6	4
11	19	13	13	12	12	17	13
1	1	1	1	1	1	1	1
2	8	2	4	1	2	5	2
6	14	11	11	10	10	12	10
1	2	1	1	1	1	1	1
1	4	2	1	1	1	3	2
1	7	4	7	4	4	7	4

	Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
Total chi-square value (18)	83,3692	44,60595	23,67952	16,07785	38,02961	34,15212	11,52603	26,79763
P Value	1,15E-07	0,017887	0,648028	0,951447	0,077395	0,16167	0,995904	0,474754

portfolio		Hull-White	Historical VAR	MLE VAR	Mixture Copula	Normal Copula	T-Copula	Frank Copula	Clayton Copula
25%	Total-Chi Square Values (6)	33,311303	16,4756	2,262482	2,951857	4,702743	3,580639	4,722854	2,320536
	P Value	0,0001179	0,05759	0,986605	0,966173	0,859413	0,936788	0,857768	0,985327
50%	Total-Chi Square Values (6)	22,396584	21,34356	5,255605	1,058397	11,40964	10,34126	2,088593	7,638687
	P Value	0,0077036	0,011209	0,811482	0,999291	0,248671	0,323571	0,989988	0,570927
100%	Total-Chi Square Values (6)	27,66131	6,786795	16,16143	12,0676	21,91722	20,23022	4,714584	16,8384
	P Value	0,0010864	0,659306	0,063586	0,209525	0,009145	0,016544	0,858445	0,051308

ÖZGEÇMİŞ

Kişisel Bilgiler

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Öğrenim Durumu

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Fen bilimleri

2004-2010 Yaşar Üniv. Fen Edebiyat Fakültesi

Matematik bölümü

2010-2013 Yaşar Üniv.Fen Edebiyat Enstitüsü

Aktüerya Bilimleri Yüksek Lisansı

Yabancı Dil : İNGİLİZCE

Programlama Dilleri : C 'ye giriş, R studio

İşletim Sistemleri : Windows

Paket Programlar : MS Word, Excel, Power Point

Hobilerim:

Bilgisayar kullanmak, bulmaca çözmek, yürümek