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FAIR SINGLE SOURCE CAPACITATED FACILITY LOCATION PROBLEM

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ABSTRACT

FAIR SINGLE SOURCE CAPACITATED FACILITY LOCATION PROBLEM

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This thesis concerns with the bi-objective Single-Source Capacitated Facility Location Problem. The first component of the objective function comprises of fixed opening cost of facilities and assignment costs of demand points to the facilities. The second component is related to the fairness of the assignments. The fairness objective is expressed in terms of the conditional β-mean concept. The characteristics of the problem are discussed and related mathematical models are given. Three solution methods; the Weighted Sum method, the Epsilon Constraint method, and the Benders Decomposition method are used to show the solutions of the problem. Benchmark problem instances are solved to test the solution methods. The outcomes are reported and solution methods are compared to each other. As outcomes of the study; the Epsilon Constraint Method is able to find a greater number of solutions than the Weighted Sum Method and the Benders Decomposition Method is found to perform better as the sizes of the problems increase.

Key Words: single-source capacitated facility location problem, fair single-source capacitated facility location problem, conditional beta mean, mixed-integer programming, weighted sum method, epsilon constraint method, benders decomposition.

ÖZ

ADİL TEK KAYNAKLI VE KAPASITELI TESIS LOKASYONU PROBLEMI

Erdem, Gamze

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Danışman: Dr. Öğr. Üyesi Adalet Öner

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Bu tez; iki amaç fonksiyonlu tek kaynaklı ve kapasiteli tesis lokasyonu problemi ile ilgilidir. Amaç fonksiyonunun ilk bileşeni tesislerin sabit açılış maliyetlerini ve talep noktalarını tesislere atama maliyetlerini içermektedir. İkinci bileşen ise, talep noktaları açısından atamaların adil bir şekilde yapılıp yapılmadığı ile ilgilidir. Atamalardaki adalet ölçüsü, "koşullu beta yakınsaması" kavramı ile ifade edilmektedir. Problemin nitelikleri ele alınmış ve matematiksel modeller verilmiştir. Üç çözüm yöntemi; Ağırlıklı Toplam metodu, Epsilon Kısıt yöntemi ve Benders Ayrıştırma algoritması, problemin sonuçlarını göstermek için kullanılmıştır. Referans problem setleri, çözüm yöntemlerini test etmek amacıyla çözülmüştür. Sonuçlar raporlanmış ve çözüm yöntemleri birbirleri ile kıyaslanmıştır. Sonuçlara göre, Epsilon Kısıt yöntemi ile Ağırlıklı Toplam metoduna göre daha fazla sayıda çözüme ulaşıldığı ve Benders Ayrıştırma algoritmasının problem büyüklükleri arttıkça daha etkin olduğu görülmüştür.

Anahtar Kelimeler: tek kaynaklı ve kapasiteli tesis lokasyonu problemi, adil tek kaynaklı ve kapasiteli tesis lokasyonu problemi, tesis lokasyonu optimizasyonu, karışık tamsayılı programlama, ağırlıklı toplam metodu, epsilon kısıt yöntemi, benders ayrıştırma algoritması, koşullu beta yakınsaması.

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Gamze Erdem

İzmir, 2021

TEXT OF OATH

I declare and honestly confirm that my study, titled "Improvements on Fair Single Source Capacitated Facility Location Problem" and presented as a Master's Thesis, has been written without applying to any assistance inconsistent with scientific ethics and traditions. I declare, to the best of my knowledge and belief, that all content and ideas drawn directly or indirectly from external sources are indicated in the text and listed in the list of references.

Gamze Erdem

Signature

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July 6, 2021

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SYMBOLS AND ABBREVIATIONS

ABBREVIATIONS:

FLP: Facility Location Problem

CFLP: Capacitated Facility Location Problem

SSCFLP: Single Source Capacitated Facility Location Problem

F-SSCFLP: Fair-Single Source Capacitated Facility Location Problem

CVaR: Conditional Value at Risk

MOO: Multi-Objective Optimization

PS: Pareto-optimal Set

PF: Pareto-optimal Front

MIP: Mixed Integer Program

CGLP: Cut Generating Linear Program

TF: Total Fixed Cost

TV: Total Variable Cost

SYMBOLS:

ρ: total cost value

σ: conditional β-mean value

β: percentage of the total demand who pay the highest unit assignment cost

λ: the weight of the total cost in the F-SSCFLP

CHAPTER 1 INTRODUCTION

Facility location decision-making is a significant administrative duty that both companies and governments should consider crucially to improve their objectives such as increasing profit margins or customer satisfaction. As the number of companies has grown exponentially during the past several decades, the decision-making process for facility locations has become very complex.

Companies are very selective while deciding the locations of their plants or facilities since this decision is very significant for their profit margins. In the private sector, the primary goal is to maximize profits and market share while working on facility location models. Furthermore, companies usually emphasize locating their facilities as close to the center of the market as possible since they get the chance to increase their market share when their facilities are near to the center of the corresponding sector's market. Retail, technology, food, automotive, clothing, airline, energy, construction are good illustrations for private sectors, and companies related to these sectors are highly sensitive to their profit margins. For instance, if an automotive company locates its facilities far from the automotive companies with different brands, then they would not be recognized by potential customers. This issue would arise due to people's tendency to shop from places where they have various alternatives and where they would be able to make comparisons to select the most suitable products for themselves. As opposed to the private sector, governments generally consider the best regional allocation of public facilities within their budgets and priority is not making the maximum profit in the public sector. Public facility location is related to government welfare and making profit is not the main issue while making decisions about public facility locations. Logical distribution of public facilities by considering budgetary constraints is the main concern while making public facility location decisions. Cost minimization, maximizing reachability of facilities, efficiency and fairness are the main goals while deciding facility locations in the public sector. Determining the location of public facilities should be in coordination with the services that they provide. Education, healthcare, police services, public transit, fire service, emergency services, social services, and electricity can be given as examples for public sectors. Besides governmental budget, politics also affect facility location decisions, especially in the public sector. For instance, for sports, political factors affect the decision-making process for sports centers' locations. If the last century is considered, it can be seen that local political leaders play a significant role in locating sports centers in central city sites. As another illustration of the effects of politics, the bonds between the allocation of healthcare facilities and-politics can be investigated. There are many constraints in terms of costs and capacities that are regulated by political powers in healthcare facility location models. However, in this study, politics and its effects on facility location models are not taken into consideration.

Customer satisfaction also plays an important role while making decisions about facility locations. In general, customer satisfaction can be measured by companies' ability to meet their customers' expectations. Customer satisfaction is related to some factors such that the service that customers get, the closeness of the facilities to customers' locations, the cost of the service, and also the ratio of the service that customers receive with the money they spend. Most importantly, customers are very sensitive about variations on the assignment costs which are paid by themselves or their counterparts to reach potential facilities. In other words, they are highly sensitive about the fairness in cost distribution between themselves and their counterparts who want to get service from the same potential facilities.

Companies or planners should be very careful while making decisions about their facilities' locations. It is common for planners to select a model which results in the minimum costs. However, they should emphasize customer satisfaction and especially fairness as it is mentioned before.

In this study, the facility location problem is studied when the fairness concept is taken into consideration. The classical facility location problem involves minimizing the total cost of opening cost of facilities and the cost of assigning the customers to the opened facilities. However, if fairness is embedded in the objective function, it turns out to be a bi-objective problem.

This thesis consists of 7 chapters. The definition of the problem is covered in the next chapter. In chapter 3, a literature review about multi-objective optimization, facility location problem, and solution methods are presented. The mathematical model for the classical facility location problem, the Fair Single Source Capacitated Facility Location Problem (F-SSCFLP) with cost and fairness objectives are given in Chapter 4 and then the toy example is given to compare the F-SSCFLP with cost and fairness objectives with the classical facility location model under the same chapter. In Chapter 5, solution methods to solve F-SSCFLP with cost and fairness objectives will be given which are the Weighted Sum method, Epsilon Constraint method, and Benders Decomposition method. In Chapter 6, computational results, as well as analyses, will be given and finally, conclusions and future work will be discussed in Chapter 7.

CHAPTER 2 PROBLEM DEFINITION

Facility Location Problem (FLP) deals with minimizing transportation costs and fixed costs while locating the facilities. Fixed costs are the costs to open (set up) a facility at a specific location. Transportation costs are the costs that reflect customers' costs for reaching or accessing a specific facility. Mostly considered examples for the facilities which are considered in FLP are schools, healthcare centers, warehouses, and plants (Silva et al., 2007). The basic framework of the FLP can be shown in Figure 1 below.

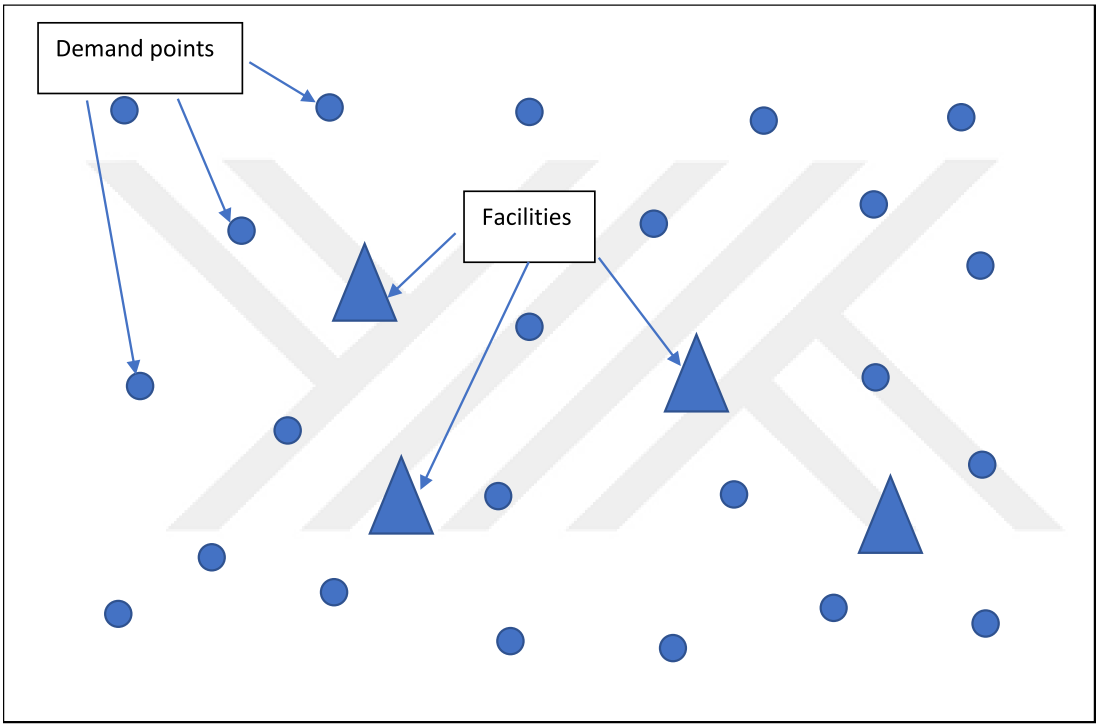

Figure2.1. Basic Framework of Facility Location problems

In Figure 1, the circles represent customers which are called demand points and the triangles represent candidate facilities. Customers are called demand points throughout this thesis since each demand point includes one or more individuals. In FLP, there are some candidate points where facilities can be opened. There are variants of FLP based on the fact that customers are served by single or multiple facilities. Furthermore, facilities may have capacities or may have limitless service potential. In Figure 1, there are four candidate facilities and customers which are expected to be served by one or more than one of these facilities. Integer Programming (IP) is generally used to exhibit the status of the facilities by assigning 1 to the open facilities and 0 to the closed facilities.

The basic variants of the facility location problems include Uncapacitated Facility Location Problem (UFLP) and Capacitated Facility Location Problem (CFLP) which are defined depending on whether the location is assumed to have limited or unlimited capacity to serve the customers. In some other variants, it is possible to open multiple facilities instead of a single facility. Those variants are also divided into further variants based on the assumption that the customer groups (demand points) would be served by only one facility or by multiple facilities.

This thesis involves the “Single Source Capacitated Facility Location Problem (SSCFLP)”. SSCFLP includes demand points and candidate facilities that are expected to serve these demand points. As mentioned previously, demand points can be regarded as sets that include one or multiple individuals. The assumptions of the problem are as follows:

i. The locations of the demand points and the number of demands are known.

ii. Multiple facilities or one facility can be opened.

iii. Each demand point should be served by a single facility.

iv. Each facility has a pre-determined capacity measured by the number of individuals.

In most works about Single Source Capacitated Facility Location optimization, only cost objective has been taken into consideration. However, providing customer satisfaction should be regarded as significant as minimizing opening and assignment costs while solving the SSCFLP.

Customer satisfaction can be measured by some metrics in marketing literature. Customer satisfaction has two main and significant components which are customers’ expectations and facilities’ ability to meet these expectations. Fairness can be regarded as the most significant measure which affects customer satisfaction since customers are highly sensitive about the fair distribution of costs between themselves and their counterparts. Fairness can be improved by minimizing the variability among the costs paid by the candidate customers.

It is important to handle both cost and fairness objectives at the same time. When fairness objective embedded into the SSCFLP, it is called the Fair Single Source Capacitated Facility Location Problem (F-SSCFLP) and it is naturally a bi-objective problem. A well-known and logical way to handle the bi-objective optimization problems is to take the weighted sum of fairness and cost functions as an objective function. By using the weighted sum method, the bi-objective SSCFLP turns into a mono-objective optimization problem. However, there are some other methods to solve F-SSCFLP, and those methods are presented in the section that describes the solution methods.

In a F-SSCFLP, the cost objective is simply the summation of total fixed costs for opening (setting up) the facilities and the variable costs that represent transportation (reaching or accessing) costs that are paid by customers. On the other hand, the fairness objective includes a conditional β-mean function which represents the costs paid by the customers who pay the highest unit costs to reach facilities. It is not expected to get optimal solutions for both objectives at the same time for any bi-objective optimization problem. Therefore, Pareto optimal solutions and representative Pareto sets are expected to be obtained as outcomes of the F-SSCFLP with cost and fairness objectives.

Although the weighted sum method is a useful method for a bi-objective F-SSCFLP, it cannot detect some Pareto efficient points. This drawback of the weighted sum method can be related to the method's inability to discover efficient points on nonconvex parts of the Pareto optimal front.

The motivation of the thesis is to develop a model which concerns locating service facilities by using efficient methods which are applicable for real-life facility location scenarios. In real life, customers prefer a facility that is close to their location and they want to get service at least at a level that other customers get, and they also want to pay for this service at most other customers do. The thesis focuses on a new modeling approach in which the fairness objective is also included. This modeling approach is called F-SSCFLP. Three different methods which are the Weighted Sum Method, the Epsilon Constraint method as well as the Benders Decomposition Method (in CPLEX) will be used to solve this new model.

CHAPTER 3 LITERATURE REVIEW

The facility location problem has been widely studied in the OR literature since it is very significant in private and public firms' decision-making processes. The facility location decision should be sustainable such that this decision should remain advantageous as the environmental facts, market trends, and the population change (Owen et al., 1998).

In the facility location problem (FLP), there are several customers and facilities and the aim is to link each customer to a facility while minimizing the total cost. The linking cost to link customer i to a facility j is usually represented with c_{ij}. There is also a cost f_j which represents the opening cost of the facility j. The total cost is simply the summation of the linking costs and the opening costs. FLP was firstly introduced by Alfred Weber in 1929. The problem is well explained in the book called "Theory of Location of Industries" where the general theory of the location is studied and explained (Weber, 1929). Especially, the location of the industrial resources is highlighted in that book. The main assumption in the study was the equality of the fuel costs and cost of raw materials. Transportation and labor costs are the two general costs in Weber's study. Material index and the labor coefficient are the terms that have been mentioned by Weber. The labor coefficient of the different industries makes it possible to make a comparison between these industries according to Weber's study.

The most basic and simple form of the Facility Location problem is the Uncapacitated Facility Location problem (UFLP). In the UFLP, facilities that are under consideration are assumed to have unlimited capacities. Machine scheduling and money dispensing facility locations can be given as examples for UFLP (Silva et al., 2007), (Cornuejols et al., 1977).

Another type of FLP is the Capacitated Facility Location Problem (CFLP). In CFLP the facilities have predetermined capacities in terms of the number of customers to be served (Hajiaghayi et al.,2003). However, the customers can be served by more than one opened facility (Silva et al., 2007).

The more complex form of the FLP is the Single Source Capacitated Facility Location Problem (SSCFLP). As a distinctive factor from CFLP, each customer can be served by only one facility (Silva et al., 2007). SSCFL problems are the NP-hard problems that consist of two parties in terms of cost distribution. The planner who decides the locations of the facilities pays the fixed costs while customers pay assignment costs (ie. transportation costs) to reach located facilities. In SSCFL problems, the purpose is minimizing costs that are paid by both parties (Filippi et al., 2019). Some Integer Linear Programming models have been developed with demand and capacity constraints to solve this type of problem (Gadegaard, 2017).

Heuristic approaches for solving SSCFL problems are proposed by some researchers in the academic literature. A Lagrangian Heuristic for SSCFLP has been emphasized in some studies since most of the heuristic methods which have been used to solve SSCFLP are based on a Lagrangian Relaxation of a SSCFLP. Holmberg et al. (1999) studied Lagrangian Heuristic (LH) to solve SSCFLP and as a result of their work, this method was found to be a logical method for solving SSCFL problems. According to the computational results of Holmberg et. al's work (1999), the LH method was able to solve the SSCFLP to optimality or it terminated with small optimality gaps. Holmberg et al. further developed the LH method by adding branch and bound method to the simplify LH method. In Holmberg et al.'s study, this method was called Lagrangian heuristic with branch and bound (LHBB).

Based on a literature survey by (Demircioğlu, 2015). it can be observed that various other methods have been utilized to solve SSCFL problems such as a greedy heuristic with a restricted neighborhood search, a Lagrangian heuristic combined with an ant colony system, a reactive GRASP heuristic and Tabu Search (TS), and other two methods which are combinations of GRASP and TS heuristics. Some studies utilize a Very Large-Scale Neighbourhood Search Algorithm (VLNS) where the neighborhood search was performed by facility movements and client exchanges.

(Guastaroba et al.,2014) reported that the use of the Kernel Search (KS) method to solve SSCFLP has been a good contribution to the literature. According to the outcomes which have been exhibited in some articles, the KS approach outperformed many other heuristic methods when it was compared with other methods based on computational time and efficiency.

Classical facility location models have been discussed up to this point. However, some studies emphasize the significance of the fairness concept in FLP.

Marsch et al. (1994) present a literature review framework for facility location problems to solve FLP with fairness objective. The main purpose of their study was to embed equity measures into FLP. In their paper, Marsch et al. defined action as locating facilities and effect as a benefit or harm caused by the closeness of facilities to customers, then based on these definitions they defined equity as a fair distribution of the effect among customers. Effects are various and they are not only dependent on distance. For instance, congestion, response time, noise, etc. In FLP, area and population are regarded as common measures while scaling these effects. Marsch et.al. emphasized previously studied measures for equity. Some of these measures are the p-center model, variance, mean absolute deviation, sum of absolute deviations, Gini coefficient, and range. Marsch et al. also compared various measures of equity in a particular framework. This framework consists of three parts which are distance measurement, scaling factor, and reference distribution selection. Marsch et al. also proposed seven criteria to choose a good equity measure for FLP. These criteria are analytic tractability, appropriateness, impartiality, the principle of transfers, the scale of invariance, Pareto optimality, and normalization. (Marsh et al., 1994)

Kalcsics et al. (2015) discussed different equity measures for facility location problems with two facilities. The fair solutions can be considered as a subset of all Pareto-efficient points. In statistical means, low variation between the distribution of travel costs was aimed to achieve fairness. Some works to minimize variance in location analysis were discussed. Other equity measures were also discussed in this study. These measures were the mean absolute deviation, the maximum weighted absolute deviation, the sum of weighted absolute differences between customers, the range, and the minimum variance (Kalcsics et al., 2015)

Ogryczak (2000) worked on achieving equitable consistency by constructing mean equity models. Mean outcome was used as an efficiency and inequality measure to ensure fairness. It was emphasized that fairness is usually obtained by minimizing inequality measures. It was also pointed out that if only relative inequality measures are minimized, individual outcomes would be worse off. Therefore, minimization of individual outcomes was studied in this work. In the model, integer weights were utilized. However, normalized weights were considered while discussing solution approaches.

Mulligan (1991) argued that distributional equality should be used for facility location problems. In this study, a real-life scenario was considered in which there should be many customers and few facilities. It was obvious that if the population is vast and far from each other, facilities cannot be equally distanced from customers. For analyzing equality solutions with a pilot illustration, one facility and three population nodes were considered in this study. Piecewise continuous curves which demonstrate location-based levels of distributional equality were utilized for different equality measures and they were compared both visually and mathematically. Distributional equality in travel distances was a primary objective in Mulligan's work. The purpose was to compare the travel cost of an individual with the travel costs of other people in the population. Distributional equality measures such as the Gini coefficient, the mean deviation, Hoover's concentration index, the variance, and Theil's entropy index, were utilized and compared in this work. The Hoover's concentration index, the mean deviation, and the variance gave similar optimal solutions. As a significant outcome of this work, distribution-based equality measures exhibited more impressive results than other equality measures, but the computational effort was higher (Demircioğlu, 2015).

Barbati et al. (2015) studied the required properties of equality measures in facility location problems. In this study, new properties were explored, and ten equality measures were analyzed in terms of their compatibility with these properties. Ten equality measures were center, range, mean absolute deviation, variance, maximum deviation, absolute difference, SumMaxDiffAbs, Schutz's index, coefficient of variation, and Gini coefficient. In most of the facility location studies, demands are given separate weights to exhibit values of these demands. However, in this work demands were given weights which are equal to 1 so that customers at each demand point were assumed to be located at exactly at the same place. The distribution of distances between each customer and their assigned facility was considered in this study. Empirical analyses were performed to exhibit characteristics of equality measures with introduced properties.

Chapman et al. (2016) studied the placement of relief distribution centers after a disaster by considering fairness with conditional value at risk which minimizes variations among transportation costs. A formulation was proposed to decide the location of distribution centers to assign each member of the population to a distribution center. When conditional value at risk was added to the formulation, it was observed that optimal solutions were guaranteed to be Pareto efficient (Chapman et al., 2016)

There is a special case of SSCFLP, where both fairness and cost minimization objectives are considered together. This type of SSCFLP is called the Fair Single Source Capacitated Facility Location Problem (F-SSCFLP). It is a bi-objective optimization problem.

The fairness concept may be best understood with some preliminary concepts such as Conditional Value at Risk (CVaR) and Value at Risk (VaR). In academic studies, Conditional Value at Risk (CVaR) has been used as a method that measures the risk. This value is roughly the average of a percentage of loss scenarios. Value at Risk (VaR) is another value that has been used for risk management. The only difference between CVaR and VaR is that the VaR exhibits a percentage of loss distribution (Serraino et al., 2013). On the other hand, the β factor has usually been used as a risk measure in CVaR content. The average value of the highest (1-β) % of the distribution is named $CVaR_{\beta}$ (Anderson et al., 2014). In CVaR literature, the β value has been the values 0.01 or 0.05 (Semenov, 2009). When there is a determined value β, then this means the loss would not exceed the lowest amount α with probability β according to the β-var concept. On the other hand, β-CVaR exhibits the expectation of losses above the value α with probability β (Rockafellar et al., 1999)

In this thesis, the "conditional β-mean" term will be used in expressing the fairness objective. The conditional β-mean can be defined as the average unit cost that is paid by the β percentile of the total demand who pay the highest unit assignment costs (Filippi, 2019). The conditional β-mean minimization makes the tail of the highest assignment costs shorter. Ogryzack et al. (2002) studied the portion of demand related to the largest or the worst outcomes that are dependent on distances. Furthermore, they considered the β quantile of the largest outcomes and defined the conditional β-mean value as the average of that value. As is mentioned in the previous paragraph, β values in CVaR literature have been usually 0.01 and 0.05. Filippi et al. (2019) compared these two commonly used β values and showed that for the Single Source Capacitated Facility Location Problems (SSCFLP), the value β=0.05 gives more sensitive results than the value β=0.01 in terms of the opening and assignment costs. Filippi et. al also showed that β=0.05 results in a higher number of valid instances since the percentage of conditional β-mean increases in β=0.05 case when it is compared with β=0.01 case (Filippi et al., 2019). There are a variety of other measures in the OR literature which emphasize the fairness objective. Although there are many academic studies about the fairness objective in the Facility Location Problems (FLP), only a few are about SSCFL problems.

Pareto efficiency is also a very significant concern in bi-objective facility location problems. A Pareto-efficient solution provides the best possible solution for each objective which are the cost and the fairness objectives in this study. In a Pareto-efficient solution, there is no possibility to improve the solution of one objective without degrading the solution of another objective.

In literature, there are many methods reported to solve the bi-objective F-SSCFLP problem in which two distinct objectives, i.e. fairness and cost minimization are considered. However, the following methods are the ones that are commonly used.

Weighted Sum Method: Weighted sum method is a significant method to provide solutions for multi-objective optimization problems. This method is a utility function in which parameters demonstrate the importance of each objective function. Value of weights exhibit significance of objective functions. An illustration for this utility function is below:

$$U = \sum_{i=1}^{k} w_i * F_i(x)$$

Theoretically, Pareto optimality is achieved by minimizing this utility function when all weights are nonnegative. However, in practice, this utility function is not satisfactory to achieve Pareto optimality. Therefore, there have been some developments in this method. For instance, Koski et al. grouped objectives with similar properties and then constructed the utility function with a reduced number of individual objective functions (Marler et al., 2004). Reducing the number of objective functions is called the partial weighting method and it is very handful for solving multi-objective optimization problems with many objective functions. In this method, parametrization has a significant role. Values of weighting factors determine the relative significance of objective functions. In the normal weighting method, Pareto optima can be missed which is a serious drawback. The partial weighting method is very valuable for eliminating this drawback. Preserving the multicriteria nature of the optimization problem should be considered while reducing the number of objective functions as much as possible (Koski et al., 1987). Koski et al. also discussed three significant drawbacks of the weighted sum method. The first disadvantage is that predetermination of weights by using satisfactory methods is not enough to guarantee the acceptability of the results. Weights should be functions of objective due to this issue. The second disadvantage is the impossibility to obtain solutions on nonconvex sections of the Pareto optimal set. The third disadvantage is the inability to reach the correct Pareto optimal set by varying weights continuously (Koski et al., 1987).

Although these disadvantages are significant, evolutionary multi-objective (EMO) algorithms are developed as remedies for these drawbacks. For instance, one of these EMO methods, a localized weighted sum (LWS) method can find solutions for nonconvex Pareto front regions in contrast to a regular weighted sum method. In addition, decomposition-based algorithms which are integrated with the LWS method can handle many conflicting objectives (Wang et al., 2018).

In all applications of the weighted sum method how the weights are related to preferences is significant since increasing the weighting factor of an individual objective function would increase its relative importance in the aggregate objective function (Filippi et al., 2019)

In most of the studies about the weighted sum method, it is recommended to use weights for individual functions which sum up to 1 to obtain a convex combination of all objective functions in an aggregate objective function. However, it is adequate to set weights to a value greater than or equal to 0 to ensure Pareto optimality. Therefore, utilization of unrestricted weights would be beneficial for determining weight values for individual functions easily (Marler et al., 2004)

Epsilon Constraint Method: This method is useful to convert a multi-objective optimization problem into a single objective optimization problem by transforming the other objectives into a constraint.

For the bi-objective optimization (BOO) problems, one of the objective functions is transformed into a constraint which is given below (see Table 3.1.)

Table 3.1. Use of Epsilon Constraint Method in BOO problems

min $f_1(x)$	min $f_2(x)$
subject to	subject to
$x \in X$	$x \in X$
$f_2(x) \leq \epsilon_2$	$f_1(x) \leq \epsilon_1$

According to the epsilon constraint method, x^* is an efficient solution if x^* solves both $P_1(\epsilon_2)$ and $P_2(\epsilon_1)$ where $P_k(\epsilon)$ is a problem which is obtained by transforming one objective into a constraint (Bérubé et al., 2009).

Benders Decomposition: This method is useful in solving large-scale Mixed Integer Programs (MIP). As its name suggests, the original problem is decomposed into a master problem and a Cut Generating Linear Program (CGLP), and the problem is solved iteratively. As the first step of each iteration, the optimal solution is found for the master problem and then the CGLP is solved by using the solution of the master problem. This procedure is repeated to the time when there is no more cut generated by CGLP. Benders Decomposition can solve problems that are similar to real-life in terms of instance sizes (Bonami et al., 2020). Benders Decomposition (BD) as a tool to solve Capacitated Facility Location problem (CFLP) was used by Fischetti et. al. Two different scenarios were considered. One scenario was representing a classical linear model, on the other hand, the other scenario was representing a model with an objective function that consists of non-separable quadratic convex terms. Although there are some negative effects of non-separability, the BD method was proved to be successful for a Capacitated Facility Location problem with non-separable quadratic convex terms. Outcomes of the method in this study were compared with the outcomes of the article of Guastaroba and Speranza (2014) which suggests the use of Kernel Search (KS) for SSCFLP. The reason for this comparison was due to the best available heuristic method for solving CFLP in the literature was the KS method of Guastaroba et al. According to the results of that comparison, while solving a CFLP, the BD approach was found to be superior to KS approach for some instances (Fischetti et al., 2016)

The most significant references which have been used and paraphrased in this chapter will be summarized below based on their titles and main properties (see Table 3.2.).

While determining the most significant references, the frequency of the use of these references and their relationship with the specific problem which is studied in this thesis are taken into account.

Table 3.2. Summary of the Literature Review

Reference	**Title**	**Description**
(Demircioğlu, E. et al. 2015)	A Survey of Discrete Facility Location Problems.	A literature review on facility location models is made.
(Filippi et al., 2019)	On Single-Source Capacitated Facility Location with Cost and Fairness Objectives.	A Single Source Capacitated Facility Location Problem with cost and fairness objectives is studied.
(Serraino, G. et al., 2013)	Conditional Value-at-Risk.	A comparison of the performances of the Conditional Value-at-Risk and Value-at-Risk is made.
(Marler, R.T. et al.,2004)	Survey of Multi-objective Optimization Methods for Engineering. Structural and Multidisciplinary Optimization.	A survey of the solution methods for the multi-objective optimization is conducted.
(Koski et al., 1987)	Norm Methods and Partial Weighting in Multi-Criterion Optimization of Structures	Methods to detect Pareto-optimal solutions are studied.

(Bonami P. et al., 2020)	Implementing Automatic Benders Decomposition in a Modern MIP Solver.	Automatic Benders Decomposition in CPLEX is studied.
(R. Wang et al., 2018)	Localized Weighted Sum Method for Many-Objective Optimization (MOO).	The Weighted-Sum method for the MOO is studied.
(Rahmaniani, R. et al., 2017)	The Benders decomposition algorithm: A literature review.	A literature review on Benders Decomposition method is made.

Those studies are then tabulated in Table 3.3 based on the main topics of the problem such as problem type or solution methods.

Table 3.3. References vs Related Topics

Topic	**Reference**
Facility Location Problems (FLP)	(Silva et al., 2007), (Demircioğlu et al., 2015)
Uncapacitated Facility Location Problems (UFLP)	(Silva et al., 2007)
Capacitated Facility Location Problems (CFLP)	(Silva et al., 2007)
Single Source Capacitated Facility Location Problems (SSCFLP)	(Filippi et al., 2019), (Silva et al., 2007)
Fair-Single Source Capacitated Facility Location Problems (F-SSCFLP)	(Filippi et al., 2019)
Solution Methods- Weighted Sum	(Marler et al., 2004), (Filippi et al., 2019)

Weighted-Sum Method	(Marler et al., 2004), (Koski et al., 1987)
Benders Decomposition (BD)	(Bonami et al., 2020)
Solution Methods- Epsilon- Constraint	(Bonami et al., 2020)
Multi-Objective Optimization (MOO)	(Wang et al., 2018)
SSCFLP and BD	(Rahmaniani et al., 2017)
Conditional β-mean	(Serraino et al., 2013)

CHAPTER 4 MATHEMATICAL MODEL

In this chapter, the mathematical models are presented for the facility location problems. First, the formulation for the classical FLP model is given and the formulation for the F-SSCFLP model is presented. Additionally, a toy example will be constructed and solved by using both the classical facility location model and the F-SSCFLP with cost and fairness objectives. The results of the toy example will be compared.

4.1. The Single Source Capacitated Facility Location Model

The mathematical model for the mono-objective single-source capacitated facility location model with cost objective is as follows.

Parameters:

F_j**:** Fixed cost of facility j ϵ J.

D_i**:** Demand of customer i ϵ I.

C_j**:** Capacity of facility j ϵ J.

V_{ij}**:** Variable cost of assigning customer i to facility j (i ϵ I Λ j ϵ J).

Decision Variables:

y_j**:** Binary variable which is equal to 1 if facility j is opened (j ϵ J), 0 otherwise.

x_{ij}**:** Binary variable which is equal to 1 if customer i (i ϵ I) is assigned to facility j (j ϵ J), 0 otherwise.

By considering the parameters and decision variables which are mentioned above, mathematical formulation for the classical facility location problem can be seen below:

$$min \sum_j y_j * F_j + \sum_i \sum_j D_i * x_{ij} * V_{ij} \qquad (1)$$

Subject to

$$\sum_j x_{ij} = 1 \qquad i \epsilon I \qquad (2)$$

$$\sum_i D_i * x_{ij} \leq C_j * y_j \qquad j \in J \tag{3}$$

$$x_{ij} \leq y_j \qquad i \in I;\ j \in J \tag{4}$$

$$x_{ij} = \{0,1\} \qquad i \in I;\ j \in J \tag{5}$$

$$y_j = \{0,1\} \qquad j \in J \tag{6}$$

The objective function minimizes the total cost which consists of total fixed costs and total variable costs. Constraints (2) and (4) together guarantee that customers get service from only 1 facility. Constraint set (3) ensures that the capacity of a facility is sufficient for the demand of customers who are assigned to that facility. Constraint sets (5) and (6) are used to exhibit binary variables.

Although minimizing the total cost is the most popular optimization technique for facility location decision-making, it is usually not enough for real-life facility location issues. In real life, apart from fixed cost and travel or customer assignment costs, customer satisfaction plays a significant role in facility location models and In this model customer satisfaction is enabled by minimizing the variations in assignment costs that are paid by customers to reach the opened facilities. Customers feel more satisfied when there is not a large variation between assignment costs that they pay and assignment costs that are paid by other customers. Therefore, with the customer satisfaction factor in mind, a new bi-objective optimization model which can solve real-life facility location problems is introduced in this study.

4.2. The F-SSCFLP with Cost and Fairness Objectives

In this bi-objective optimization model, one objective exists for the minimization of the total cost similar to the classical facility location models and the other objective function exists for the maximization of the equality among customers. Maximizing fairness is formulated as minimizing the conditional β-mean since the conditional β-mean represents inequality among customers. If $V_i(x)$ is defined to be the summation of assignment costs of all demand points which are originated from customer i, then $V_i(x)$ would be equal to $\sum_j V_{ij} * x_{ij}$ where V_{ij} and x_{ij} are the parameters representing the variable cost of the demand point i to reach the facility j and the decision variables representing the assignment status of the demand point i to the facility j respectively. By definition, the conditional β-mean equals the average unit cost that is paid by the β percentile of the total demand who pay the highest unit costs (Filippi et al., 2019).

When two objectives which are minimizing cost and minimizing conditional β-mean (e.g. maximizing fairness) are considered, Pareto optimality comes on the scene since it is not possible to solve two objectives to their optimal values at the same time in bi-objective optimization models. In bi-objective optimization, Pareto optimality means the inability to improve one objective function value without diminishing another objective function value. In bi-objective or multi-objective optimization models, there is always a trade-off between different objective functions. For this specific model, if the cost objective is solved to the optimality, then for this solution it would be impossible to obtain the minimum value of the conditional β-mean function, in other words, the maximum level of fairness. Similarly, if the minimum value of the conditional β-mean function or equally maximum level of fairness is reached, then it would not be possible to obtain the minimum total cost. Therefore, in this study, the aim is to find Pareto efficient points for the bi-objective SSCFLP with cost and fairness criteria which is modeled as below:

Parameters:

F_j: Fixed cost of facility j ϵ J.

D_i: Demand of customer i ϵ I.

C_j**:** Capacity of facility j ϵ J.

V_{ij}**:** Variable cost of assigning customer i to facility j (i ϵ I and j ϵ J).

β: A scalar ϵ [0,1] represents the percentile of all customers who pay the highest assignment costs.

α: A scalar ϵ [0,1] defined to consider (1-β) % of the customer demand in the objective function.

λ: A scalar ϵ [0,1] represents the percentage weight of the F-SSCFLP model. In this case, λ represents the weight of total cost whereas (1- λ) represents the weight of the fairness objective.

Decision Variables:

y_j: Binary variable which is equal to 1 if facility j is opened (j ϵ J), 0 otherwise.

x_{ij}: Binary variable which is equal to 1 if customer i (i ϵ I) is assigned to facility j (j ϵ J), 0 otherwise.

Dual Decision Variables:

u: Dual variable for conditional β mean constraint.

v_i: Dual variable for customer demand constraint for each customer i (i ϵ I).

Expressions:

TF: An expression to represent the total amount of fixed costs in the objective function.

TV: An expression to represent the total amount of variable costs in the objective function.

D_β: An expression to represent the total demand of customers who pay the highest cost in the objective function.

Mathematical Formulation:

$$min\ \lambda * (TF + \alpha * TV) + (1 - \lambda) * (D_\beta * u + \sum_i D_i * v_i) \tag{7}$$

subject to:

$$TF = \sum_j y_j * F_j \qquad j \in J \tag{8}$$

$$TV = \sum_i \sum_j D_i * x_{ij} * V_{ij} \qquad i \in I, j \in J \tag{9}$$

$$D_\beta = \left\lceil \beta * \sum_i D_i \right\rceil \qquad i \in I \tag{10}$$

$$\sum_j x_{ij} = 1 \qquad i \in I, j \in J \tag{11}$$

$$\sum_i D_i * x_{ij} \leq C_j * y_j \qquad i \in I, j \in J \tag{12}$$

$$x_{ij} \leq y_j \qquad i \in I, j \in J \tag{13}$$

$$u + v_i \geq (\sum_j V_{ij} * x_{ij})/D_\beta \qquad i \in I, j \in J \qquad (14)$$

$$x_{ij} = \{0,1\} \qquad i \in I, j \in J \qquad (15)$$

$$y_j = \{0,1\} \qquad j \in J \qquad (16)$$

$$v_i \geq 0 \qquad i \in I \qquad (17)$$

The objective function minimizes the weighted sum of the total cost objective and fairness objective. The total cost objective consists of a sum of facility opening costs (TF) and variable costs of customers (α*TV). Fairness objective, on the other hand, represents the total demand of the customers who pay the highest assignment costs which is $(D_\beta * \mathrm{u} + \sum_i D_i * v_i)$. Here conditional β mean method is implemented to observe the cost paid by the customers who pay the highest assignment costs. In this thesis, β is assumed to be equal to 0.05 to consider 5% of total demand as the demand of customers who pay high assignment costs. Although the results will be reported with β=0.05, the same model has also been solved for β=0.01 and it has been proved that β=0.05 gives more sensitive results.

Constraint (8) is constructed to define TF which is an expression in the objective function. Constraint (9) is constructed to define TV which is an expression in the objective function. Constraint (10) is constructed to define D_β which is also an expression in the objective function. D_β represents the demand of customers who pay the highest assignment costs which is the β percentile of the total demand. The value is rounded to the upper integer since demand should be an integer number.

Constraint set (11) with the constraint set (13) guarantees that the total demand of each customer is satisfied by only 1 facility. Constraint set (12) ensures that the total demand of customers assigned to a facility does not exceed the capacity of that facility. Constraint set (13) ensure that customer i can be assigned to facility j if and only if facility j is open.

Constraint set (14) represents the constraint consisting of the dual variables. The formulation for the highest assignment costs which are paid β% of total demand was as follows before the utilization of duality:

$$max\ \frac{1}{D_\beta}\sum_i z_i * \sum_j V_{ij} * x_{ij} \qquad (18)$$

s.t.

$$\sum_i z_i = D_\beta \qquad i\ \epsilon\ I \qquad (19)$$

$$0 \le z_i \le D_i,\ \ z_i\ is\ integer \qquad i\ \epsilon\ I \qquad (20)$$

Since $\sum_j V_{ij} * x_{ij}$ is unimodular, integrality of z_i can be relaxed. After duality is applied, the formulation takes the following form:

$$min\ D_\beta * u\ + \sum_i D_i * v_i \qquad (21)$$

s.t.

$$u + v_i \ge \frac{1}{D_\beta}\sum_j V_{ij} * x_{ij} \qquad i\ \epsilon\ I \qquad (22)$$

$$v_i \ge 0 \qquad i\ \epsilon\ I \qquad (23)$$

Constraint sets (15) and (16) are binary constraints for variables x_{ij} and y_j, while constraint set (17) ensures that all dual variables v_i should be positive.

4.3. The Toy Example

For a better understanding of the effect of the fairness objective on facility location decisions, a toy example is constructed and solved for both the classical facility location model and the bi-objective F-SSCFL model with cost and fairness objectives. In the toy example, there are 5 facilities and 15 customers or in other words, demand points. Demands corresponding to demand points are random numbers between 0 and 100. Capacities of the facilities are set to values between 100 and 400. Fixed costs are determined based on the capacity of each facility in a logical manner. The facilities that have higher capacities have higher fixed costs than the facilities that have low capacities. Data and results for the toy example can be observed below with corresponding explanations.

Data of the toy example consists of 5 facilities and 15 customers (e.g. demand points). Demand points should be considered like wholesale points at which there is a variant number of customers. This situation will be more obvious in the explanation of Table 4.3.

Table 4.1. The capacity of facilities

1	2	3	4	5
250	350	250	100	150

Table 4.1 explains the capacities of each facility in terms of the number of customers. Facility 1,2,3,4 and 5 can serve 250, 350, 250, 100, and 150 customers respectively. In this measure, the number of customers should be regarded as the number of people (individuals) not as demand points as explained above.

Table 4.2. Fixed Cost of facilities

1	2	3	4	5
150000	200000	150000	70000	90000

Table 4.2 gives the opening cost of each facility. Opening costs include all costs which are required to construct and maintain respective facilities. The unit of measurement is USD.

Table 4.3. Demands of Customers

1	2	3	4	5	6	7	8
70	20	50	50	80	15	20	75

9	10	11	12	13	14	15
13	90	50	12	13	75	13

Table 4.3 explains that 15 customers should be thought of as demand points. There are more than 15 customers. For instance, demand point 1 represents 70 people who want to get service from the candidate facilities. Similarly, demand point 13 represents 13

people who want to get service from the facilities which will be opened. When all demand points are considered, 646 people are candidate customers for the facilities which are planned to be opened. In other words, the total demand is 646.

Table 4.4. Variable Costs

i \ j	1	2	3	4	5
1	10	4	13	12	5
2	5	10	1	3	5
3	11	13	17	8	4
4	11	13	19	11	5
5	12	11	1	3	7
6	1	13	11	8	15
7	12	12	21	2	7
8	21	17	11	3	12
9	12	11	13	4	9
10	11	12	15	7	13
11	12	8	5	13	17
12	8	15	4	7	13
13	15	7	18	13	7
14	7	15	9	17	12
15	13	14	13	7	8

Table 4.4 exhibits variable costs which are assignment costs of customers to facilities. Assignment costs are travel costs and all other costs that are paid by customers to reach

a specific facility.

Firstly, the results of the classical model will be tabulated (see Table 4.5). In the classical model, there is only one objective which is minimizing the total cost of locating facilities while considering constraints for the assignment of customers to the candidate facilities as explained in detail in Section 4.1.

Table 4.5. Results of the toy example for the classical model

Total cost	394407		
Total Variable Costs (TV)	4407		
Total Fixed Costs (TF)	390000		
Facilities opened	1	3	5
Demand points assigned to facilities	1,6,10,14	2,5,8,9,11,12	3,4,7,13,15

Table 4.1. shows the total costs, total fixed costs, facilities that are opened, and demand points that are assigned to the respective opened facilities. After observing the results of the classical model, the bi-objective facility location problem with cost and fairness objectives is solved by using the same data.

In F-SSCFLP there are cost and fairness objective functions. The parameter λ gives the weight of the cost objective function while (1-λ) gives the weight of the fairness function. In the results, ρ value represents the total cost function value and δ represents the conditional β-mean value (the measure of fairness). The results of the F-SSCFLP can be seen in Table 4.6.

Table 4.6. Points for different λ values with objective value and opened facilities

λ	**ρ**	**δ**	**Obj value**	**Facilities opened**	**Demand points assigned to facilities**
0	664190	11	11	1	1,6,14
				2	11,13
				3	2,5,8,12
				4	10
				5	3,4,7,9,15
0,5	394407	11,788	197209,4	1	1,6,10,14
				3	2,5,8,9,11,12
				5	3,4,7,13,15
1	394407	33	394407	1	1,6,10,14
				3	2,5,8,9,11,12
				5	3,4,7,13,15

As it is depicted in Table 4.6, 3 different values of the total cost weights (λ) is used to show the model's outcomes for the case when no importance is given to the cost objective (λ=0), the cost and the fairness objectives are given equal importance (λ=0.5) and no importance is given to the fairness objective (λ=1). It can be seen that when the weight of the cost objective increases, the conditional β-mean value (δ) also increases which is negatively correlated with the fairness of the model. Therefore, when the weight, or in other words, the significance of the cost objective function of the model increases, the fairness of the model decreases. For λ=0.5, the δ is less than the δ value for λ=1 but the same number of the facilities are opened. This is because of the size of the toy example. As it was discussed in Chapter 3, the Fair Single Source Capacitated Facility Location Problem (F-SSCFLP) is used for the real-life facility location model. Additionally, it is impossible to open less than 3 facilities in the toy example since the total demand is 646 for the toy example and the capacities of any 2 facilities are not sufficient for this amount of demand. To illustrate, the maximum capacity belongs to facility 2 and the second maximum capacity belongs to facilities 1 and 3 which are equal and 250. Therefore, the capacity of two facilities can be 600 at a maximum which is less than the total demand of 646. However, when λ=0, since all of the significance is given to the fairness, the δ value reaches its minimum, or the fairness reaches to its maximum, therefore all of the facilities are opened and the total cost is very high.

As a summary, for the classical facility location model, the toy example results in the total cost of 394407 with 3 out of 5 facilities open. On the other hand, when the same data is solved for the F-SSCFLP with three different weights, the results of λ=1 are the same as the results of the classical model. This result was expected since all of the importance is given to the cost objective when λ=1 so that the F-SSCFLP becomes the classical facility location problem. As a second inference, when the F-SSCFLP is solved, the model tends to result in a larger number of facilities opened and a higher amount of total cost as the λ and δ get smaller. Since the size of the toy example is too small, an increment in the number of facilities opened and the total cost can be seen when all of the importance is given to fairness (λ=0). However, the negative correlation between the cost and the conditional β-mean values will be obvious with the experimental analyses with the large-scale data in Chapter 6.

The results for the F-SSCFLP with cost and fairness objectives model by using both the Weighted Sum Method and the Epsilon Constraint Method will also be obtained and analyzed in Chapter 5 after explanations of each solution method.

CHAPTER 5 SOLUTION METHODS

In this section, the solution methods of the facility location problems are discussed. Since the F-SSCFLP is a multi-objective optimization problem, first it would be useful to describe multi-objective optimization (MOO) problems and their properties.

MOO problems: Problems that contain more than one objective function are called the MOO problems. The general formulation for MOO problems is as follows:

$Min_x\, Z(x)= (z_1(x)..., z_m(x))$

subject to $x \in S$

$Z: S \rightarrow R^m$

where S is a feasible space and R^m is an objective space.

To completely understand the multi-objective optimization problems, significant terms which are frequently used in these problems will be mentioned below: (Wang et al., 2018), (Stanimirovi´c et al., 2011), (Tamiz, 1996)

Pareto optimal solution: A solution x^* is regarded as Pareto optimal if and only if there does not exist any other feasible solution $x \in S$ such that $z(x) \leq z(x^*)$.

Pareto optimal set (PS): A set that includes all Pareto solutions is called the Pareto optimal set.

Pareto optimal front (PF): The set, $PF = \{Z(x) \in R_m | x \in PS\}$ which includes all Pareto optimal vectors is called the Pareto optimal front.

Pareto efficiency: An objective function value is said to be "Pareto efficient" if it cannot get a better value without worsening a value of another objective function value.

Pareto inefficiency: In contrast to Pareto efficiency, an objective function value is called "Pareto inefficient" if it can get better without degrading another objective function value.

Ideal point: An objective vector $i^* = (i_1^*, \ldots, i_m^*)$ where i_j^* is the infimum (greatest lower bound) of $Z_i(x)$ for every $j \in \{1, 2, ..., m\}$.

When the above definitions are considered, the most significant one is Pareto Efficiency since it is used to detect Pareto optimal solutions in this study. To depict the Pareto Efficiency, the change in one objective function value with respect to the change in another objective function value for a bi-objective optimization problem can be seen in Figure 5.1. The Pareto efficient points in Figure 5.1 are the solutions to one of the benchmark problems that are used to solve the model studied in this study by using the Epsilon-Constraint method. Although the Epsilon Constraint Method will be explained in detail later, it would be beneficial to present the graphical representation of the relationship between two objective functions for a bi-objective optimization problem here. Other line graphs that depict the trade-off between two objective functions can be found in APPENDIX 2.

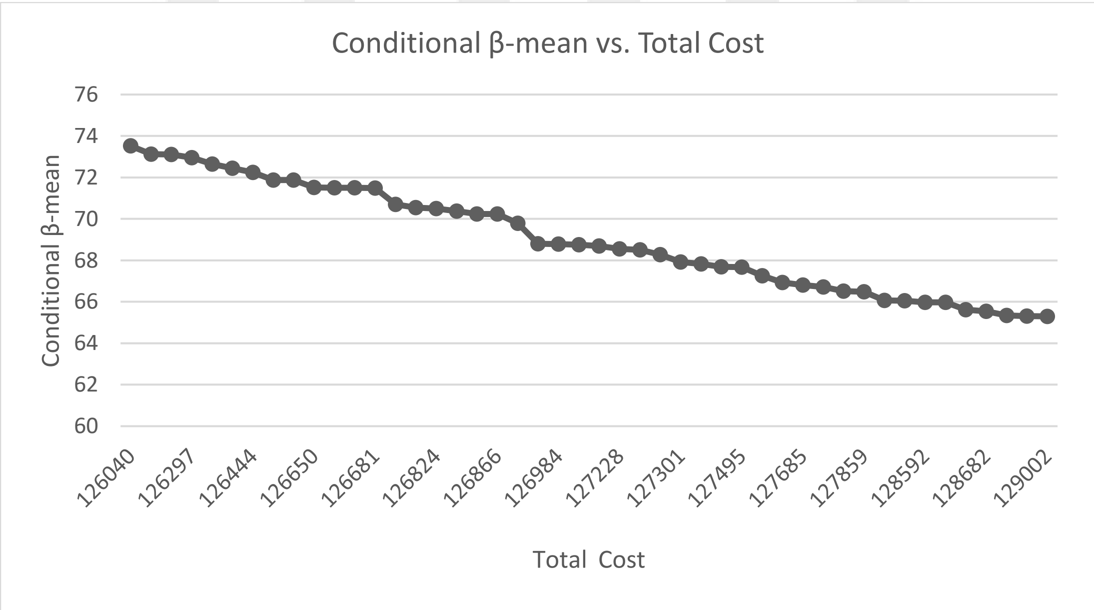

Figure 5.1. The trade-off between two objective functions

When all of the Pareto efficient points are put on a graph as in Figure 5.1., the trade-off between two objective function values (Conditional β-mean and Total Cost) is obvious. As it is seen in Figure 5.1, Total Cost is increasing while the Conditional β-mean value is decreasing. This outcome is expected since an objective function value is regarded as "Pareto efficient" when it gets better only if it degrades another objective function value according to the definition of "Pareto efficiency" as mentioned above.

After mentioning some properties of the MOO problems and giving related definitions, the most popular methods reported in the literature to solve the F-SSCFLP as a bi-objective optimization problem will be described briefly below.

5.1. The Weighted Sum Method

In the weighted sum method, separate objective functions are linearly summed up to one objective function with a weight vector. This method is commonly used to solve multi-objective problems.

The main characteristic of the Weighted Sum Method can be described below in Figure 5.1. The parameter λ_i represents the weight of the respective objective function.

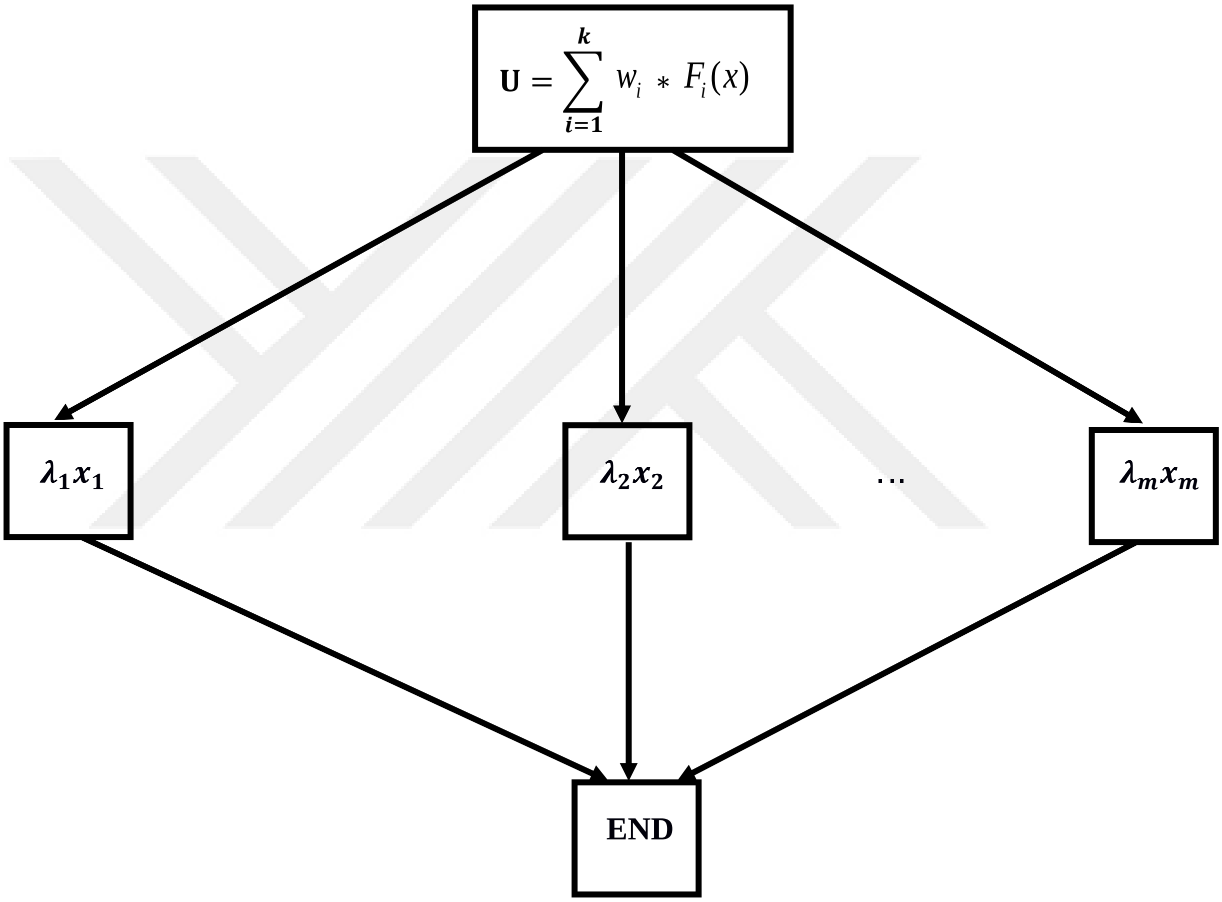

Figure 5.2. The Weighted Sum Method

In early applications of the weighted sum method, weights were defined before starting the search or they were set during the search in an interactive way. Afterward, evolutionary multi-objective algorithms emerged. Evolutionary multi-objective algorithms are very beneficial since there are some disadvantages of the early weighted sum method while approximating the Pareto Front for a multi-objective problem. Some of the disadvantages are specified as below:

i) A priori selection of the weights, that seems satisfactory, may not guarantee an acceptable solution,

ii) The Weighted Sum Method does not give the Pareto efficient points that are on the nonconvex parts of the Pareto optimal set,

iii) Even distribution of the Pareto optimal points cannot be guaranteed by changing the weights of each objective function continuously.

Since F-SSCFLP is also a multi-objective problem, the use of the Weighted Sum Method will be illustrated with a toy example which has been constructed in Chapter 4. In this toy example, various λ_i values have been used. The results can be seen in Table 5.1.

Table 5.1. Sigma and Rho values with the Weighted Sum Method

λ	**ρ**	**δ**	**Objective value**	**Facilities Opened**
0	**664190**	**11**	11	1,2,3,4,5
0,1	**394410**	**11,788**	39451,31	1,3,5
0,2	394410	11,788	78890,83	1,3,5
0,3	394410	11,788	118330,4	1,3,5
0,4	394410	11,788	157769,9	1,3,5
0,5	394410	11,788	197209,4	1,3,5
0,6	394410	11,788	236648,9	1,3,5
0,7	394420	11,788	276097,5	1,3,5

0,8	394420	11,788	315538,4	1,3,5
0,9	**394420**	**11,788**	354979,2	1,3,5
1	**394407**	**33**	394407	1,3,5

In Table 5.1, 4 efficient points are detected at first and are shown in bold but according to Pareto efficiency which was described in Chapter 3, (394410, 11,788) outperforms (394420, 11,788). Therefore points (664190, 11), (394410, 11,788), (394407,33) are solutions that are obtained by using the Weighted Sum Method.

By using the Weighted Sum Method for this bi-objective problem, 3 Pareto efficient points are detected. The first of these solutions has $M_\beta(x)=11$ with a total cost of 664190 and all facilities open (5 out of 5). The second point has $M_\beta(x)=11.788$ with a total cost of 394410 and 3 out of 5 facilities open. Finally, the third point has $M_\beta(x)=33$ with a total cost of 394407 and 3 out of 5 facilities open. The efficient points (664190, 11), (394410, 11,788), (394407,33) for the SSCFLP proves that as $M_\beta(x)$ value decreases more facilities are opened and fairness among customers increases.

5.2. The Epsilon Constraint Method

Epsilon Constraint method is a mathematical method that is useful for solving multi-objective optimization problems. A multi-objective optimization problem can be transformed into a single objective problem by adding other objectives as constraints to the model (Becerra et al., 2006)

The general formulation of the method is as follows:

$$min\ F_i(x)$$

$$subject\ to\ F_j(x)\ \leq\ \varepsilon_j\quad for\ all\ j = 1\ ,2, \ldots, m, j\ \neq\ i,$$

$$x\ \in\ S$$

for the multi-objective model such that

$$min \sum_j F_j(x) \qquad forall\ j = 1,2, \ldots, m$$

$x \in S$

In this formulation, a vector (ε_1, ε_2, ..., ε_m) represents the maximum attainable value for every single objective. S is the feasible region. To obtain the subset of the Pareto optimal set for a specific multi-objective problem, a person who wants to solve that problem can change maximum attainable values in a vector (ε_1, ε_2, ..., ε_m) and therefore obtain a progressive optimization process for each updated vector.

The advantages of the Epsilon Constraint method over the Weighted Sum method can be seen below (Mavrotas, 2009).

1. The Weighted Sum Method operates in the feasible region and thus gives only extreme solutions while the Epsilon Constraint Method can change the original feasible region and gives also non-extreme efficient solutions. The Epsilon Constraint Method proposes a variety of efficient solutions while the Weighted Sum Method gives the same extreme solution with different combinations of weights.
2. In the Weighted Sum Method, the scaling of the objective function is very significant while in the Epsilon Constraint Method, there is no need to do scaling.

The toy example in Chapter 4 is solved by using the Epsilon Constraint Method and the results can be seen in Table 5.2 and Table 5.3.

Table 5.2. Sigma and Ro values with the Epsilon Constraint Method

ρ	δ
394407	33
394407	22
394407	15
394407	13
394407	12,5
394407	12,3

394407	12,1
394407	12,05
394407	12,03
394407	12,01
394410	12
394407	11,8
394407	**11,79**
463644	**11,78**
463644	11,77
463644	11,75
463644	11,7
463644	11,5
463644	11,47
463644	**11,46**
463655	**11,45**
463655	11,4
463655	11,3
463655	11

Table 5.2 exhibits the implementation of the Epsilon Constraint method with the toy example data. In this table, ρ equals the total cost objective of the bi-objective model and σ equals to the $M_\beta(x)$ (conditional β-mean) which is the fairness objective of the model. In this table, Pareto efficient points are exhibited in bold. Pareto efficient points are detected by following the concept of Pareto efficiency. If one point or solution is called "Pareto efficient", then as one objective of that solution (e.g. σ) decreases or get

better, the other objective of that solution (e.g. ρ) should decrease or worsen or vice versa since it is not possible to improve both objective values at the same time in bi-objective optimization problems.

Table 5.3. Overall results of the Epsilon Constraint Method

Efficient points				
ρ	**δ**	**TF**	**TV**	**facilities open**
394407	**11,79**	390000	4407	1,3,5
463644	**11,46**	460000	3644	1,3,4,5
463655	**11,45**	460000	3655	1,3,4,5

Table 5.3 can be regarded as the summary of table 5.2. All Pareto efficient points that were detected in Table 5.2 are depicted in Table 5.3. As it can be observed from the table, the efficient point (463644, 11.78) which was detected in Table 5.2 is omitted in Table 5.3 since the efficient point (463644, 11.46) is superior to the efficient point (463644, 11.78). These two points have the same ρ value while the σ value of the first point is smaller or equivalently better than the σ value of the latter point. Table 5.3 also gives information about Total Fixed costs (TF), Total Variable costs (TV), and numbers of opened facilities corresponding to each Pareto efficient point. The first Pareto efficient point has $M_\beta(x)$=11.79 with a total cost of 394407 and 3 out of 5 facilities open. The second point has $M_\beta(x)$=11.46 with a total cost of 463644 and 4 out of 5 facilities open. Finally, the third point has $M_\beta(x)$=11.45 with a total cost of 463655 and 4 out of 5 facilities open. As it can be seen from the results, as $M_\beta(x)$ decreases (e.g. the fairness level increases), the total cost increases with a higher number of facilities open. Since this is a toy example and the data are too small, the Epsilon Constraint method was able to detect only 3 Pareto efficient points. However, as it will be seen in Chapter 6, the Epsilon Constraint method can give results with a larger number of Pareto efficient points which is proved by experiments with the data consisting of 30 candidate facilities and 200 demand points to 80 candidate facilities and 400 demand points.

5.3. The Benders Decomposition Method

Let's consider an MIP such that:

$$min\ cx\ +\ dy \tag{25}$$

$$Ax \geq b \tag{26}$$

$$Tx + Qy \geq r \tag{27}$$

$$x, y \geq 0\ \ and\ \ x \epsilon \mathbb{Z}^n \tag{28}$$

$$c\ \epsilon\ \mathbb{Q}^n, d\ \epsilon\ \mathbb{Q}^p, A\ \epsilon\ \mathbb{Q}^{m_1 x\ n}, T\ \epsilon\ \mathbb{Q}^{m_2 x\ n}\ and\ Q\ \epsilon\ \mathbb{Q}^{m_2 x\ p}$$

Let's assume the variable η represents the total contribution of y variables in the objective function. Then the model is converted to the following modified model:

$$min\ cx\ +\ \eta \tag{29}$$

$$Ax \geq b \tag{30}$$

$$< Benders\ Cuts > \tag{31}$$

$$x \epsilon Z_+^n, \eta\ free \tag{32}$$

and the modified model is called the Master Problem (MP). The CGLP is composed in combination with the solution of the master problem. The process is iterative but is pruned by the time the CGLP is not able to find new cuts. According to the Benders Theorem, when the problem is pruned, the corresponding solution is proven to be the optimal solution.

Set of Benders Cuts (31) are empty at first. Constraint sets (30)-(32) are assumed to be feasible or the original problem is infeasible which is a stopping criterion in the Benders Decomposition method.

(x^*, η^*) is assumed to be the solution of the master problem if y satisfies the linking constraints (27) for the x=x^* value. The following theorem shows the derivation of Benders cut or else proving the optimality.

Benders Theorem:

Let (x^*, η^*) be the optimal solution of (29)-(32) and the CGLP is defined as:

$$min\ dy \tag{33}$$

$$Tx^* +\ Qy\ \geq\ r \tag{34}$$

$$y \geq 0 \tag{35}$$

if (33)-(35) is infeasible, then there is $\pi \in Q_+^{m_2}$ such that

$$\pi T x \geq \pi r \tag{36}$$

and $\pi T x^* < \pi r$

or if (33)-(35) has an optimal solution y^*,

then $dy^* \leq \eta^*$ and (x^*, y^*) is an optimal solution for (25)-(28). Otherwise, there is $\pi \in Q_+^{m_2}$ such that the equation

$$\eta + \pi T x \geq \pi r \tag{37}$$

and cuts off (x^*, η^*).

According to the Benders theorem, (36) is the feasibility cut, and (37) is the optimality cut (Bonami et al., 2020).

The following chart would be beneficial to visualize the logic of the Benders Decomposition:

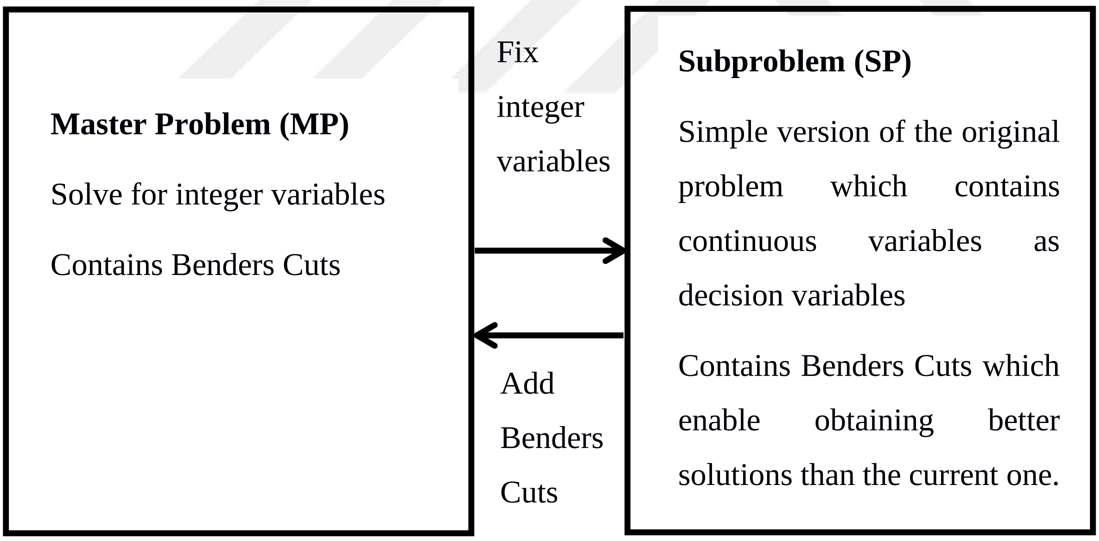

Figure 5.3. Master Problem vs. Subproblem

The dual problem gives clues about the optimality or the feasibility of the original problem. The dual problem can also give some bound to the candidate solutions.

The stepwise utilization of the Benders Decomposition for the F-SSCFLP is as follows:

Step1: A master problem and a subproblem should be identified. A master problem should be the complicated one and the subproblem should be easier (Boschetti et al., 2009).

An illustration for the master problem can be as follows:

$$z_{MP} = min \sum_j F_j y_j + z_{SP}(y) \tag{38}$$

$$y_j \epsilon \{0,1\} \qquad j\epsilon J \tag{39}$$

the subproblem (SP) z $_{SP}$ is:

$$z_{SP}(y) = min \sum_i \sum_j D_i x_{ij} V_{ij} \tag{40}$$

$$\sum_j x_{ij} = 1 \qquad i \epsilon I \tag{41}$$

$$\sum_i D_i * x_{ij} \leq C_j * y_j \quad j\epsilon J \tag{42}$$

$$x_{ij} \leq y_j \qquad i \epsilon I;\ j \epsilon J \tag{43}$$

$$x_{ij} = \{0,1\} \qquad i \epsilon I;\ j \epsilon J \tag{44}$$

Step 2: The master problem should be solved. After adding Bender's cuts, the problem is still NP-hard but easier to solve.

Step 3: The subproblem should be solved. The x_{ij} should be an integer.

Step 4: Add Bender's cuts generated by the subproblem to the master problem. To obtain the subproblem's dual, constraints (44) should be relaxed into x_{ij}≥0, $i \epsilon I;\ j \epsilon J$. Dual variables w_i^{l}, $j \epsilon J$ is associated with constraints (41), dual variables w_j^{ll} are associated with constraints (42) and w_{ij}^{lll} are associated with constraints (43).

Then the subproblem is as following:

$$z_{SP}(y) = max \sum_i w_i^{l} + \sum_j w_j^{ll} C_j y_j + \sum_i \sum_j w_{ij}^{lll} y_j \tag{45}$$

$$w_i^{l} + D_i w_j^{ll} + w_{ij}^{lll} \leq D_i V_{ij} \quad i\epsilon I; j\epsilon J \tag{46}$$

$$w_j^{ll} \leq 0 \tag{47}$$

$$w_{ij}^{lll} \leq 0 \qquad (47)$$

Then the formulation for the master problem is as follows:

$$min\ z$$

$$z \geq \sum_j (F_j + C_j w_j^{II} + \sum_i w_{ij}^{III})y_i + \sum_i w_i^{I} \tag{49}$$

$$y_j \in \{0,1\} \qquad j \in J \tag{50}$$

The basic idea of the Benders Decomposition: ("Benders Decomposition in IBM CPLEX")

$$min\ c^T x + d^T y \tag{51}$$

$$Ax \geq b \tag{52}$$

$$Tx + By \geq e \tag{53}$$

$$x \geq 0\ and\ x\ integer \tag{54}$$

$$y \geq 0 \tag{55}$$

where

x: integer or continuous primary variables

y: secondary continuous variables

and $Tx + By \geq e$ are the linking constraints.

when the variable $\eta = d^T y$ is added to the objective function and projected onto the (x, η) space, the model becomes:

$$min\ c^T x + \eta \tag{56}$$

$$Ax \geq b \tag{57}$$

$$v^T(e - Tx) \leq \eta, \qquad v \in V \tag{58}$$

$$r^T(e - Tx) \leq 0, \qquad r \in R \tag{59}$$

$$x \geq 0, \quad integer \tag{60}$$

$$where\ V, R\ are\ the\ extreme\ rays\ of\ Q = \{\pi \in R^m : \pi^T B \leq d^T, \pi \geq 0\}$$

The illustration of the Master Problem and the Subproblem is made with the figure below

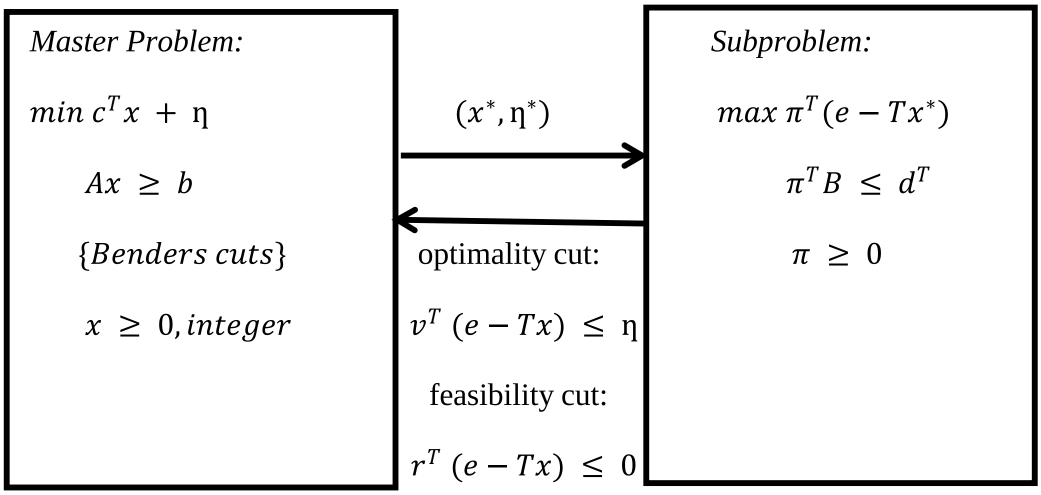

Figure 5.4. Illustration of the Master Problem vs. Subproblem (Tramontani, 2018)

The two-stage approach for the Benders Decomposition is tabulated in Table 5.4.

Table 5.4. Two-stage approach for the Benders Decomposition (Tramontani, 2018)

Stage	**Cut Generating LP (CGLP)**	**Dual of**	**Cut**
1	$min\ d_i{}^T y_i$ $B_i y_i \geq e_i - T_i x^*$ $y_i \geq 0$	$max\ \pi^T (e_i - T_i x^*)$ $\pi^T B_i \leq d_i T$ $\pi \geq 0$	*Optimality*
2	$min\ s$ $B_i y_i + Is \geq e_i - T_i x^*$ $y_i \geq 0$ $s \geq 0$	$max\ \pi^T (e_i - T_i x^*)$ $\pi^T B_i \leq 0$ $I^T \pi \leq 1$ $\pi \geq 0$	*Feasibility*

F-SSCFLP and Benders Decomposition

In this thesis Benders Decomposition (BD) in CPLEX is used to solve large-scale problems. However, in classical implementations of the BD method, the original problem (OP) is divided into two parts, namely the master problem (MP) and the subproblem (SP). Therefore, the implementation of the BD method on F-SSCFLP which has been studied by Filippi et al. (2019) is given in this part. According to this implementation, the linear variable v_i is decided to be the variable of the SP while other variables y_j, x_{ij} and u are decided to be the decision variables of the MP. Consequently, $y_j=\bar{y}_j$, $x_{ij}=\bar{x}_{ij}$ and $u=\bar{u}$. Then the mathematical model for the SP takes the form:

$$min \sum_i D_i * v_i \tag{61}$$

s.t.

$$v_i \geq \frac{1}{D_\beta}\sum_i \sum_j c_{ij}\,\bar{x}_{ij} - \bar{u} \qquad i \in I;\ j \in J \tag{62}$$

$$v_i \geq 0 \qquad i \in I \tag{63}$$

The dual of the SP should be solved according to the Benders Decomposition approach. Therefore, the dual of the SP is as follows:

$$max \sum_i \left(\frac{1}{D_\beta}\sum_i \sum_j c_{ij}\,\bar{x}_{ij} - \bar{u}\right) * z_i \tag{64}$$

s.t.

$$0 \leq z_i \leq d_i \qquad i \in I \tag{65}$$

If the SP is assumed to be bounded then the feasible region of the dual of the SP would not be empty. Otherwise, the feasible region of the dual of the SP would be empty and the OP would also be unbounded. If it is assumed that the feasible region of the dual of the SP is not empty. Since the solution of the SP is always bounded and therefore feasible. This can be proved as follows:

When constraint sets (62) and (63) are combined, it can be observed that;

$$\frac{1}{D_\beta}\sum_i \sum_j c_{ij}\,\bar{x}_{ij} - \bar{u} \geq 0$$

which means that the SP is bounded since the SP is a minimization problem. According to the Benders Decomposition algorithm, if the SP is bounded and therefore feasible, only optimality cuts should be constructed and added to the SP (Rahmaniani et al., 2017)

Consequently, the optimal solutions can be demonstrated as:

$$z_i^* = \begin{cases} d_i \; if \; \frac{1}{D_\beta} \sum_i \sum_j c_{ij} \, \bar{x}_{ij} > \bar{u}, \\ 0 \qquad\qquad\qquad otherwise \end{cases}$$

Since the solution of the SP is feasible, there is no need to add feasibility cuts and thus only optimality cuts should be added to the MP. According to these considerations, the MP can be structured as follows:

$$min \; \lambda(\sum_j F_j * \; y_j + \alpha \sum_i \sum_j V_{ij} * D_i * x_{ij}) + (1-\lambda)(D_\beta \, u + \omega) \tag{66}$$

subject to

$$\sum_i z_i^{\theta} (\frac{1}{D_\beta} \sum_j V_{ij} * x_{ij} - u) \leq \omega, \quad \theta \in \Theta \tag{67}$$

$x, y \in XY$

Θ is the set of optimality cuts.

As it can be observed from the above objective function and constraints, the Benders decomposition method can be applied to F-SSCFL problems by constructing a new model from scratch. However, CPLEX proposes a default strategy for Benders decomposition which performs very well to solve facility location problems. Therefore, this strategy will be explained and used to solve the problems with a large number of facilities and customers. Then, the results which are obtained with this default method will be compared with the results which have been obtained with other methods in terms of optimality gap and CPU time.

Benders Decomposition in CPLEX

If the user does not provide any annotation then CPLEX automatically splits the problem into two parts. Integer variables are designated to the master problem while continuous variables are designated to the CGLP. For Mixed Integer Linear Programs (MILP), by default CPLEX decomposes the model by using three different procedures. These procedures are USER, WORKERS, and FULL procedures. In the USER procedure, CPLEX attempts to decompose the model by using the annotations which are specified by the user. In the WORKERS procedure, there are two steps. Initially, CPLEX decomposes the model according to the user's annotations and then it improves the decomposition by decomposing the specified subproblems. While using the WORKERS procedure, it is recommended to make annotations to assign specific variables to the master problem and other variables to the subproblem so that CPLEX would be able to decompose the problem further. In the FULL procedure, CPLEX automatically decomposes the problem without considering any annotations which are specified by the user. All integer variables are assigned to the master problem, all continuous variables are assigned to the subproblem and the problem is further decomposed by CPLEX if it is possible. Other than explained three procedures, the Benders Strategy parameter in CPLEX can take two other values which are -1 (OFF) or 0 (AUTO). When the Benders Strategy parameter is -1, then the conventional branch and cut procedure are applied to the model. On the other hand, when the Benders Strategy parameter is 0, then the model is run with the conventional branch and cut if there is no annotation, otherwise, the same procedure in the WORKERS procedure is applied to the model. The Benders Strategy parameter values with the corresponding explanations can be seen in Table 5.5. In this thesis, the FULL procedure is used for large-scale instances since it can be very complicated to solve SSCFL problems when the number of customers and facilities is very large.

Table 5.5. Benders Strategy Parameter in CPLEX (Tramontani, A., 2018)

Parameter #	Explanation
1 (USER)	Use the decomposition which is provided in the annotation.
2 (WORKERS)	Use the annotation solely for the identification of the master and the secondary variables. Then, the Automatic Workers decomposition is made to decompose the second stage matrix into disjoint blocks.
3 (FULL)	Any annotation is not used. Integer variables are simply assigned to the master block and other variables are assigned to the workers and then workers decomposition is used.
-1 (OFF)	Any annotation is not used and the conventional branch & cut is applied.

0 (AUTO), default	If there is no annotation, then the conventional branch & cut is applied. Otherwise, the procedure is the same as the WORKERS procedure.

CHAPTER 6 COMPUTATIONAL RESULTS

Experimental analysis was performed based on two different SSCFLP benchmark instance sets. These sets are called “Yang” and “TBED1” sets, and they are presented in the studies (Yang et al, 2012) and (Avella et al., 2007) respectively.

Both of the sets are can be downloaded on the University of Brescia website (Brescia University, 2021). The features of the instance sets are given in the following table below.

Table 6.1. Benchmark Problem Sets

Instance Set	Number of Instances Included	Size of the Problems (Number of Facilities x Number of Demand Points)	Solution Methods Used to Solve the Problems in This Study
Yang	20	30 x 200	Weighted Sum Epsilon Constraint Benders Decomposition
		60 x 200	
		60 x 300	
		80 x 400	
TBED1	20	1000 x 1000	Benders Decomposition

As it can be seen in Table 6.1, the Weighted Sum Method (WS) and the Epsilon Constraint method were used to solve the small to large scale Yang instances from 30 candidate facilities and 200 demand points, up to 80 candidate facilities and 400 demand points. The Benders Decomposition (BD) method was used to solve the same problem instances as well as the instances included in TBED1 with 1000 candidate facilities and 1000 demand points.

During the experimental analyses, firstly, both β=0.01 and β=0.05 values have been used to make logical inferences about three methods which are the Weighted-Sum (WS) method, the Epsilon Constraint method, and the Benders Decomposition (BD) method as well as to make logical comparisons between these three methods. There is a consensus in the literature that β=0.01 and β=0.05 values are the most reasonable values to study. However, according to the results of the analyses, β=0.05 gives more sensitive results when compared with the results of β=0.01. Therefore, the results of the three methods will be reported in this section given that β=0.05.

6.1. The Weighted Sum Method

The outcomes of the Weighted Sum Method (WS) are presented in the following tables. In each table, there are two rows for each problem instance to represent the Pareto optimal solutions for that problem. Table 6.2 describes the outcomes of the problem instances with 30 candidate facilities and 200 demand points.

Table 6.2. WS Outcomes (Problem Size 30x200)

Instance #	Total Cost	Conditional β-mean	Total Fixed Cost (TF)	Change in Total Cost	Change in Conditional β-mean	Change in Total Fixed Cost
30-200-1	178560	62,76	48151	-0,18	0,15	-0,30
30-200-1	145730	71,90	33854			
30-200-2	166470	65,10	46091	-0,24	0,13	-0,33
30-200-2	126040	73,52	30733			
30-200-3	160270	73,15	48424	-0,20	0,06	-0,31
30-200-3	128220	77,40	33554			
30-200-4	180040	66,76	49104	-0,24	0,08	-0,41
30-200-4	136090	72,10	29018			
30-200-5	190830	81,60	49870	-0,29	0,03	-0,42
30-200-5	136090	84,42	29018			

In table 6.2., the results of five instances with 30 candidate facilities and 200 demand points are reported. While solving the model with the WS method, λ values have been incremented by the scale of 0.1 and then Pareto efficiency has been used to detect Pareto efficient points. The Total Cost column represents the summation of the opening cost of the facilities and the total assignment costs of the customers to the facilities. On the other hand, the Conditional β-mean column represents the fairness function which should increase when the fairness of the model decreases and vice versa. Additionally, the Total Cost Change column gives the percentage change (increment/decrement) of the total costs belonging to the efficient points of the same instance. Similarly, the Conditional β-mean Change column represents the percentage change of the conditional β-mean values of the same instance.

As a significant inference from the table, it can be observed that as the percentage change in the conditional β-mean value is positive, the percentage change in the total cost is negative. This proves the negative correlation between the total costs and conditional β-mean values. Additionally, there is also a negative correlation between the Total Fixed Costs (TF) and the conditional β-mean values.

The bar chart below describes the relative changes in Total Cost and Total Fixed Costs with respect to the changes in conditional β-mean values. The relative change is the absolute value of the division of the cost changes by the change in conditional β-mean.

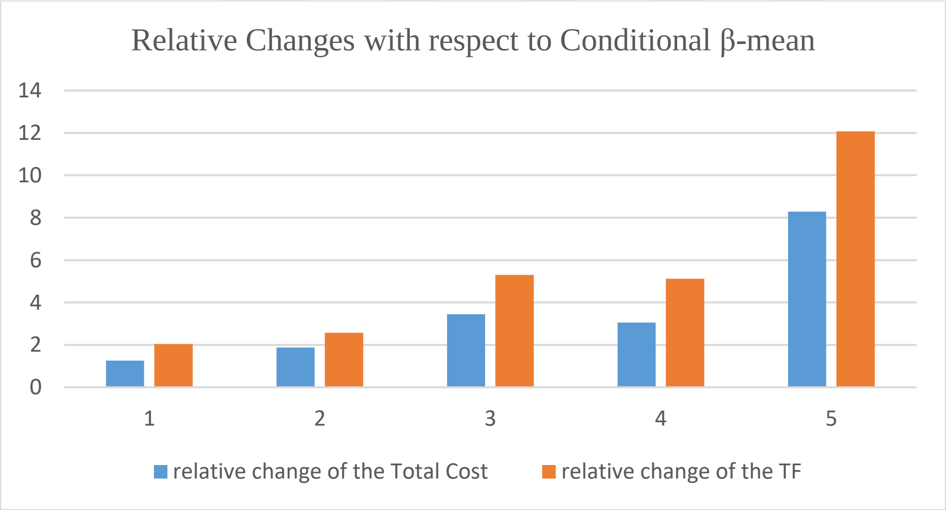

Figure 6.1. Relative Changes with respect to Conditional β-mean values (30x200)

According to Figure 6.1, it can be seen that there is a larger relative change of the TF with respect to conditional β-mean values when compared with the relative change of the Total Cost with respect to conditional β-mean values.

Next, Table 6.3 describes the outcomes of the problem instances with 60 candidate facilities and 200 demand points.

Table 6.3. WS Outcomes (Problem Size 60x200)

Instance #	Total Cost	Conditional β-mean	Total Fixed Cost (TF)	Change in Total Cost	Change in Conditional β-mean	Change in Total Fixed Cost
60-200-1	192030	55,82	93484	-0,36	0,02	-0,56
60-200-1	123380	57,13	40982			
60-200-2	175020	39,82	93282	-0,31	0,11	-0,57
60-200-2	120840	44,37	40495			
60-200-3	181030	41,95	88389	-0,42	0,09	-0,56
60-200-3	105590	45,61	3852			
60-200-4	187360	44,60	95381	-0,38	0,10	-0,61
60-200-4	116698	49,02	37062			
60-200-5	152390	40,13	84470	-0,33	0,12	-0,57
60-200-5	101414	44,91	36194			

The negative correlation between the Total Cost as well as Total Fixed Cost (TF) and the conditional β-mean for 60 candidate facilities and 200 demand points can be seen in Table 6.2. The relative changes in Total Costs and Total Fixed Costs with respect to conditional β-mean values for Yang instances with 60 candidate facilities and 200 demand points can be seen as a bar-chart below:

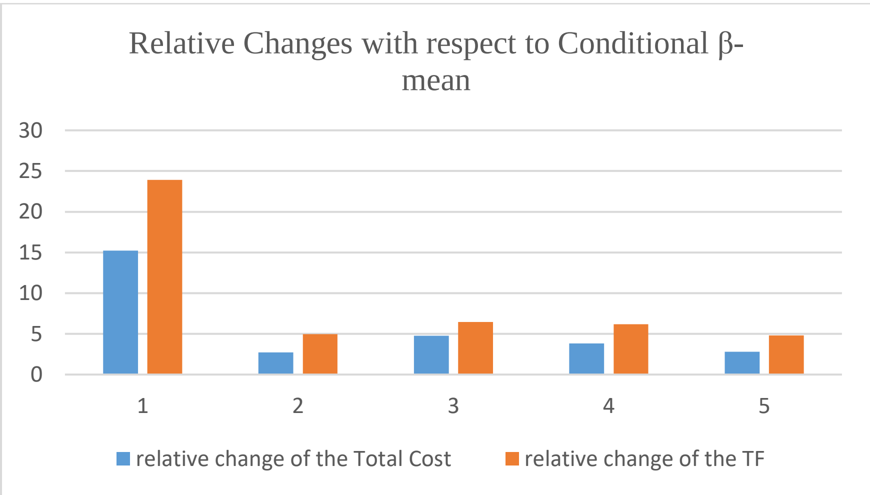

Figure 6.2. Relative Changes with respect to Conditional β-mean values (60x200)

According to Figure 6.2, the relative changes of the TF values with respect to the changes in conditional β-mean values are more than the relative changes of the Total Cost values with respect to the conditional β-mean values for each instance with the size of 60 candidate facilities and 200 demand points.

Next, the outcomes of the problem instances with 60 candidate facilities and 300 demand points are described in Table 6.4.

Table 6.4. WS Outcomes (Problem Size 60x300)

Instance #	Total Cost	Conditional β-mean	Total Fixed Cost (TF)	Change in Total Cost	Change in Conditional β-mean	Change in Total Fixed Cost
60-300-1	256920	49,03	113550	-0,31	0,06	-0,57
60-300-1	176380	51,92	48869			
60-300-2	254420	42,85	116670	-0,38	0,06	-0,57
60-300-2	158582	45,47	49588			
60-300-3	272920	49,62	140640	-0,32	0,10	-0,64
60-300-3	185382	54,55	50526			
60-300-4	262980	56,01	124770	-0,33	0,04	-0,63
60-300-4	176920	58,34	46330			
60-300-5	262580	39,23	146490	-0,36	0,21	-0,61
60-300-5	167473	47,58	56917			

Table 6.4 depicts the solutions of the data with 60 candidate facilities and 300 demand points by using the WS method. It can be inferred from the table that there is a negative correlation between Total Costs as well as TF and the conditional β-mean values.

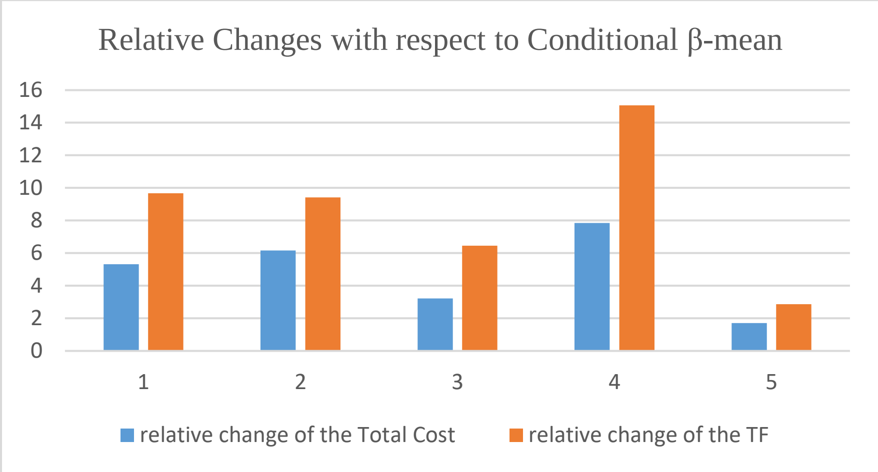

Figure 6.3. Relative Changes with respect to Conditional β-mean values (60x300)

According to Figure 6.3, the relative change of the TF with respect to the conditional β-mean value is higher than the relative change of the Total Cost with respect to the change in conditional β-mean value for each instance. Therefore, most of the increase in the total cost arises from opening costs.

Finally, the outcomes of the problem instances with 80 candidate facilities and 400 demand points are described in Table 6.5.

Table 6.5. WS Outcomes (Problem Size 80x400)

Instance #	Total Cost	Conditional β-mean	Total Fixed Cost (TF)	Change in Total Cost	Change in Conditional β-mean	Change in Total Fixed Cost
80-400-1	309930	29,65	181900	-0,38	0,27	-0,62
80-400-1	193231	37,64	69862			
80-400-2	366440	40,96	219100	-0,40	0,14	-0,69
80-400-2	219100	46,73	67652			
80-400-3	315620	36,02	157450	-0,37	0,07	-0,60
80-400-3	199880	38,65	62354			
80-400-4	363650	38,37	219490	-0,44	0,09	-0,68
80-400-4	204760	41,90	70710			
80-400-5	282580	37,19	150700	-0,33	0,11	-0,57
80-400-5	190310	41,42	65017			

According to Table 6.5, when the instances with 80 candidate facilities and 400 demand points are solved, it is observed that the total cost change is in the opposite direction with the conditional β-mean change. The relative changes in total costs and fixed cost with respect to the changes in conditional β-mean values can be seen as a bar chart in Figure 6.4.

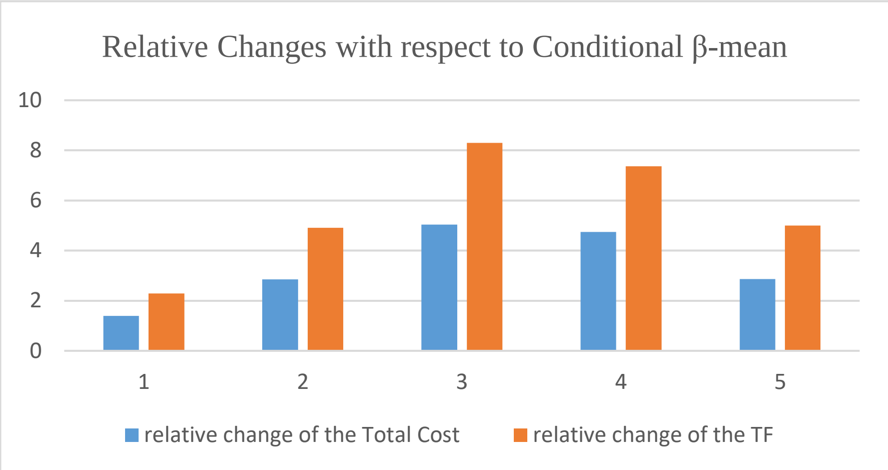

Figure 6.4. Relative Changes with respect to Conditional β-mean values (80x400)

As an overall assessment of the all tables and figures above, when the same instance is taken into account, it can be seen that as the conditional β-mean increases, the total cost value decreases and vice versa.

This result had been expected before making this experimental analysis on the SSCFLP since as the conditional β-mean value increases, the fairness of the model decreases therefore a smaller number of facilities would be opened. Similarly, as the conditional β-mean value decreases, the fairness of the model increases therefore a larger number of facilities would be opened.

Notice that the percentage change and relative change in Total Variable Cost (TV) which is the assignment cost of the demand points to the facilities (i.e., operating cost) is not given in the tables and figures above. However extended analysis has also been done regarding Total Variable Cost (TV) along with the information given above. The extended analysis is given in APPENDIX 1.

6.2. The Epsilon Constraint Method

The Epsilon Constraint Method is implemented on the same Yang problem instances as the WS method. Average of Total Costs, Average Conditional β-mean values, and their respective changes can be seen in the following tables.

The entries in the tables indicate average values since a large number of Pareto efficient points have been detected for each instance by using the Epsilon Constraint method. However, extended results and related graphs for each problem instance are tabulated in APPENDIX 2.

The first table below, Table 6.6 represents the results for the problem instances with 30 candidate facilities and 200 demand points.

Table 6.6: Epsilon Constraint Method Results (Problem Size 30x200)

Instance #	Average Total Cost	Average Conditional β-mean	Average TF	Average TV	Average Total Cost Change	Average Conditional β-mean Change	Average TF change	Average TV change
1	146569,56	66,17	33416,50	113153,89	0,0009	-0,0077	0,0001	0,0011
2	127290,85	69,04	31133,50	96157,35	0,0005	-0,0026	0,0005	0,0006
3	128910,60	75,03	34796,13	94114,47	0,0013	-0,0039	0,0065	-0,0006
4	136172,06	71,40	29018,00	107154,07	0,0005	-0,0074	0,0000	0,0006
5	132395,86	83,06	32638,86	99757,00	0,0034	-0,0050	0,0106	0,0010

Table 6.6 demonstrates the average total cost, average conditional β-mean, average total opening cost of facilities (TF), average total assignment costs of customers to the facilities (TV), and their percentage changes respectively for the instances consisting of 30 candidate facilities and 200 demand points by using the Epsilon Constraint method.

Based on the results, it is seen that as the average conditional β-mean change is negative, then the average total cost change is positive. Similarly, the average TF change is also positive. This is an expected result since as the fairness of the model increases, it is expected to have more open facilities and therefore higher opening cost.

Next, Table 6.7 represents the results for the problem instances with 60 candidate facilities and 200 demand points.

Table 6.7. Epsilon Constraint Method Results (Problem Size 60x200)

Instance #	Average Total Cost	Average Conditional β-mean	Average TF	Average TV	Average Total Cost Change	Average Conditional β-mean Change	Average TF change	Average TV change
1	123694	68,98	42109,50	81584,50	0,0052	-0,2933	0,0550	-0,0196
2	123103	41,91	43724,75	79378,44	0,0032	-0,0071	0,0116	-0,0010
3	106136	42,79	40890,14	65246,00	0,0016	-0,0132	0,0166	-0,0070
4	117309	46,87	38146,17	79162,75	0,0009	-0,0075	0,0048	-0,0008
5	102619	41,97	40153,37	62465,68	0,0017	-0,0062	0,0097	-0,0029

According to the results in Table 6.7, as the average conditional β-mean value decreases, the average total cost increases, and also the average total fixed cost increases. The increments in both the total costs and the fixed costs are due to opening more facilities as the fairness of the model increases.

Next, the following table, Table 6.8 presents the results of the problem size of 30 candidate facilities and 200 demand points.

Table 6.8. Epsilon Constraint Method Results (Problem Size 60x300)

Instance #	Average Total Cost	Average Conditional β-mean	Average TF	Average TV	Average Total Cost Change	Average Conditional β-mean Change	Average TF change	Average TV change
1	176828,08	50,67	51859,25	124968,83	0,0010	-0,0052	0,0115	-0,0033
2	159790,07	43,72	53162,13	106627,93	0,0013	-0,0041	0,0144	-0,0050
3	188504,19	51,83	56863,05	131641,14	0,0007	-0,0067	0,0043	-0,0008
4	177608,91	56,91	47904,91	129704,00	0,0010	-0,0038	0,0055	-0,0005
5	170598,27	42,76	63925,83	106672,44	0,0019	-0,0047	0,0093	-0,0027

Table 6.8 shows the average results of Yang instances with 60 candidate facilities and 300 demand points by using the Epsilon Constraint method. According to the results, as the average conditional β-mean value decreases, the average Total Cost and the average Total Fixed Cost (TF) increase. The average TF, as well as Total Cost, are increasing while the conditional β-mean value is decreasing since more facilities are opened as the fairness of the model increases.

Finally, Table 6.9 shows the results of the problem size of 80 candidate facilities and 400 demand points.

Table 6.9. Epsilon Constraint Method Results (Problem Size 80x400)

Instance #	Average Total Cost	Average Conditional β-mean	Average TF	Average TV	Average Total Cost Change	Average Conditional β-mean Change	Average TF change	Average TV change
1	199181,43	32,81	84773,73	114893,91	0,0006	-0,0016	0,0030	0,0006
2	213538,18	43,19	76844,98	136693,20	0,0006	-0,0014	0,0042	-0,0012
3	200485,06	37,00	64202,47	136282,59	0,0009	-0,0043	0,0074	-0,0021
4	206973,00	39,66	75673,95	131298,95	0,0011	-0,0023	0,0049	-0,0009
5	191754,83	38,87	67694,76	124060,07	0,0008	-0,0038	0,0037	-0,0008

Table 6.9 gives the average results of Yang instances with 80 candidate facilities and 400 demand points by using the Epsilon Constraint method. According to these results, the average Total Cost and the average Total Fixed Cost (TF) increase as the conditional β-mean value decreases. This is normal since as the conditional β-mean value decreases, the fairness of the model increases and it is expected to open more facilities and thus TF and Total cost increase.

The bar charts in Figure 6.5 represent the relative change of the average Total Cost change with respect to the change of the average conditional β-mean value and the relative change of the Average TF with respect to the change of the conditional β-mean value. The relative change of the Average Total Cost with respect to the average conditional β-mean value is the division of the average Total Cost change by the average conditional β-mean change. On the other hand, the relative change of the Average TF with respect to the change of the conditional β-mean value is the division of the average Total Fixed Cost change by the average conditional β-mean change.

The blue-colored bars represent the relative change of the Average Total Costs with respect to the change of the average conditional β-mean value while the orange-colored bars represent the relative change of the Average TF with respect to the change of the average conditional β-mean value.

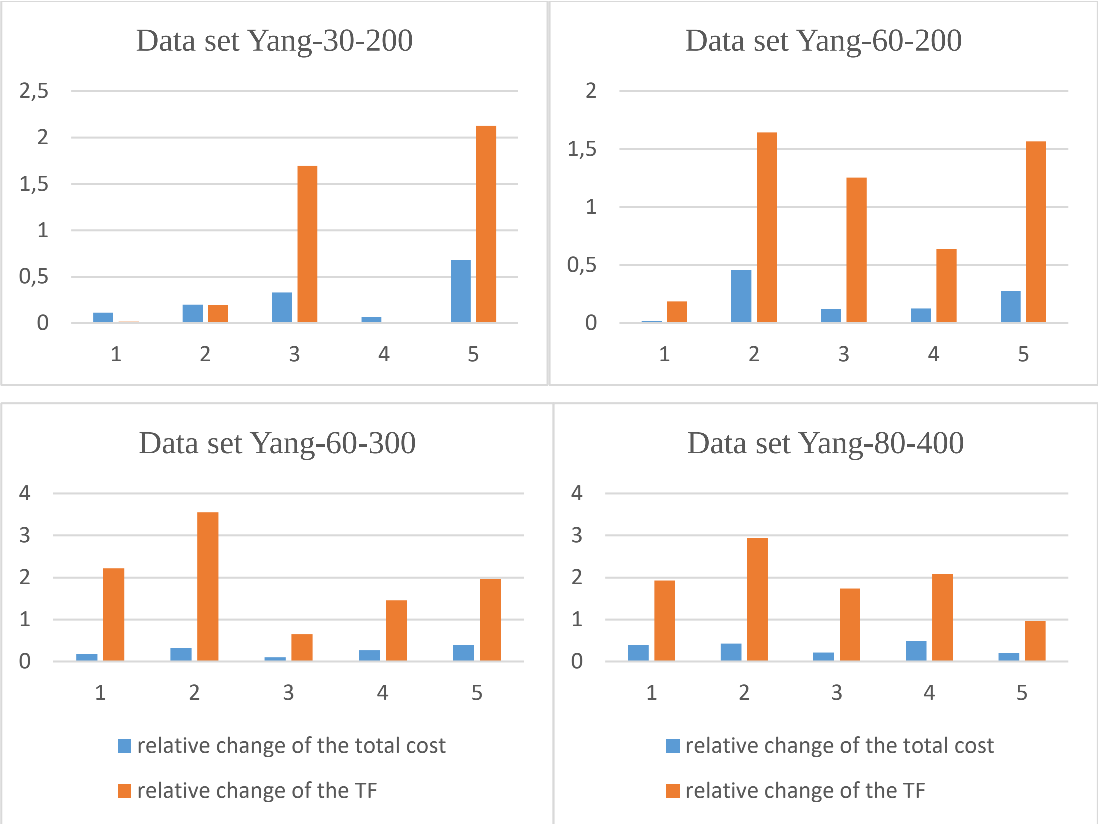

Figure 6.5. Average changes with respect to changes in average conditional β-mean

According to Figure 6.5, it can be seen that the relative change in the average Total Fixed Cost is usually higher than the relative change in the Average Total Cost. There are two exceptions for this conclusion in problems 1 and 4 of Yang instances with 30 candidate facilities and 200 customers (Yang-30-200).

The Total Cost and Total Fixed Cost have a negative correlation with the conditional β-mean values. As it has been explained after each table, it is logical to see an increase in the total fixed cost/opening cost while the conditional β-mean value is decreasing since decrement in the conditional β-mean value means increased fairness of the model. Since the model minimizes the average unit cost paid by the 5% of the total demand who pay the highest assignment costs by its fairness objective, it is inclined to result in a larger number of facilities opened and decreased variability among the variable/assignment costs. Therefore, as the fairness of the model increases (i.e. the conditional β-mean value decreases), the model gives solutions with higher total fixed costs.

In a conclusion, the Epsilon Constraint Method can detect a larger number of Pareto efficient points while being consistent with the cost and fairness objective relationship which has been proved in WS method results.

6.3. The Benders Decomposition Method

Benders Decomposition Method is considered as another additional solution method. The Benders Decomposition tool that is called "Benders Strategy FULL" in IBM OPL Optimization Suite (CPLEX) has been utilized for the Yang problem set as well as the TBED1 problem set which includes larger instances with 1000 candidate facilities and 1000 demand points. Since the TBED1 problem set includes instances with considerably larger sizes and hence it takes more time to solve those problems, a time limit of 2 hours is defined on the solver. All of the instances terminated with the time limit.

For the Yang data, the Benders Decomposition method is added to the model while solving with the Epsilon Constraint Method since the Epsilon Constraint method finds a larger number of Pareto efficient points as previously reported in this chapter. However, for the data with 1000 candidate facilities and 1000 demand points, the Benders Decomposition method is added to the model while solving the model with λ=0.5. The reason for not using the Epsilon Constraint method for TBED1 problem set is that the Epsilon Constraint method is not time efficient while implementing on very large-scale instances.

The results of the Benders Decomposition method are presented in the following tables. Those tables include the gap% values of each efficient point to compare objective function values' closeness to the optimal solutions with and without the use of the Benders Decomposition method. If the gap% values of the efficient points obtained by using the Benders Decomposition method are equal to or smaller than the gap% values obtained without using the Benders Decomposition method, then these gap% values are highlighted with gray.

Since the relative gap% value is intensively used to examine Benders Decomposition Method's performance, it would be beneficial to give the formulation of the relative gap% in CPLEX. The formulation of the relative gap% is as follows:

CLPEX relative gap% = $\frac{|Best\ Bound - Best\ Integer|}{10^{-10} + |Best\ Integer|}$ (Nodet, 2019).

After presenting the formulation of the gap% in CPLEX, the first table, Table 6.10 presents the results for problem 1 of size 30 candidate facilities and 200 demand points.

Table 6.10. Results with and without Benders (Problem 1 of size 30x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,03	145741	0,03	145741
2	0	145749	0	145749
3	0	145765	0,02	145765
4	0,01	145929	0,02	145929
5	0,01	145945	0,01	145945
6	0,01	146163	0,03	146163
7	0,05	146241	0,04	146241
8	0,01	146246	0,07	146246
9	0,03	146158	0,04	146444
10	0,03	146449	0,03	146449
11	0,03	146483	0,05	146483
12	0,04	146566	0,01	146566
13	0,01	146769	0	146769
14	0,04	146852	0,02	146852
15	0,03	147542	0,04	147542
16	0,01	147625	0,03	147629
17	0	147828	0,01	147828
18	0,01	147915	0	147911

Table 6.10 shows that when the Benders Decomposition is implemented, 13 Pareto efficient points out of 18 Pareto efficient points resulted in smaller o equal gap% values. 9 of these 13 points have smaller gap% values and therefore closer to the optimal solution when the Benders Strategy FULL is implemented.

Next, Table 6.11 shows the comparison of gap% values for problem 2 with 30 candidate facilities and 200 demand points.

Table 6.11. Results with and without Benders (Problem 2 of size 30x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,03	126040	0,03	126040
2	0,05	126124	0,1	126117
3	0,05	126117	0,06	126124
4	0,07	126297	0,03	126297
5	0,12	126367	0,07	126367
6	0,06	126374	0,04	126374
7	0,02	126444	0,05	126444
8	0,05	126570	0,02	126570
9	0,05	126593	0,05	126593
10	0,02	126650	0,03	126650
11	0,02	126671	0,01	126673
12	0,05	126673	0,04	126673
13	0,07	126682	0,03	126681
14	0,05	126679	0,03	126674
15	0,06	126727	0,01	126727
16	0,06	126824	0,04	126824
17	0,04	126848	0,02	126848
18	0,02	126864	0,01	126864
19	0,07	126864	0,04	126866
20	0,04	126906	0,02	126906
21	0,08	126984	0,02	126984
22	0,08	126990	0,02	126984
23	0,04	127023	0,04	127023
24	0,03	127189	0,01	127189
25	0,02	127228	0,12	127228
26	0,02	127239	0,06	127239
27	0,01	127266	0,02	127266
28	0,02	127301	0,02	127301
29	0,02	127332	0,01	127332
30	0,02	127406	0,02	127406
31	0,01	127495	0,01	127495
32	0,05	127516	0,08	127516
33	0,02	127647	0,03	127647
34	0,02	127685	0,03	127685
35	0,07	127686	0,02	127688
36	0,04	127838	0,04	127838

37	0,08	127859	0,03	127859
38	0,03	128510	0,03	128510
39	0,01	128591	0,02	128599
40	0,02	128592	0,05	128592
41	0,04	128641	0,07	128641
42	0,02	128679	0,02	128679
43	0,02	128680	0,03	128682
44	0,06	128829	0,03	128829
45	0,05	128853	0,07	128853
46	0,02	129002	0,02	129002

According to Table 6.11, 25 Pareto efficient points out of 46 Pareto efficient points resulted in smaller or equal gap% values under the implementation of the Benders Decomposition method. As the optimality gap decreases, the solution gets closer to the optimal solution. 15 of these efficient points get smaller and thus get closer to the optimal solution.

Next, Table 6.12 presents the comparison of gap% values for problem 3 with 30 candidate facilities and 200 demand points.

Table 6.12. Results with and without Benders (Problem 3 of size 30x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	128216	0,05	128216
2	0,01	128232	0,02	128232
3	0,01	128252	0,02	128252
4	0	128268	0,01	128268
5	0,01	128308	0,06	128308
6	0,01	128323	0,03	128323
7	0,01	128359	0,01	128359
8	0,01	128955	0,04	128955
9	0,01	128970	0,01	128970
10	0	128991	0,03	128991
11	0,01	129023	0,04	129023
12	0,45	129736	0,03	129736
13	0,04	129751	0,01	129751
14	0	129768	0,06	129768
15	0,63	130507	0,83	130507

Table 6.12 shows that when the Benders Decomposition is implemented, 13 Pareto efficient points out of 15 Pareto efficient points resulted in smaller or equal gap% values. 11 of these points result in smaller gap% values which means these points are getting closer to the optimal solution.

Next, the comparison of gap% values for problem 3 with 30 candidate facilities and 200 demand points is presented in Table 6.13.

Table 6.13. Results with and without Benders (Problem 4 of size 30x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	136087	0,04	136087
2	0,04	136209	0,04	136209
3	0	136209	0,04	136220

Based on the results in Table 6.13, when the Benders Decomposition method was implemented, all of the Pareto efficient points have at most equal %gap values when compared with the gap% values without the implementation of the Benders Decomposition method. 2 of these points have smaller gap% values and thus closer to the optimal solution.

The final problem, problem 5 of instances of size 30 candidate facilities and 200 demand points is presented in the following table, Table 6.14.

Table 6.14. Results with and without Benders (Problem 5 of size 30x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	131534	0,01	131534
2	0	131648	0	131648
3	0	131948	0	131948
4	0,01	132112	0	132102
5	0	132606	0	132606
6	0	132720	0	132720
7	0	134213	0	134213

Based on the results in Table 6.14, it can be said that there is no improvement when the Benders Decomposition method is utilized for instance 5 with 30 candidate facilities and 200 demand points. This is expected since all efficient points except for one point result with gap% values that are equal to 0.

After analyzing all problems of size 30 candidate facilities and 200 demand points, the analyses of the problems of size 60 candidate facilities and 200 demand points can be seen in the following tables.

Firstly, the results for Problem 1 of size 60 candidate facilities and 200 demand points are given in Table 6.15

Table 6.15. Results with and without Benders (Problem 1 of size 60x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,06	123376	0,1	123376
2	0,11	124012	0,15	124012

In Table 6.15, it is seen that gap% values improved for all efficient points and thus these points get closer to the optimal solution.

Next, the results for Problem 2 of size 60 candidate facilities and 200 demand points are depicted in Table 6.16.

Table 6.16. Results with and without Benders (Problem 2 of size 60x200).

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0	120843	0,07	120843
2	0	120880	0,06	120880
3	0	121070	0,01	121070
4	0,01	121075	0,01	121075

5	0,01	121248	0,01	121248
6	0,13	121697	0	121697
7	0,16	121847	0,19	121847
8	0,06	122296	0	122296
9	0,4	122920	0	122920
10	0,34	122932	0,3	122932
11	0,08	123495	0,1	123495
12	0,47	124012	0	124012
13	0,54	125847	0	125847
14	0,5	125943	0,01	125943
15	0,02	126725	0	126725
16	0,06	126821	0	126821

Table 6.16 shows that when the Benders Decomposition is implemented, 7 Pareto efficient points out of 16 Pareto efficient points resulted in smaller or equal gap% values. 5 of these points have smaller gap% values and thus get closer to the optimal solution.

Next, Table 6.17 gives the results for problem 3 of size 60 candidate facilities and 200 demand points.

Table 6.17. Results with and without Benders (Problem 3 of size 60x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,12	105588	0,07	105588
2	0,04	105661	0,07	105661
3	0,05	105990	0,07	105990
4	0,03	106071	0,13	106071
5	0,09	106512	0,08	106512
6	0,07	106512	0,04	106512
7	0,02	106619	0,18	106619

According to Table 6.17, when the Benders Decomposition is implemented, 4 Pareto efficient points out of 7 Pareto efficient points resulted in smaller gap% values and they get closer to the optimal solution.

Next, the results for Problem 4 of size 60 candidate facilities and 200 demand points are depicted in Table 6.18.

Table 6.18. Results with and without Benders (Problem 4 of size 60x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0	116698	0	116698
2	0	116751	0	116751
3	0	116878	0	116878
4	0	117006	0,02	117006
5	0,01	117133	0,04	117133
6	0,12	117416	0,2	117416
7	0,06	117469	0	117469
8	0	117475	0,01	117475
9	0,01	117533	0	117528
10	0	117655	0,01	117660
11	0	117783	0,01	117783
12	0	117910	0	117910

Table 6.18 shows that 9 points out of 12 efficient points result with smaller or equal gap% values under the implementation of the Benders Decomposition method than gap% values without implementation of the Benders Decomposition method. 5 of these points have smaller gap% values when the Benders Strategy FULL is used and thus get closer to the optimal solution.

Finally, the results for Problem 4 of size 60 candidate facilities and 200 demand points are depicted in Table 6.19.

Table 6.19. Results with and without Benders (Problem 5 of size 60x200)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	101414	0	101414
2	0	101478	0,04	101478
3	0,02	101636	0	101636
4	0,01	101748	0	101748
5	0	101968	0,1	101968
6	0	102017	0,21	102017

7	0,15	102176	0,02	102176
8	0	102340	0,07	102340
9	0	102404	0,04	102404
10	0,01	102562	0	102562
11	0,01	102674	0,12	102674
12	0	102844	0,08	102844
13	0,07	102857	0	102857
14	0,01	103064	0	103064
15	0,01	103077	0	103077
16	0,02	103312	0,01	103312
17	0,01	103480	0	103480
18	0	104116	0	104116
19	0	104595	0	104595

Table 6.19 shows that when the Benders Decomposition is implemented, 9 Pareto efficient points out of 19 Pareto efficient points resulted in smaller or equal gap% values. 7 of these points have smaller gap% values when the Benders Decomposition method is used and therefore get closer to the optimal solution.

After making the Benders Decomposition analyses on all problems of size 60 candidate facilities and 200 demand points, the same analysis will be performed on problems of size 60 candidate facilities and 300 demand points in the following tables.

Firstly, the results for Problem 1 of size 60 candidate facilities and 300 demand points will be given in Table 6.20.

Table 6.20. Results with and without Benders (Problem 1 of size 60x300)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	176381	0,03	176381
2	0,01	176381	0,02	176387
3	0,02	176465	0,01	176459
4	0,04	176459	0,02	176465
5	0,04	176492	0,06	176492
6	0,01	176492	0,05	176499
7	0,06	176608	0,02	176608
8	0,03	176695	0,04	176695
9	0,01	176733	0,07	176733
10	0,07	176733	0,09	176747

11	0,22	178232	0,05	178232
12	0,03	178232	0,05	178239

According to Table 6.20, 8 out of 12 Pareto efficient points have smaller gap% values when the Benders Decomposition method is implemented. Therefore, these points get closer to the optimal solution when the Benders Decomposition Method is used.

Next, the results for Problem 2 of size 60 candidate facilities and 300 demand points are given in Table 6.21.

Table 6.21. Results with and without Benders (Problem 2 of size 60x300)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0	158587	0	158582
2	0,01	158647	0,03	158647
3	0	158668	0,01	158668
4	0	158856	0,01	158856
5	0	158901	0	158901
6	0	159142	0	159142
7	0	159142	0,01	159152
8	0,02	159639	0,25	159639
9	0	160128	0	160128
10	0,01	160369	0	160369
11	0	160449	0	160449
12	0	160863	0	160863
13	0	160943	0	160943
14	0	161022	0	161022
15	0	161490	0	161490

Table 6.21 shows that there are 15 Pareto efficient points in total. 14 Pareto efficient points give smaller or the same gap% values when the Benders Decomposition method is used. 5 of these points have smaller gap% values and thus get closer to the optimal solution. This result is expected since most of the solutions without implementing the Benders Decomposition method already have gap% values equal to 0. The Benders Decomposition method is added to see if there is a possibility of further improvement.

Subsequently, the results for Problem 3 of size 60 candidate facilities and 300 demand points are available in Table 6.22.

Table 6.22. Results with and without Benders (Problem 3 of size 60x300)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	185382	0,01	185386
2	0,01	185382	0	185382
3	0	185485	0,02	185485
4	0,01	185516	0,01	185516
5	0,02	185592	0,01	185592
6	0,04	185650	0,02	185650
7	0,05	185726	0,02	185726
8	0,02	185919	0,05	185919
9	0,01	185999	0,05	185995
10	0,05	186070	0,02	186070
11	0,01	186105	0,02	186105
12	0	186146	0,08	186146
13	0,02	186181	0,02	186181
14	0,07	186322	0,02	186322
15	0,08	186591	0,03	186591
16	0	186637	0,05	186637
17	0,02	186672	0,06	186681
18	0,04	186672	0,03	186672
19	0,02	186759	0,02	186742
20	0,04	186777	0,01	186777
21	0,1	187183	0,03	187181
22	0,08	187222	0,02	187216
23	0,03	187455	0,03	187455
24	0,07	187672	0,04	187668
25	0,02	187703	0,04	187703
26	0,34	187859	0,06	187859
27	0,01	187872	0,08	187867
28	0,02	187898	0,03	187894
29	0,06	187976	0,09	187972
30	0,03	188188	0,03	188188
31	0,03	188411	0,04	188411
32	0,18	188631	0,06	188627
33	0,07	188870	0,12	188866
34	0,17	188898	0,08	188898
35	0,01	189089	0,02	189089

36	0,01	189310	0,02	189305
37	0,04	189337	0,04	189337
38	0,07	189550	0,02	189550
39	0,11	189766	0,09	189766
40	0,18	189766	0,27	189781
41	0,01	189989	0,2	189989
42	0,01	189989	0,09	189991
43	0,01	189989	0,03	190004
44	0,02	190205	0,1	190205
45	0,03	191358	0,36	191358
46	0,03	191404	0,12	191404
47	0,01	191586	0,07	191571
48	0,03	191609	0,09	191605
49	0,1	191615	0,04	191615
50	0,08	191652	0,04	191652
51	0,06	191863	0,07	191863
52	0,1	191863	0,03	191867
53	0	192010	0	192010
54	0,02	192257	0,05	192257
55	0,01	192304	0	192304
56	0	192304	0,01	192321
57	0	192515	0	192515

Table 6.22 shows that 36 out of 57 Pareto efficient points give smaller or the same gap% values when the Benders Decomposition method is used. 27 of these points have smaller gap% values and thus get closer to the optimal solution.

The results for Problem 4 of size 60 candidate facilities and 300 demand points are shown in Table 6.23.

Table 6.23. Results with and without Benders (Problem 4 of size 60x300)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,03	176913	0,03	176913
2	0,01	177014	0,01	177014
3	0,03	177220	0,03	177220
4	0,03	177220	0,02	177226
5	0,05	177472	0,02	177478
6	0,04	177595	0,03	177595

7	0,03	177619	0,02	177619
8	0,06	177678	0,02	177678
9	0,02	177897	0,1	177905
10	0,05	178331	0,03	178331
11	0,02	178719	0,02	178719

Table 6.23, shows that 5 out of 11 Pareto efficient points give smaller or the same gap% values when the Benders Decomposition method is used. 1 of these points results in a smaller gap% value and thus gets closer to the optimal solution.

Finally, the results of Problem 5 of size 60 candidate facilities and 300 demand points are presented in Table 6.24.

Table 6.24. Results with and without Benders (Problem 5 of size 60x300)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0	167473	0,03	167473
2	0,01	167533	0,04	167533
3	0,07	167630	0,13	167630
4	0,04	167960	0,03	167690
5	0,07	167959	0,04	167962
6	0,32	168261	0,03	168261
7	0,14	168261	0,37	168267
8	0,04	168261	0,08	168265
9	0,03	168289	0,35	168283
10	0,24	168318	0	168318
11	0,27	168318	0,32	168328
12	0,2	168340	0,04	168340
13	0,05	168350	0	168350
14	0,16	168615	0,42	168615
15	0,17	168637	0,46	168637
16	0,02	168924	0,06	168918
17	0,09	168940	0,06	168940
18	0,1	168944	0,07	168944
19	0,06	168966	0,05	168966
20	0,03	169010	0,25	168997
21	0,16	169004	0,02	169004
22	0,1	169026	0,02	169026
23	0,22	169061	0,07	169061

24	0,01	170062	0,05	170062
25	0,1	170066	0,02	170066
26	0,34	170077	0,04	170083
27	0,04	170088	0,08	170088
28	0,06	170132	0,14	170119
29	0,05	170126	0,04	170132
30	0,24	170126	0,02	170126
31	0,09	170148	0,01	170148
32	0,06	172226	0,01	172226
33	0	172608	0	172608
34	0	173143	0	173143
35	0,01	173525	0	173525
36	0,02	175337	1,1	175337
37	0	175723	0	175723
38	0,02	178161	0,03	178161
39	0,14	179177	0,02	179177
40	0	179595	0	179595
41	0,03	180402	0	180402

Table 6.24 shows that 20 out of 41 Pareto efficient points detected to result in at most equal gap% values when the BD is implemented. 16 of these points have smaller gap% values when the Benders Decomposition is used and thus get closer to the optimal solution.

After analyzing the results of all problems of size 60 candidate facilities and 300 demand points, the results of the problems of size 80 candidate facilities and 400 demand points can be seen in the following tables.

Table 6.25. Results with and without Benders (Problem 1 of size 80x400)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,02	193231	0,02	193231
2	0,01	193281	0,04	193281
3	0,02	193306	0,03	193297
4	0,03	193324	0,03	193324
5	0,06	193374	0,04	193374
6	0,02	193379	0,06	193379
7	0,01	193390	0,07	193390
8	0,02	193429	0,04	193429

9	0,03	193440	0,03	193440
10	0,07	193463	0,03	193463
11	0,02	193498	0,02	193498
12	0,01	193505	0,03	193505
13	0,04	193525	0,04	193513
14	0,01	193525	0,06	193525
15	0,01	193556	0,12	193556
16	0,07	193575	0,03	193575
17	0,04	193606	0,02	193606
18	0,01	193611	0,01	193611
19	0,03	193622	0,03	193622
20	0,02	193622	0,01	193672
21	0,02	193730	0,06	193731
22	0,03	193737	0,02	193737
23	0,02	193757	0,02	193757
24	0,04	193796	0,03	193796
25	0,04	193807	0,01	193807
26	0,04	193849	0,01	193846
27	0,06	194050	0,04	194047
28	0,06	194062	0,04	194062
29	0,01	194141	0,05	194141
30	0,01	194432	0,03	194429
31	0,02	194543	0,07	194543
32	0,02	194558	0,02	194558
33	0,01	194637	0,01	194637
34	0,03	194724	0,02	194724
35	0,03	195078	0,14	195078
36	0,01	195093	0,06	195093
37	0,03	195146	0,03	195146
38	0,03	195172	0,03	195172
39	0,05	195259	0,01	195259
40	0	195652	0,06	195652
41	0	195678	0,03	195678
42	0,02	195731	0	195731
43	0,01	195765	0,03	195765
44	0,03	195799	0,03	195799
45	0,02	196058	0,02	196061
46	0,03	196172	0,05	196172
47	0,01	196226	0,07	196226
48	0,02	196237	0,01	196237
49	0,01	196305	0,02	196305

50	0,01	196353	0,03	196353
51	0,02	196498	0,15	196498
52	0,01	196584	0,01	196584
53	0,02	196712	0,11	196712
54	0,02	196719	0,01	196719
55	0,02	196825	0,05	196825
56	0,02	196835	0,12	196835
57	0,01	196893	0,04	196893
58	0,01	196903	0,18	196903
59	0,01	197029	0,02	197029
60	0,01	197038	0,04	197038
61	0,01	197124	0,06	197124
62	0,03	197250	0,03	197250
63	0,01	197259	0,07	197259
64	0,02	197404	0,13	197404
65	0,01	197424	0,02	197424
66	0,02	197434	0,02	197434
67	0,02	197462	0,05	197462
68	0,02	197472	0,09	197472
69	0,01	197569	0,08	197569
70	0,03	197613	0,16	197607
71	0,01	197693	0,01	197693
72	0,01	197790	0,02	197790
73	0,02	197828	0,01	197828
74	0,02	197935	0,05	197935
75	0,01	197993	0,04	197993
76	0,02	198003	0,04	198003
77	0,02	198138	0,19	198138
78	0,02	198224	0,01	198224
79	0,03	198359	0,02	198359
80	0,02	198859	0,02	198859
81	0,03	198883	0,04	198883
82	0	199034	0,01	199034
83	0,01	199072	0,01	199062
84	0,03	199072	0,1	199072
85	0,03	199169	0,08	199169
86	0,03	199207	0,03	199207
87	0,03	199255	0,03	199255
88	0,03	199390	0,01	199390
89	0,01	199428	0,06	199428
90	0,07	199535	0,1	199535

91	0,02	199593	0,06	199593
92	0,02	199603	0,22	199603
93	0,07	199686	0,03	199686
94	0,03	199738	0,29	199738
95	0,03	199754	0,12	199754
96	0,01	199824	0,01	199834
97	0,01	199959	0,06	199959
98	0,03	200483	0,18	200483
99	0,06	201112	0,03	201112
100	0,03	201180	0,02	201180
101	0,02	201180	0,04	201189
102	0,03	201315	0,02	201315
103	0,04	201428	0,01	201428
104	0,04	201587	0,02	201587
105	0,01	201587	0,02	201593
106	0,04	201953	0,04	201953
107	0,08	202111	0,16	202123
108	0,01	202121	0,06	202111
109	0,03	202711	0,29	202711
110	0,02	202765	0,12	202765
111	0,06	202808	0,02	202808
112	0,02	202891	0,04	202897
113	0,08	202943	0,36	202943
114	0,04	203241	0,01	203241
115	0,05	203309	0,01	203309
116	0,04	203804	0,16	203804
117	0,01	204053	0,32	204065
118	0,03	204053	0,04	204053
119	0,09	204121	0,03	204121
120	0,05	204616	0,37	204616
121	0,03	204648	0,05	204648
122	0,08	204716	0,13	204716
123	0,07	205001	0,09	205001
124	0,05	205092	0,07	205092
125	0	205160	0,63	205160
126	0,02	205460	1,21	205460
127	0,18	205528	0,49	205528
128	0,02	205904	0,18	205904
129	0,03	205972	0,19	205972
130	0,17	206340	0,12	206340
131	0,22	206408	0,45	206408

132	0,27	206852	0	206852
133	0,07	206940	0,04	206937
134	0,06	206940	0,37	206950
135	0,02	207443	0,37	207443
136	0,01	207511	0,12	207511
137	0,05	208093	0,08	208089
138	0,06	208323	0,38	208323
139	0,2	208391	0,04	208391
140	0,29	209201	0,18	209201
141	0,2	209201	0,07	209207
142	0,17	209269	0,23	209269
143	0,04	210203	0,02	210203
144	0,04	210800	0,02	210781
145	0,14	210785	0,01	210783
146	0,25	211015	0,13	211015

Table 6.25 shows that there are 105 highlighted gap values. Therefore, 105 efficient points out of 146 efficient points result with smaller or equal gap% values when the Benders Decomposition is implemented. 81 of these points result in smaller gap% values when the Benders Decomposition method is implemented and thus get closer to the optimal solution.

Subsequently, the results of problem 2 are described in Table 6.26.

Table 6.26. Results with and without Benders (Problem 2 of size 80x400)

Efficient Point #	Gap Values with Benders	Total Costs with Benders	Gap Values without Benders	Total Costs without Benders
1	0,01	210187	0,03	210187
2	0,02	210219	0,02	210219
3	0,05	210353	0,08	210353
4	0,07	210364	0,11	210364
5	0,04	210402	0,06	210402
6	0,04	210424	0,01	210424
7	0,05	210450	0,04	210450
8	0,08	210600	0,09	210600
9	0,04	210605	0,06	210605
10	0,02	210627	0,08	210627

11	0	210653	0,01	210653
12	0,07	210822	0,06	210813
13	0,08	210947	0,02	210947
14	0,11	210959	0,04	210959
15	0,06	210959	0,07	210963
16	0,1	210985	0,03	210985
17	0,03	211135	0,1	211135
18	0,04	211140	0,06	211140
19	0,09	211162	0,02	211162
20	0,03	211188	0,09	211188
21	0,02	211348	0,04	211348
22	0,04	211500	0,03	211482
23	0,01	211522	0,07	211522
24	0,03	211682	0,04	211686
25	0,01	211682	0,04	211682
26	0,15	211730	0,11	211730
27	0,02	211752	0,06	211752
28	0,02	211778	0,22	211778
29	0,06	211938	0,02	211938
30	0,11	212112	0,04	212112
31	0,04	212272	0,01	212272
32	0,01	212481	0,07	212473
33	0,07	212477	0,08	212477
34	0,11	212521	0,14	212521
35	0,03	212525	0,04	212525
36	0,06	212690	0,01	212681
37	0,05	212685	0,05	212685
38	0,01	212855	0,08	212855
39	0,01	212859	0,04	212859
40	0,05	213015	0,08	213015
41	0,01	213019	0,02	213019
42	0	213063	0,09	213063
43	0,14	213067	0,12	213067
44	0,04	213111	0,02	213111
45	0,02	213115	0,03	213115
46	0,17	213271	0,04	213271
47	0,02	213275	0,09	213275
48	0,04	213445	0,01	213445
49	0,01	213449	0,01	213449

50	0,01	213651	0,04	213561
51	0,03	213600	0,03	213595
52	0,06	213600	0,06	213604
53	0,03	213671	0,03	213666
54	0,04	213835	0,06	213815
55	0,01	213835	0,07	213835
56	0,03	213858	0,02	213858
57	0,02	213995	0,02	213995
58	0,1	214051	0,02	214043
59	0,02	214196	0	214192
60	0,02	214329	0,03	214329
61	0	214338	0,07	214338
62	0,01	214352	0,02	214352
63	0,03	214363	0,04	214363
64	0,03	214400	0,04	214400
65	0,07	214400	0,11	214409
66	0,02	214425	0,02	214425
67	0,05	214448	0,02	214448
68	0,13	214448	0,01	214457
69	0,02	214473	0,07	214473
70	0,12	214585	0,03	214585
71	0,09	214608	0,05	214608
72	0,06	214633	0	214633
73	0,04	214814	0,05	214804
74	0	214928	0,03	214928
75	0,13	214953	0,03	214953
76	0,07	215024	0,03	215024
77	0	215195	0,08	215195
78	0,1	215237	0,09	215237
79	0,09	215262	0	215262
80	0,09	215333	0,07	215333
81	0,02	215504	0,04	215504
82	0,09	215909	0,14	215909
83	0,05	215914	0	215914
84	0,09	216085	0,19	216085
85	0,12	216439	0,1	216439
86	0,04	217537	0,5	217537
87	0,02	217542	0,26	217542
88	0,24	217896	0,18	217896

89	0,41	219263	0	219263
90	0,11	219617	0,15	219617
91	0	220426	0	220426
92	0	221503	0	221503
93	0	222312	0	222312

According to the results of Table 6.26, 59 points out of 93 Pareto efficient points resulted in smaller or equal gap% values when the Benders Decomposition method is implemented. 48 of these points have smaller gap% values when the Benders Decomposition method is implemented and thus these points get closer to the optimal solution.

Next, the results of problem 3 of size 80 candidate facilities and 400 demand points are presented in Table 6.27.

Table 6.27. Results with and without Benders (Problem 3 of size 80x400)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	199882	0,01	199882
2	0,01	199949	0,03	199954
3	0,01	199954	0,02	199949
4	0,05	200087	0,04	200079
5	0	200147	0,1	200147
6	0,01	200201	0,05	200201
7	0,01	200224	0,08	200214
8	0,01	200268	0,01	200268
9	0	200334	0,04	200334
10	0,01	200388	0,02	200388
11	0,01	200388	0,01	200534
12	0,01	200542	0,04	200542
13	0,02	200593	0,01	200588
14	0,01	200654	0,01	200654
15	0	200708	0,01	200708
16	0	200998	0,01	200998
17	0,01	202806	0,12	202806

According to Table 6.26, 17 Pareto efficient points are detected by using the Epsilon

Constraint method. 15 out of these 17 points have smaller or equal gap% values when the Benders Decomposition method is implemented. 11 out of 15 points have smaller gap% values and thus get closer to the optimal solution.

Next, the results of problem 4 of size 80 candidate facilities and 400 demand points are depicted in Table 6.28.

Table 6.28. Results with and without Benders (Problem 4 of size 80x400)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,01	204746	0,01	204746
2	0,03	204801	0,02	204801
3	0,03	204838	0,05	204838
4	0,06	204857	0,04	204857
5	0,04	204868	0,01	204868
6	0,01	204935	0,03	204934
7	0,05	204960	0,07	204960
8	0,02	204980	0,03	204980
9	0,01	204989	0,03	204993
10	0,02	205028	0,01	205028
11	0,04	205035	0,01	205035
12	0,06	205038	0,01	205038
13	0,02	205072	0,01	205072
14	0,1	205072	0,05	205081
15	0	205115	0,05	205118
16	0,05	205152	0,01	205152
17	0,05	205594	0,04	205594
18	0,1	205622	0,08	205622
19	0,03	206325	0,05	206323
20	0,04	206360	0,17	206360
21	0,01	206391	0,04	206391
22	0,06	206428	0,06	206428
23	0,02	206474	0,03	206471
24	0,04	206950	0,01	206941
25	0,12	206978	0	206978
26	0,04	207691	0,03	207679
27	0,16	207716	0	207716
28	0,04	208579	0,33	208570
29	0,24	208718	0,02	208718
30	0,03	209051	0	209063

31	0,02	209280	0,02	209271
32	0,09	209308	0,26	209308
33	0	209308	0,01	209317
34	0	210298	0,22	210298
35	0,2	210647	0,06	210656
36	0,09	211637	0	211637
37	0	212587	0,01	212590
38	0	213530	0,01	213542

Table 6.28 shows that when the Benders Decomposition is implemented, 19 Pareto efficient points resulted in the same or smaller gap% values. 16 of these points have smaller optimality gap% values and thus these points get closer to the optimal solution when the Benders Decomposition method is implemented.

Finally, the results for Problem 5 of size 80 candidate facilities and 400 demand points are presented in Table 6.29.

Table 6.29. Results with and without Benders (Problem 5 of size 80x400)

Efficient Point #	Gap% Values with Benders	Total Costs with Benders	Gap% Values without Benders	Total Costs without Benders
1	0,04	190314	0,03	190311
2	0,03	190337	0,04	190333
3	0,04	190380	0,03	190379
4	0,05	190403	0,02	190403
5	0,01	190574	0,1	190574
6	0,01	190633	0,03	190634
7	0,04	190644	0,03	190643
8	0,02	190706	0,05	190707
9	0,01	190860	0,05	190859
10	0,03	190905	0,02	190904
11	0,06	191119	0,02	191119
12	0,02	191241	0,13	191241
13	0,02	191394	0,03	191395
14	0,05	191439	0,03	191439
15	0,03	191654	0,03	191654
16	0,04	191781	0,05	191780
17	0,07	192142	0,05	192141
18	0,03	192315	0,05	192315
19	0,01	192338	0,02	192338

20	0,01	192338	0,03	192339
21	0,03	192398	0,02	192398
22	0,04	192492	0,1	192508
23	0,01	192524	0,02	192525
24	0,02	192676	0,03	192676
25	0,02	192717	0,04	192717
26	0,02	193027	0,01	193028
27	0,01	193254	0,05	193252
28	0,22	193850	0,04	193850
29	0,01	194428	0,04	194428

Table 6.29 shows that 18 out of 29 Pareto efficient points give equal or better gap% values (i.e. smaller gap% values) when the Benders Decomposition method. 17 of these points' gap% values decrease when the Benders Strategy FULL is added to the model. Thus, these points get closer to the optimal solution with the Benders Decomposition method.

Besides Yang instances, benchmark instances with 1000 candidate facilities and 1000 customers (TBED1) are also solved for the F-SSCFLP model that is studied in this thesis. The objective function values and the gap% values for the F-SSCFLP with and without the implementation of the Benders Decomposition (BD) method are compared in Table 6.30.

Table 6.30. Results with and without Benders (TBED1)

Instance #	Objective function value without BD	Gap% value without BD	Objective function value with BD	Gap% value with BD
1	26152,23	5,06	26278,81	5,53
2	26858,87	5,42	26399,69	3,75
3	25145,94	5,87	24849,75	4,73
4	25975,09	5,65	25573,36	4,17
5	26734,38	4,73	26847,28	5,13
6	14305,94	2,81	14314,28	2,85
7	14032,52	3,01	13861,99	1,85
8	14000,02	2,48	14130,55	3,36
9	13915,91	3,69	13728,66	2,37

10	13944,57	2,62	13911,72	2,38
11	11312,26	2,3	11276,82	1,99
12	11247,54	1,81	11222,21	1,56
13	11491,72	1,75	11482,41	1,67
14	11326,02	1,66	11460,31	2,81
15	11444,29	1,91	11479,36	1,68
16	10718,93	0,7	11325,57	1,65
17	10650,25	0,5	10639,95	0,37
18	10390,11	0,4	10382,92	0,26
19	10399,11	1,45	10341,26	0,91
20	10870,65	1,05	10834,5	0,65

Based on the results in Table 6.30, when the Automatic Benders Strategy was added to the F-SSCFLP, the gap% values decreased for most of the instances (14 out of 20 instances) On the other hand, if the gap% value for the F-SSCFLP with the implementation of the Automatic Benders Strategy is less than the gap% value for the F-SSCFLP without implementation of the Automatic Benders Strategy, then those gap% values have been highlighted with dark grey.

As it can be observed from the overall analysis, when Benders Decomposition (CPLEX Benders Strategy) is utilized, the gap% values get smaller for most of the Pareto efficient points. The decrement in the gap% values is especially larger for Yang instances with 80 candidate facilities and 400 demand points and TBED1 problems. In a conclusion, Benders Strategy gives near-optimal solutions when it is implemented, especially on large-scale instances.

The average number of efficient points, the average number of efficient points that result in equal gap% values, and smaller gap% values for each problem set with different sizes are tabulated in Table 6.31.

Table 6.31. Overall Benders Decomposition Analysis

Benchmark Problem Set (Problem Size)	Average # of Pareto Efficient Points	Average # of Equal Gap% Values with BD	Average # of Smaller Gap% Values with BD	Average # of Smaller Gap%/ Average # of All Points
Yang (30x200)	17,8	4,6	7,4	0,42
Yang (60x200)	11,2	1,6	4,6	0,41

Yang (60x300)	27,2	7,8	11,4	0,42
Yang (80x400)	64,6	8,6	34,6	0,54
TBED1 (1000x1000)	20	0	14	0,70

When the last column of Table 6.31 is considered, it can be seen that the ratios of the average number of instances with smaller gap% values to the average number of all Pareto efficient points are approximately same for problems with size 30 candidate facilities and 200 demand points up to 60 candidate facilities and 300 demand points. However, that ratio increases when problems with size 80 candidate facilities and 400 demand points are solved and reaches its maximum when TBED1 problems with size 1000 candidate facilities and 1000 demand points are solved. Therefore, it can be inferred from Table 6.31 that the Benders Decomposition method performs better for large-scale problems.

CHAPTER 7 CONCLUSIONS AND FUTURE WORK

In this thesis, the F-SSCFLP is studied. The definition of the problem is made and the characteristics are described. The solution methods are investigated, the most common ones, i.e., the Weighted Sum Method, the Epsilon Constraint Method, and the Benders Decomposition Method, are defined in detail. Then benchmark problems having intermediate and large sizes, ranging from 30 candidate facilities and 200 demand points up to 1000 candidate facilities and 1000 demand points, are used to test those methods. The outcomes are reported, analyzed & commented on.

One of the conclusions to be drawn, the Epsilon Constraint method gives a greater number of Pareto efficient points within reasonable CPU time when this method is compared with the Weighted Sum Method. Therefore, it is beneficial to use the Epsilon Constraint method instead of the Weighted Method.

As another outcome, when the Benders Decomposition method is applied to the large-scale instances, the gap% value of the objective function value decreases for most of the problems. However, the gap% value decrement is not so obvious for the problems of size up to 60 candidate facilities and 300 demand points. It means that the Benders Decomposition method becomes more efficient as the size of the problem increases.

As future work, the model which is used in this thesis can be applied to real-life instances. It might be an interesting application in which the cost coefficients are defined depending on the real-world environment.

As a second improvement, Kernel Search can be applied to the large-scale instances as proposed by Guastaroba et al. (2014) and utilized by Filippi et al. (2016) for a bi-objective Mixed Integer Linear Programming.

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APPENDIX 1 – The Weighted Sum Method Solutions (Percentage Change in TF and TV)

Table A1.1. WS Overall Outcomes (Problem Size 30x200)

Instance #	Total Cost	Conditional β-mean	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
30-200-1	178560	62,76	48151	130409	-0,18	0,15	-0,30	-0,14
30-200-1	145730	71,90	33854	111876				
30-200-2	166470	65,10	46091	120379	-0,24	0,13	-0,33	-0,21
30-200-2	126040	73,52	30733	95307				
30-200-3	160270	73,15	48424	111846	-0,20	0,06	-0,31	-0,15
30-200-3	128220	77,40	33554	94666				
30-200-4	180040	66,76	49104	130936	-0,24	0,08	-0,41	-0,18
30-200-4	136090	72,10	29018	107072				
30-200-5	131530	84,42	32345	99185	0,03	-0,15	-0,10	0,08
30-200-5	136090	72,10	29018	107072				

Table A1.2. WS Overall Outcomes (Problem Size 60x200)

Instance #	Total Cost	Conditional β-mean	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
60-200-1	192030	55,82	93484	98546	-0,36	0,02	-0,56	-0,16
60-200-1	123380	57,13	40982	82398				
60-200-2	175020	39,82	93282	81738	-0,31	0,11	-0,57	-0,02
60-200-2	120840	44,37	40495	80345				
60-200-3	181030	41,95	88389	92641	-0,42	0,09	-0,56	-0,28
60-200-3	105590	45,61	38529	67061				
60-200-4	187360	44,60	95381	91979	-0,38	0,10	-0,61	-0,13
60-200-4	116698	49,02	37062	79636				
60-200-5	152390	40,13	84470	67920	-0,33	0,12	-0,57	-0,04
60-200-5	101414	44,91	36194	65220				

Table A1.3. WS Overall Outcomes (Problem Size 60x300)

Instance #	Total Cost	Conditional β-mean	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
60-300-1	256920	49,03	113550	143370	-0,31	0,06	-0,57	-0,11
60-300-1	176380	51,92	48869	127511				
60-300-2	254420	42,85	116670	137750	-0,38	0,06	-0,57	-0,21
60-300-2	158582	45,47	49588	108994				
60-300-3	272920	49,62	140640	132280	-0,32	0,10	-0,64	0,02
60-300-3	185382	54,55	50526	134856				
60-300-4	262980	56,01	124770	138210	-0,33	0,04	-0,63	-0,06
60-300-4	176920	58,34	46330	130590				
60-300-5	262580	39,23	146490	116090	-0,36	0,21	-0,61	-0,05
60-300-5	167473	47,58	56917	110556				

Table A1.4. WS Overall Outcomes (Problem Size 80x400)

Instance #	Total Cost	Conditional β-mean	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
80-400-1	309930	29,65	181900	128030	-0,38	0,27	-0,62	-0,04
80-400-1	193231	37,64	69862	123369				
80-400-2	366440	40,96	219100	147340	-0,40	0,14	-0,69	-0,03
80-400-2	219100	46,73	67652	142540				
80-400-3	315620	36,02	157450	158170	-0,37	0,07	-0,60	-0,13
80-400-3	199880	38,65	62354	137526				
80-400-4	363650	38,37	219490	144160	-0,44	0,09	-0,68	-0,07
80-400-4	204760	41,90	70710	134050				
80-400-5	282580	37,19	150700	131880	-0,33	0,11	-0,57	-0,05
80-400-5	190310	41,42	65017	125293				

APPENDIX 2 - Epsilon Constraint Method Solutions

Table A2.1. Instance 1 with 30 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	145741	71,83	33854	111890	-	-	-	-
2	145749	69,65	33854	111900	5,5E-05	-0,0303494	0	8,94E-05
3	145765	69,5	33854	111910	0,00011	-0,0021536	0	8,94E-05
4	145929	68,11	32979	112950	0,00113	-0,02	-0,025846	0,009293
5	145945	67,68	32979	112970	0,00011	-0,0063133	0	0,000177
6	146163	66,71	32979	113180	0,00149	-0,0143322	0	0,001859
7	146241	66,7	32979	113260	0,00053	-0,0001499	0	0,000707
8	146246	66,54	32979	113270	3,4E-05	-0,0023988	0	8,83E-05
9	146444	65,99	33854	112590	0,00135	-0,0082657	0,026532	-0,006003
10	146449	65,86	33854	112600	3,4E-05	-0,00197	0	8,88E-05
11	146483	65,33	32979	113500	0,00023	-0,0080474	-0,025846	0,007993
12	146566	65,05	32979	113590	0,00057	-0,0042859	0	0,000793
13	146769	64,54	33854	112920	0,00139	-0,0078401	0,026532	-0,005898
14	146852	64,31	33854	113000	0,00057	-0,0035637	0	0,000708
15	147542	63,78	32979	114560	0,0047	-0,0082413	-0,025846	0,013805
16	147629	63,5	32979	114650	0,00059	-0,0043901	0	0,000786
17	147828	63	33854	113970	0,00135	-0,007874	0,026532	-0,005931
18	147911	62,99	33854	114060	0,00056	-0,0001587	0	0,00079
average	146569,56	66,17	33416,50	113153,89	0,0009	-0,0077	0,0001	0,0011

Table A2.2. Instance 2 with 30 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	126040	73,52	30733	95307	-	-	-	-
2	126117	73,12	31153	94964	0,00061	-0,005441	0,01367	-0,0036
3	126124	73,11	31153	94971	5,6E-05	-0,000137	0	7,4E-05
4	126297	72,96	30733	95564	0,00137	-0,002052	-0,01348	0,00624
5	126367	72,65	31153	95214	0,00055	-0,004249	0,01367	-0,00366
6	126374	72,45	31153	95221	5,5E-05	-0,002753	0	7,4E-05
7	126444	72,24	31153	95291	0,00055	-0,002899	0	0,00074
8	126570	71,88	31153	95417	0,001	-0,004983	0	0,00132
9	126593	71,87	31153	95440	0,00018	-0,000139	0	0,00024
10	126650	71,52	30733	95917	0,00045	-0,00487	-0,01348	0,005
11	126673	71,51	30733	95940	0,00018	-0,00014	0	0,00024
12	126673	71,5	30733	95940	0	-0,00014	0	0
13	126681	71,49	30733	95948	6,3E-05	-0,00014	0	8,3E-05
14	126674	70,7	30733	95941	-5,5E-05	-0,01105	0	-7,3E-05
15	126727	70,54	30047	96680	0,00042	-0,002263	-0,02232	0,0077
16	126824	70,5	32146	94678	0,00077	-0,000567	0,06986	-0,02071
17	126848	70,38	30733	96115	0,00019	-0,001702	-0,04396	0,01518
18	126864	70,24	30733	96131	0,00013	-0,001989	0	0,00017
19	126866	70,23	30733	96133	1,6E-05	-0,000142	0	2,1E-05
20	126906	69,79	30733	96173	0,00032	-0,006265	0	0,00042
21	126984	68,8	32315	94669	0,00061	-0,014185	0,05148	-0,01564
22	126984	68,79	32315	94669	0	-0,000145	0	0
23	127023	68,75	32315	94708	0,00031	-0,000581	0	0,00041
24	127189	68,69	32315	94874	0,00131	-0,000873	0	0,00175
25	127228	68,56	32315	94913	0,00031	-0,001893	0	0,00041
26	127239	68,51	30733	96506	8,6E-05	-0,000729	-0,04896	0,01678
27	127266	68,28	32315	94951	0,00021	-0,003357	0,05148	-0,01611
28	127301	67,92	32315	94986	0,00028	-0,005272	0	0,00037
29	127332	67,83	32315	95017	0,00024	-0,001325	0	0,00033
30	127406	67,69	30733	96673	0,00058	-0,002064	-0,04896	0,01743
31	127495	67,68	30733	96762	0,0007	-0,000148	0	0,00092
32	127516	67,25	30733	96783	0,00016	-0,006353	0	0,00022
33	127647	66,94	31153	96494	0,00103	-0,00461	0,01367	-0,00299
34	127685	66,81	30733	96952	0,0003	-0,001942	-0,01348	0,00475
35	127688	66,72	30733	96955	2,3E-05	-0,001347	0	3,1E-05
36	127838	66,52	30733	97105	0,00117	-0,002998	0	0,00155

37	127859	66,49	31153	96706	0,00016	-0,000451	0,01367	-0,00411
38	128510	66,07	30733	97777	0,00509	-0,006317	-0,01348	0,01107
39	128599	66,06	30733	97866	0,00069	-0,000151	0	0,00091
40	128592	65,98	30733	97859	-5,4E-05	-0,001211	0	-7,2E-05
41	128641	65,97	31153	97488	0,00038	-0,000152	0,01367	-0,00379
42	128679	65,62	30733	97946	0,0003	-0,005305	-0,01348	0,0047
43	128682	65,54	30733	97949	2,3E-05	-0,001219	0	3,1E-05
44	128829	65,34	30733	98096	0,00114	-0,003052	0	0,0015
45	128853	65,31	31153	97700	0,00019	-0,000459	0,01367	-0,00404
46	129002	65,3	31153	97849	0,00116	-0,000153	0	0,00153
average	127290,85	69,04	31133,50	96157,35	0,0005	-0,0026	0,0005	0,0006

Table A2.3. Instance 3 with 30 candidate facilities and 200 demand points

Efficient Point #	**Total Costs**	**Conditional β-mean values**	**TF**	**TV**	**Total Cost Change**	**Conditional β-mean Change**	**TF change**	**TV change**
1	128216	77,4	33554	94662	-	-	-	-
2	128232	77,39	33554	94678	0,00012	-0,0001292	0	0,00017
3	128252	77,1	34507	93745	0,00016	-0,0037473	0,0284	-0,00985
4	128268	77,09	34507	93761	0,00012	-0,0001297	0	0,00017
5	128308	77,02	33554	94754	0,00031	-0,000908	-0,02762	0,01059
6	128323	74,98	33554	94769	0,00012	-0,0264866	0	0,00016
7	128359	74,91	34507	93852	0,00028	-0,0009336	0,0284	-0,00968
8	128955	74,22	34677	94278	0,00464	-0,0092111	0,00493	0,00454
9	128970	74,21	34677	94293	0,00012	-0,0001347	0	0,00016
10	128991	74,14	35630	93361	0,00016	-0,0009433	0,02748	-0,00988
11	129023	73,57	35630	93393	0,00025	-0,0076882	0	0,00034
12	129736	73,5	35630	94106	0,00553	-0,0009515	0	0,00763
13	129751	73,36	35630	94121	0,00012	-0,0019048	0	0,00016
14	129768	73,34	35630	94138	0,00013	-0,0002726	0	0,00018
15	130507	73,29	36701	93806	0,00569	-0,0006818	0,03006	-0,00353
average	128910,60	75,03	34796,13	94114,47	0,0013	-0,0039	0,0065	-0,0006

Table A2.4. Instance 4 with 30 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	136087,1	72,1	29018	107069,1	-	-	-	-
2	136209,1	71,05	29018	107191,1	0,000896	-0,0145631	0	0,001139
3	136220	71,04	29018	107202	8,01E-05	-0,0001407	0	0,000102
average	136172,06	71,40	29018,00	107154,07	0,0005	-0,0074	0,0000	0,0006

Table A2.5. Instance 5 with 30 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	131534	84,43	32345	99189	-	-	-	-
2	131648	83,54	32345	99303	0,00087	-0,0105	0	0,00115
3	131948	83,37	32345	99603	0,00228	-0,002	0	0,00302
4	132102	83,36	32345	99757	0,00117	-0,0001	0	0,00155
5	132606	82,82	32345	100261	0,00382	-0,0065	0	0,00505
6	132720	81,94	32345	100375	0,00086	-0,0106	0	0,00114
7	134213	81,93	34402	99811	0,01125	-0,0001	0,0636	-0,0056
average	132395,86	83,06	32638,86	99757,00	0,0034	-0,0050	0,0106	0,0010

Table A2.6. Instance 1 with 60 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	123376	80,83	40982	82394	-	-	-	-
2	124012	57,12	43237	80775	0,0052	-0,2933	0,0550	-0,0196
average	123694,00	68,98	42109,50	81584,50	0,0052	-0,2933	0,0550	-0,0196

Table A2.7. Instance 2 with 60 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	120843	44,37	40495	80348	-	-	-	-
2	120880	44,11	40495	80385	0,0003	-0,0059	0,0000	0,0005
3	121070	44,06	41375	79695	0,0016	-0,0011	0,0217	-0,0086
4	121075	44	41375	79700	0,0000	-0,0014	0,0000	0,0001
5	121248	42,75	41672	79576	0,0014	-0,0284	0,0072	-0,0016
6	121697	42,36	41399	80298	0,0037	-0,0091	-0,0066	0,0091
7	121847	42,13	43569	78278	0,0012	-0,0054	0,0524	-0,0252
8	122296	41,69	43296	79000	0,0037	-0,0104	-0,0063	0,0092
9	122920	41,59	45408	77512	0,0051	-0,0024	0,0488	-0,0188
10	122932	41,58	43296	79636	0,0001	-0,0002	-0,0465	0,0274
11	123495	41,09	43611	79884	0,0046	-0,0118	0,0073	0,0031
12	124012	41	43057	80955	0,0042	-0,0022	-0,0127	0,0134
13	125847	40,09	47511	78336	0,0148	-0,0222	0,1034	-0,0324
14	125943	39,95	47511	78432	0,0008	-0,0035	0,0000	0,0012
15	126725	39,88	47763	78962	0,0062	-0,0018	0,0053	0,0068
16	126821	39,87	47763	79058	0,0008	-0,0003	0,0000	0,0012
average	123103,19	41,91	43724,75	79378,44	0,0032	-0,0071	0,0116	-0,0010

Table A2.8. Instance 3 with 60 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	105588	45,62	38529	67059	-	-	-	-
2	105661	42,62	40677	64984	0,0007	-0,0658	0,0558	-0,0309
3	105990	42,41	39899	66091	0,0031	-0,0049	-0,0191	0,0170
4	106071	42,37	41509	64562	0,0008	-0,0009	0,0404	-0,0231
5	106512	42,36	40799	65713	0,0042	-0,0002	-0,0171	0,0178
6	106512	42,1	42409	64103	0,0000	-0,0061	0,0395	-0,0245
7	106619	42,05	42409	64210	0,0010	-0,0012	0,0000	0,0017
average	106136,14	42,79	40890,14	65246,00	0,0016	-0,0132	0,0166	-0,0070

Table A2.9. Instance 4 with 60 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	116698	49,02	37062	79636	-	-	-	-
2	116751	48,28	37062	79689	0,0005	-0,0151	0,0000	0,0007
3	116878	48,13	37062	79816	0,0011	-0,0031	0,0000	0,0016
4	117006	47,99	37062	79944	0,0011	-0,0029	0,0000	0,0016
5	117133	47,85	37062	80071	0,0011	-0,0029	0,0000	0,0016
6	117416	47,29	38677	78739	0,0024	-0,0117	0,0436	-0,0166
7	117469	46,54	38677	78792	0,0005	-0,0159	0,0000	0,0007
8	117475	46,3	39018	78457	0,0001	-0,0052	0,0088	-0,0043
9	117528	45,45	39018	78510	0,0005	-0,0184	0,0000	0,0007
10	117660	45,31	39018	78642	0,0011	-0,0031	0,0000	0,0017
11	117783	45,14	39018	78765	0,0010	-0,0038	0,0000	0,0016
12	117910	45,13	39018	78892	0,0011	-0,0002	0,0000	0,0016
average	117308,92	46,87	38146,17	79162,75	0,0009	-0,0075	0,0048	-0,0008

Table A2.10. Instance 5 with 60 candidate facilities and 200 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	101414	44,91	36194	65220	-	-	-	-
2	101478	44,41	37377	64101	0,0006	-0,0111	0,0327	-0,0172
3	101636	43,93	38021	63615	0,0016	-0,0108	0,0172	-0,0076
4	101748	43,42	39204	62544	0,0011	-0,0116	0,0311	-0,0168
5	101968	43,41	39603	62365	0,0022	-0,0002	0,0102	-0,0029
6	102017	43,29	39538	62479	0,0005	-0,0028	-0,0016	0,0018
7	102176	43,11	40417	61759	0,0016	-0,0042	0,0222	-0,0115
8	102340	42,39	37204	65136	0,0016	-0,0167	-0,0795	0,0547
9	102404	41,89	38387	64017	0,0006	-0,0118	0,0318	-0,0172
10	102562	41,41	39031	63531	0,0015	-0,0115	0,0168	-0,0076
11	102674	41,4	40214	62460	0,0011	-0,0002	0,0303	-0,0169
12	102844	40,88	41791	61053	0,0017	-0,0126	0,0392	-0,0225
13	102857	40,76	41726	61131	0,0001	-0,0029	-0,0016	0,0013
14	103064	40,63	42190	60874	0,0020	-0,0032	0,0111	-0,0042
15	103077	40,62	42125	60952	0,0001	-0,0002	-0,0015	0,0013

16	103312	40,38	42125	61187	0,0023	-0,0059	0,0000	0,0039
17	103480	40,29	42125	61355	0,0016	-0,0022	0,0000	0,0027
18	104116	40,15	42821	61295	0,0061	-0,0035	0,0165	-0,0010
19	104595	40,14	42821	61774	0,0046	-0,0002	0,0000	0,0078
average	102619,05	41,97	40153,37	62465,68	0,0017	-0,0062	0,0097	-0,0029

Table A2.11. Instance 1 with 60 candidate facilities and 300 demand points

Efficient Point #	**Total Costs**	**Conditional β-mean values**	**TF**	**TV**	**Total Cost Change**	**Conditional β-mean Change**	**TF change**	**TV change**
1	176381	51,93	48869	127512	-	-	-	-
2	176387	51,92	48869	127518	0,0000	-0,0002	0,0000	0,0000
3	176459	51,71	51356	125103	0,0004	-0,0040	0,0509	-0,0189
4	176465	51,7	51356	125109	0,0000	-0,0002	0,0000	0,0000
5	176492	50,93	50247	126245	0,0002	-0,0149	-0,0216	0,0091
6	176499	50,92	50247	126252	0,0000	-0,0002	0,0000	0,0001
7	176608	50,46	51820	124788	0,0006	-0,0090	0,0313	-0,0116
8	176695	50,45	51820	124875	0,0005	-0,0002	0,0000	0,0007
9	176733	50,01	52759	123974	0,0002	-0,0087	0,0181	-0,0072
10	176747	50	54332	122415	0,0001	-0,0002	0,0298	-0,0126
11	178232	49,03	55318	122914	0,0084	-0,0194	0,0181	0,0041
12	178239	49,027	55318	122921	0,0000	-0,0001	0,0000	0,0001
average	176828	50,673917	51859,3	124969	0,00096	-0,0051941	0,01152	-0,0033

Table A2.12. Instance 2 with 60 candidate facilities and 300 demand points

Efficient Point #	**Total Costs**	**Conditional β-mean values**	**TF**	**TV**	**Total Cost Change**	**Conditional β-mean Change**	**TF change**	**TV change**
1	158582	45,47	49588	108994	-	-	-	-
2	158647	45,38	49588	109059	0,0004	-0,0020	0,0000	0,0006
3	158668	45,1	49650	109018	0,0001	-0,0062	0,0013	-0,0004
4	158856	43,82	49385	109471	0,0012	-0,0284	-0,0053	0,0042
5	158901	43,73	49385	109516	0,0003	-0,0021	0,0000	0,0004
6	159142	43,64	51128	108014	0,0015	-0,0021	0,0353	-0,0137
7	159152	43,63	51128	108024	0,0001	-0,0002	0,0000	0,0001
8	159639	43,56	54952	104687	0,0031	-0,0016	0,0748	-0,0309

9	160128	43,25	52038	108090	0,0031	-0,0071	-0,0530	0,0325
10	160369	43,15	53781	106588	0,0015	-0,0023	0,0335	-0,0139
11	160449	43,11	56337	104112	0,0005	-0,0009	0,0475	-0,0232
12	160863	43,09	55049	105814	0,0026	-0,0005	-0,0229	0,0163
13	160943	43	57605	103338	0,0005	-0,0021	0,0464	-0,0234
14	161022	42,95	57677	103345	0,0005	-0,0012	0,0012	0,0001
15	161490	42,94	60141	101349	0,0029	-0,0002	0,0427	-0,0193
average	159790,07	43,72	53162,13	106627,93	0,0013	-0,0041	0,0144	-0,0050

Table A2.13. Instance 3 with 60 candidate facilities and 300 demand points

Efficient Point #	**Total Costs**	**Conditional β-mean values**	**TF**	**TV**	**Total Cost Change**	**Conditional β-mean Change**	**TF change**	**TV change**
1	185386	76,07	50526	134860	-	-	-	-
2	185382	54,56	50526	134856	0,0000	-0,2828	0,0000	0,0000
3	185485	54,04	52365	133120	0,0006	-0,0095	0,0364	-0,0129
4	185516	53,69	52365	133151	0,0002	-0,0065	0,0000	0,0002
5	185592	53,59	52365	133227	0,0004	-0,0019	0,0000	0,0006
6	185650	53,37	52365	133285	0,0003	-0,0041	0,0000	0,0004
7	185726	53,27	52365	133361	0,0004	-0,0019	0,0000	0,0006
8	185919	53,22	52365	133554	0,0010	-0,0009	0,0000	0,0014
9	185995	53,12	52365	133630	0,0004	-0,0019	0,0000	0,0006
10	186070	52,9	52365	133705	0,0004	-0,0041	0,0000	0,0006
11	186105	52,83	52365	133740	0,0002	-0,0013	0,0000	0,0003
12	186146	52,8	52365	133781	0,0002	-0,0006	0,0000	0,0003
13	186181	52,73	52365	133816	0,0002	-0,0013	0,0000	0,0003
14	186322	52,56	53506	132816	0,0008	-0,0032	0,0218	-0,0075
15	186591	52,41	53506	133085	0,0014	-0,0029	0,0000	0,0020
16	186637	52,37	51192	135445	0,0002	-0,0008	-0,0432	0,0177
17	186681	52,35	51192	135489	0,0002	-0,0004	0,0000	0,0003
18	186672	52,3	51192	135480	0,0000	-0,0010	0,0000	-0,0001
19	186742	52,09	53506	133236	0,0004	-0,0040	0,0452	-0,0166
20	186777	52,02	53506	133271	0,0002	-0,0013	0,0000	0,0003
21	187181	51,81	53660	133521	0,0022	-0,0040	0,0029	0,0019
22	187216	51,74	53660	133556	0,0002	-0,0014	0,0000	0,0003
23	187455	51,71	54780	132675	0,0013	-0,0006	0,0209	-0,0066
24	187668	51,68	56658	131010	0,0011	-0,0006	0,0343	-0,0125
25	187703	51,61	56658	131045	0,0002	-0,0014	0,0000	0,0003

26	187859	51,5	54934	132925	0,0008	-0,0021	-0,0304	0,0143
27	187867	51,49	54289	133578	0,0000	-0,0002	-0,0117	0,0049
28	187894	51,43	54934	132960	0,0001	-0,0012	0,0119	-0,0046
29	187972	51,42	56603	131369	0,0004	-0,0002	0,0304	-0,0120
30	188188	51,18	56665	131523	0,0011	-0,0047	0,0011	0,0012
31	188411	50,93	56757	131654	0,0012	-0,0049	0,0016	0,0010
32	188627	50,9	56819	131808	0,0011	-0,0006	0,0011	0,0012
33	188866	50,87	57939	130927	0,0013	-0,0006	0,0197	-0,0067
34	188898	50,8	59755	129143	0,0002	-0,0014	0,0313	-0,0136
35	189089	50,62	58031	131058	0,0010	-0,0035	-0,0289	0,0148
36	189305	50,59	58093	131212	0,0011	-0,0006	0,0011	0,0012
37	189337	50,51	59909	129428	0,0002	-0,0016	0,0313	-0,0136
38	189550	50,49	61029	128521	0,0011	-0,0004	0,0187	-0,0070
39	189766	50,47	61091	128675	0,0011	-0,0004	0,0010	0,0012
40	189781	50,46	61091	128690	0,0001	-0,0002	0,0000	0,0001
41	189989	50,24	61183	128806	0,0011	-0,0044	0,0015	0,0009
42	189991	50,23	61183	128808	0,0000	-0,0002	0,0000	0,0000
43	190004	50,22	61183	128821	0,0001	-0,0002	0,0000	0,0001
44	190205	50,18	61245	128960	0,0011	-0,0008	0,0010	0,0011
45	191358	50,15	59784	131574	0,0061	-0,0006	-0,0239	0,0203
46	191404	50,13	60645	130759	0,0002	-0,0004	0,0144	-0,0062
47	191571	50,12	60904	130667	0,0009	-0,0002	0,0043	-0,0007
48	191605	50,08	62430	129175	0,0002	-0,0008	0,0251	-0,0114
49	191615	50,04	60707	130908	0,0001	-0,0008	-0,0276	0,0134
50	191652	50,03	62523	129129	0,0002	-0,0002	0,0299	-0,0136
51	191863	49,95	62585	129278	0,0011	-0,0016	0,0010	0,0012
52	191867	49,94	62585	129282	0,0000	-0,0002	0,0000	0,0000
53	192010	49,84	61058	130952	0,0007	-0,0020	-0,0244	0,0129
54	192257	49,77	63704	128553	0,0013	-0,0014	0,0433	-0,0183
55	192304	49,73	63797	128507	0,0002	-0,0008	0,0015	-0,0004
56	192321	49,72	63797	128524	0,0001	-0,0002	0,0000	0,0001
57	192515	49,71	63859	128656	0,0010	-0,0002	0,0010	0,0010
average	188504,19	51,83	56863,05	131641,14	0,0007	-0,0067	0,0043	-0,0008

Table A2.14. Instance 4 with 60 candidate facilities and 300 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	176913	58,35	46330	130583	-	-	-	-
2	177014	57,49	46330	130684	0,0006	-0,0147	0,0000	0,0008
3	177220	57,07	46330	130890	0,0012	-0,0073	0,0000	0,0016
4	177226	57,08	46330	130896	0,0000	0,0002	0,0000	0,0000
5	177478	56,98	48823	128655	0,0014	-0,0018	0,0538	-0,0171
6	177595	56,92	48823	128772	0,0007	-0,0011	0,0000	0,0009
7	177619	56,91	47623	129996	0,0001	-0,0002	-0,0246	0,0095
8	177678	56,52	48823	128855	0,0003	-0,0069	0,0252	-0,0088
9	177905	56,34	49896	128009	0,0013	-0,0032	0,0220	-0,0066
10	178331	56,19	48823	129508	0,0024	-0,0027	-0,0215	0,0117
11	178719	56,18	48823	129896	0,0022	-0,0002	0,0000	0,0030
average	177608,91	56,91	47904,91	129704,00	0,0010	-0,0038	0,0055	-0,0005

Table A2.15. Instance 5 with 60 candidate facilities and 300 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	167473	47,59	56917	110556	-	-	-	-
2	167533	46,9	56917	110616	0,0004	-0,0145	0,0000	0,0005
3	167630	46,65	56961	110669	0,0006	-0,0053	0,0008	0,0005
4	167690	45,97	56961	110729	0,0004	-0,0146	0,0000	0,0005
5	167962	45,43	57212	110750	0,0016	-0,0117	0,0044	0,0002
6	168261	45,27	57878	110383	0,0018	-0,0035	0,0116	-0,0033
7	168267	45,26	57878	110389	0,0000	-0,0002	0,0000	0,0001
8	168265	45,22	57878	110387	0,0000	-0,0009	0,0000	0,0000
9	168283	45,08	57878	110405	0,0001	-0,0031	0,0000	0,0002
10	168318	45,03	58458	109860	0,0002	-0,0011	0,0100	-0,0049
11	168328	45,02	60091	108237	0,0001	-0,0002	0,0279	-0,0148
12	168340	44,78	58458	109882	0,0001	-0,0053	-0,0272	0,0152
13	168350	44,62	60091	108259	0,0001	-0,0036	0,0279	-0,0148
14	168615	44,25	60135	108480	0,0016	-0,0083	0,0007	0,0020
15	168637	44,03	60135	108502	0,0001	-0,0050	0,0000	0,0002
16	168918	43,58	60635	108283	0,0017	-0,0102	0,0083	-0,0020
17	168940	43,35	60635	108305	0,0001	-0,0053	0,0000	0,0002

18	168944	43,34	60886	108058	0,0000	-0,0002	0,0041	-0,0023
19	168966	43,07	60886	108080	0,0001	-0,0062	0,0000	0,0002
20	168997	42,96	60091	108906	0,0002	-0,0026	-0,0131	0,0076
21	169004	42,65	60886	108118	0,0000	-0,0072	0,0132	-0,0072
22	169026	42,38	60886	108140	0,0001	-0,0063	0,0000	0,0002
23	169061	42,26	61466	107595	0,0002	-0,0028	0,0095	-0,0050
24	170062	41,91	64194	105868	0,0059	-0,0083	0,0444	-0,0161
25	170066	41,9	64445	105621	0,0000	-0,0002	0,0039	-0,0023
26	170083	41,89	61785	108298	0,0001	-0,0002	-0,0413	0,0253
27	170088	41,66	64445	105643	0,0000	-0,0055	0,0431	-0,0245
28	170119	41,52	64774	105345	0,0002	-0,0034	0,0051	-0,0028
29	170132	41,25	64445	105687	0,0001	-0,0065	-0,0051	0,0032
30	170126	41,21	64445	105681	0,0000	-0,0010	0,0000	-0,0001
31	170148	41	64445	105703	0,0001	-0,0051	0,0000	0,0002
32	172226	39,98	68014	104212	0,0122	-0,0249	0,0554	-0,0141
33	172608	39,94	70984	101624	0,0022	-0,0010	0,0437	-0,0248
34	173143	39,82	68344	104799	0,0031	-0,0030	-0,0372	0,0312
35	173525	39,72	71314	102211	0,0022	-0,0025	0,0435	-0,0247
36	175337	39,63	71792	103545	0,0104	-0,0023	0,0067	0,0131
37	175723	39,53	74762	100961	0,0022	-0,0025	0,0414	-0,0250
38	178161	39,47	79689	98472	0,0139	-0,0015	0,0659	-0,0247
39	179177	39,41	80879	98298	0,0057	-0,0015	0,0149	-0,0018
40	179595	39,33	80397	99198	0,0023	-0,0020	-0,0060	0,0092
41	180402	39,32	81587	98815	0,0045	-0,0003	0,0148	-0,0039
average	170598,27	42,76	63925,83	106672,44	0,0019	-0,0047	0,0093	-0,0027

Table A2.16. Instance 1 with 80 candidate facilities and 400 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	193231	37,64	69862	123369	-	-	-	-
2	193281	37,52	72713	120568	0,0003	-0,0032	0,0408	-0,0227
3	193297	37,25	69862	123435	0,0001	-0,0072	-0,0392	0,0238
4	193324	37,1	69862	123462	0,0001	-0,0040	0,0000	0,0002
5	193374	36,96	72713	120661	0,0003	-0,0038	0,0408	-0,0227
6	193379	36,82	69862	123517	0,0000	-0,0038	-0,0392	0,0237
7	193390	36,7	69862	123528	0,0001	-0,0033	0,0000	0,0001
8	193429	36,69	72713	120716	0,0002	-0,0003	0,0408	-0,0228

9	193440	36,57	72713	120727	0,0001	-0,0033	0,0000	0,0001
10	193463	36,49	71574	121890	0,0001	-0,0022	-0,0157	0,0096
11	193498	36,48	72219	121279	0,0002	-0,0003	0,0090	-0,0050
12	193505	36,41	69862	123643	0,0000	-0,0019	-0,0326	0,0195
13	193513	36,36	74425	119088	0,0000	-0,0014	0,0653	-0,0368
14	193525	36,13	71574	121955	0,0001	-0,0063	-0,0383	0,0241
15	193556	35,95	71574	121982	0,0002	-0,0050	0,0000	0,0002
16	193575	35,91	75070	118505	0,0001	-0,0011	0,0488	-0,0285
17	193606	35,82	74425	119181	0,0002	-0,0025	-0,0086	0,0057
18	193611	35,67	71574	122037	0,0000	-0,0042	-0,0383	0,0240
19	193622	35,66	71574	122048	0,0001	-0,0003	0,0000	0,0001
20	193672	35,37	74425	119247	0,0003	-0,0081	0,0398	-0,0229
21	193731	35,33	73931	119799	0,0003	-0,0011	-0,0066	0,0046
22	193737	35,27	71574	122163	0,0000	-0,0017	-0,0319	0,0197
23	193757	34,89	73931	119826	0,0001	-0,0108	0,0329	-0,0191
24	193796	34,83	73931	119865	0,0002	-0,0017	0,0000	0,0003
25	193807	34,71	76782	117025	0,0001	-0,0034	0,0386	-0,0237
26	193846	34,7	76782	117064	0,0002	-0,0003	0,0000	0,0003
27	194047	34,5	76782	117265	0,0010	-0,0058	0,0000	0,0017
28	194062	34,46	74425	119637	0,0001	-0,0012	-0,0307	0,0202
29	194141	34,45	74425	119716	0,0004	-0,0003	0,0000	0,0007
30	194429	34,23	77537	116892	0,0015	-0,0064	0,0418	-0,0236
31	194543	34,16	77537	187006	0,0006	-0,0020	0,0000	0,5998
32	194558	34,12	75180	119378	0,0001	-0,0012	-0,0304	-0,3616
33	194637	34,01	75180	119457	0,0004	-0,0032	0,0000	0,0007
34	194724	33,93	75180	119544	0,0004	-0,0024	0,0000	0,0007
35	195078	33,9	77537	117541	0,0018	-0,0009	0,0314	-0,0168
36	195093	33,89	75180	119913	0,0001	-0,0003	-0,0304	0,0202
37	195146	33,84	75505	119641	0,0003	-0,0015	0,0043	-0,0023
38	195172	33,74	75180	119992	0,0001	-0,0030	-0,0043	0,0029
39	195259	33,66	75180	120079	0,0004	-0,0024	0,0000	0,0007
40	195652	33,63	75180	120472	0,0020	-0,0009	0,0000	0,0033
41	195678	33,54	76926	118752	0,0001	-0,0027	0,0232	-0,0143
42	195731	33,51	75694	120037	0,0003	-0,0009	-0,0160	0,0108
43	195765	33,46	74425	121340	0,0002	-0,0015	-0,0168	0,0109
44	195799	33,43	75694	120105	0,0002	-0,0009	0,0171	-0,0102
45	196061	33,42	81072	114989	0,0013	-0,0003	0,0710	-0,0426
46	196172	33,41	81072	115100	0,0006	-0,0003	0,0000	0,0010
47	196226	33,34	77440	118786	0,0003	-0,0021	-0,0448	0,0320
48	196237	33,33	77440	118797	0,0001	-0,0003	0,0000	0,0001
49	196305	33,23	77440	118865	0,0003	-0,0030	0,0000	0,0006

50	196353	33,18	78715	117638	0,0002	-0,0015	0,0165	-0,0103
51	196498	33,13	81072	115426	0,0007	-0,0015	0,0299	-0,0188
52	196584	33,12	80344	116240	0,0004	-0,0003	-0,0090	0,0071
53	196712	33,1	81586	115126	0,0007	-0,0006	0,0155	-0,0096
54	196719	33,06	80344	116368	0,0000	-0,0012	-0,0152	0,0108
55	196825	33,03	79229	117596	0,0005	-0,0009	-0,0139	0,0106
56	196835	32,99	81586	115249	0,0001	-0,0012	0,0297	-0,0200
57	196893	32,94	79229	117664	0,0003	-0,0015	-0,0289	0,0210
58	196903	32,93	81586	115317	0,0001	-0,0003	0,0297	-0,0199
59	197029	32,92	82366	114663	0,0006	-0,0003	0,0096	-0,0057
60	197038	32,87	81586	115452	0,0000	-0,0015	-0,0095	0,0069
61	197124	32,86	80858	116266	0,0004	-0,0003	-0,0089	0,0071
62	197250	32,85	81638	115612	0,0006	-0,0003	0,0096	-0,0056
63	197259	32,8	80858	116401	0,0000	-0,0015	-0,0096	0,0068
64	197404	32,75	83975	113429	0,0007	-0,0015	0,0385	-0,0255
65	197424	32,74	80523	116901	0,0001	-0,0003	-0,0411	0,0306
66	197434	32,72	82880	114554	0,0001	-0,0006	0,0293	-0,0201
67	197462	32,71	81618	115844	0,0001	-0,0003	-0,0152	0,0113
68	197472	32,7	83975	113497	0,0001	-0,0003	0,0289	-0,0203
69	197569	32,66	82880	114689	0,0005	-0,0012	-0,0130	0,0105
70	197607	32,63	83975	113632	0,0002	-0,0009	0,0132	-0,0092
71	197693	32,62	83247	114446	0,0004	-0,0003	-0,0087	0,0072
72	197790	32,59	82152	115638	0,0005	-0,0009	-0,0132	0,0104
73	197828	32,56	83247	114581	0,0002	-0,0009	0,0133	-0,0091
74	197935	32,54	85269	112666	0,0005	-0,0006	0,0243	-0,0167
75	197993	32,49	85269	112724	0,0003	-0,0015	0,0000	0,0005
76	198003	32,48	85269	112734	0,0001	-0,0003	0,0000	0,0001
77	198138	32,42	85269	112869	0,0007	-0,0018	0,0000	0,0012
78	198224	32,37	84541	113683	0,0004	-0,0015	-0,0085	0,0072
79	198359	32,31	84541	113818	0,0007	-0,0019	0,0000	0,0012
80	198859	32,3	83357	115502	0,0025	-0,0003	-0,0140	0,0148
81	198883	32,24	84541	114342	0,0001	-0,0019	0,0142	-0,0100
82	199034	32,21	85379	113655	0,0008	-0,0009	0,0099	-0,0060
83	199062	32,2	84117	114945	0,0001	-0,0003	-0,0148	0,0114
84	199072	32,19	86474	112598	0,0001	-0,0003	0,0280	-0,0204
85	199169	32,16	85379	113790	0,0005	-0,0009	-0,0127	0,0106
86	199207	32,12	86474	112733	0,0002	-0,0012	0,0128	-0,0093
87	199255	32,11	84651	114604	0,0002	-0,0003	-0,0211	0,0166
88	199390	32,05	84651	114739	0,0007	-0,0019	0,0000	0,0012
89	199428	32,02	85746	113682	0,0002	-0,0009	0,0129	-0,0092
90	199535	32	87768	111767	0,0005	-0,0006	0,0236	-0,0168

91	199593	31,99	85411	114182	0,0003	-0,0003	-0,0269	0,0216
92	199603	31,94	87768	111835	0,0001	-0,0016	0,0276	-0,0206
93	199686	31,93	87768	111918	0,0004	-0,0003	0,0000	0,0007
94	199738	31,88	87768	111970	0,0003	-0,0016	0,0000	0,0005
95	199754	31,87	87768	111986	0,0001	-0,0003	0,0000	0,0001
96	199834	31,85	87040	112789	0,0004	-0,0006	-0,0083	0,0072
97	199959	31,79	87040	112919	0,0006	-0,0019	0,0000	0,0012
98	200483	31,68	87040	113443	0,0026	-0,0035	0,0000	0,0046
99	201112	31,67	89943	111169	0,0031	-0,0003	0,0334	-0,0200
100	201180	31,61	89943	111237	0,0003	-0,0019	0,0000	0,0006
101	201189	31,6	89943	111246	0,0000	-0,0003	0,0000	0,0001
102	201315	31,54	89943	111372	0,0006	-0,0019	0,0000	0,0011
103	201428	31,53	92284	109144	0,0006	-0,0003	0,0260	-0,0200
104	201587	31,49	90703	110884	0,0008	-0,0013	-0,0171	0,0159
105	201593	31,48	90703	110890	0,0000	-0,0003	0,0000	0,0001
106	201953	31,47	91948	110005	0,0018	-0,0003	0,0137	-0,0080
107	202123	31,44	90703	111420	0,0008	-0,0010	-0,0135	0,0129
108	202111	31,42	90703	111408	-0,0001	-0,0006	0,0000	-0,0001
109	202711	31,41	92715	109996	0,0030	-0,0003	0,0222	-0,0127
110	202765	31,36	92715	110050	0,0003	-0,0016	0,0000	0,0005
111	202808	31,35	93606	109202	0,0002	-0,0003	0,0096	-0,0077
112	202897	31,32	93606	109285	0,0004	-0,0010	0,0000	0,0008
113	202943	31,31	93606	109337	0,0002	-0,0003	0,0000	0,0005
114	203241	31,23	94851	108390	0,0015	-0,0026	0,0133	-0,0087
115	203309	31,17	94851	108458	0,0003	-0,0019	0,0000	0,0006
116	203804	31,15	92754	112050	0,0024	-0,0006	-0,0221	0,0331
117	204065	31,14	95618	108447	0,0013	-0,0003	0,0309	-0,0322
118	204053	31,09	95618	108435	-0,0001	-0,0016	0,0000	-0,0001
119	204121	31,03	95618	108503	0,0003	-0,0019	0,0000	0,0006
120	204616	31	93521	111095	0,0024	-0,0010	-0,0219	0,0239
121	204648	30,99	98044	106604	0,0002	-0,0003	0,0484	-0,0404
122	204716	30,93	98044	106672	0,0003	-0,0019	0,0000	0,0006
123	205001	30,9	97012	107989	0,0014	-0,0010	-0,0105	0,0123
124	205092	30,88	95657	109435	0,0004	-0,0006	-0,0140	0,0134
125	205160	30,82	95657	109503	0,0003	-0,0019	0,0000	0,0006
126	205460	30,81	98811	106649	0,0015	-0,0003	0,0330	-0,0261
127	205528	30,75	98811	106717	0,0003	-0,0019	0,0000	0,0006
128	205904	30,73	96424	109480	0,0018	-0,0007	-0,0242	0,0259
129	205972	30,67	96424	109548	0,0003	-0,0020	0,0000	0,0006
130	206340	30,61	100210	106130	0,0018	-0,0020	0,0393	-0,0312
131	206408	30,55	100210	106198	0,0003	-0,0020	0,0000	0,0006

132	206852	30,54	97818	109034	0,0022	-0,0003	-0,0239	0,0267
133	206937	30,53	97818	109119	0,0004	-0,0003	0,0000	0,0008
134	206950	30,52	97818	109132	0,0001	-0,0003	0,0000	0,0001
135	207443	30,51	99617	107826	0,0024	-0,0003	0,0184	-0,0120
136	207511	30,4	99617	107894	0,0003	-0,0036	0,0000	0,0006
137	208089	30,38	101010	107079	0,0028	-0,0007	0,0140	-0,0076
138	208323	30,37	101010	107313	0,0011	-0,0003	0,0000	0,0022
139	208391	30,2	101010	107381	0,0003	-0,0056	0,0000	0,0006
140	209201	30,19	102120	107081	0,0039	-0,0003	0,0110	-0,0028
141	209207	30,18	102120	107087	0,0000	-0,0003	0,0000	0,0001
142	209269	30,12	102120	107149	0,0003	-0,0020	0,0000	0,0006
143	210203	30,11	102780	107423	0,0045	-0,0003	0,0065	0,0026
144	210781	30,07	104180	106601	0,0027	-0,0013	0,0136	-0,0077
145	210783	30,06	104180	106603	0,0000	-0,0003	0,0000	0,0000
146	211015	30,05	104180	106835	0,0011	-0,0003	0,0000	0,0022
average	199181	32,8130137	84773,7	114894	0,00061	-0,0016	0,00298	0,00061

Table A2.17. Instance 2 with 80 candidate facilities and 400 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	210187	46,74	67652	142535	-	-	-	-
2	210219	46,73	67889	142330	0,0002	-0,0002	0,0035	-0,0014
3	210353	46,47	67652	142701	0,0006	-0,0056	-0,0035	0,0026
4	210364	46,09	70017	140347	0,0001	-0,0082	0,0350	-0,0165
5	210402	45,82	70254	140148	0,0002	-0,0059	0,0034	-0,0014
6	210424	45,68	69133	141291	0,0001	-0,0031	-0,0160	0,0082
7	210450	45,64	69133	141317	0,0001	-0,0009	0,0000	0,0002
8	210600	45,49	68896	141704	0,0007	-0,0033	-0,0034	0,0027
9	210605	45,39	70254	140351	0,0000	-0,0022	0,0197	-0,0095
10	210627	45,38	69133	141494	0,0001	-0,0002	-0,0160	0,0081
11	210653	45,21	69133	141520	0,0001	-0,0037	0,0000	0,0002
12	210813	45,1	70996	139817	0,0008	-0,0024	0,0269	-0,0120
13	210947	45,09	70739	140208	0,0006	-0,0002	-0,0036	0,0028
14	210959	45,03	70739	140220	0,0001	-0,0013	0,0000	0,0001
15	210963	45,02	70739	140224	0,0000	-0,0002	0,0000	0,0000
16	210985	44,98	70739	140246	0,0001	-0,0009	0,0000	0,0002
17	211135	44,82	70502	140633	0,0007	-0,0036	-0,0034	0,0028

18	211140	44,73	71860	139280	0,0000	-0,0020	0,0193	-0,0096
19	211162	44,59	70739	140423	0,0001	-0,0031	-0,0156	0,0082
20	211188	44,55	70739	140449	0,0001	-0,0009	0,0000	0,0002
21	211348	44,54	72602	138746	0,0008	-0,0002	0,0263	-0,0121
22	211482	44,42	70739	140743	0,0006	-0,0027	-0,0257	0,0144
23	211522	44,38	70739	140783	0,0002	-0,0009	0,0000	0,0003
24	211686	44,37	72602	139084	0,0008	-0,0002	0,0263	-0,0121
25	211682	44,27	72602	139080	0,0000	-0,0023	0,0000	0,0000
26	211730	44,23	73635	138095	0,0002	-0,0009	0,0142	-0,0071
27	211752	44,22	72514	139238	0,0001	-0,0002	-0,0152	0,0083
28	211778	44,05	72514	139264	0,0001	-0,0038	0,0000	0,0002
29	211938	43,94	74377	137561	0,0008	-0,0025	0,0257	-0,0122
30	212112	43,89	72514	139598	0,0008	-0,0011	-0,0250	0,0148
31	212272	43,78	74377	137895	0,0008	-0,0025	0,0257	-0,0122
32	212473	43,73	72263	140210	0,0009	-0,0011	-0,0284	0,0168
33	212477	43,72	75861	136616	0,0000	-0,0002	0,0498	-0,0256
34	212521	43,55	71142	141379	0,0002	-0,0039	-0,0622	0,0349
35	212525	43,53	74740	137785	0,0000	-0,0005	0,0506	-0,0254
36	212681	43,46	76603	136078	0,0007	-0,0016	0,0249	-0,0124
37	212685	43,42	76603	136082	0,0000	-0,0009	0,0000	0,0000
38	212855	43,39	71142	141713	0,0008	-0,0007	-0,0713	0,0414
39	212859	43,38	74740	138119	0,0000	-0,0002	0,0506	-0,0254
40	213015	43,28	73005	140010	0,0007	-0,0023	-0,0232	0,0137
41	213019	43,27	76603	136416	0,0000	-0,0002	0,0493	-0,0257
42	213063	43,24	74038	139025	0,0002	-0,0007	-0,0335	0,0191
43	213067	43,17	77636	135431	0,0000	-0,0016	0,0486	-0,0259
44	213111	43,08	72917	140194	0,0002	-0,0021	-0,0608	0,0352
45	213115	43,07	72917	140198	0,0000	-0,0002	0,0000	0,0000
46	213271	42,95	74780	138491	0,0007	-0,0028	0,0255	-0,0122
47	213275	42,94	78378	134897	0,0000	-0,0002	0,0481	-0,0260
48	213445	42,89	72917	140528	0,0008	-0,0012	-0,0697	0,0417
49	213449	42,87	76515	136934	0,0000	-0,0005	0,0493	-0,0256
50	213561	42,84	79499	134062	0,0005	-0,0007	0,0390	-0,0210
51	213595	42,75	80046	133549	0,0002	-0,0021	0,0069	-0,0038
52	213604	42,74	80046	133558	0,0000	-0,0002	0,0000	0,0001
53	213666	42,71	78925	134741	0,0003	-0,0007	-0,0140	0,0089
54	213815	42,7	77062	136753	0,0007	-0,0002	-0,0236	0,0149
55	213835	42,68	78152	135683	0,0001	-0,0005	0,0141	-0,0078
56	213858	42,54	75143	138715	0,0001	-0,0033	-0,0385	0,0223
57	213995	42,53	80015	133980	0,0006	-0,0002	0,0648	-0,0341
58	214043	42,42	78894	135149	0,0002	-0,0026	-0,0140	0,0087

59	214192	42,37	75143	139049	0,0007	-0,0012	-0,0475	0,0289
60	214329	42,35	80015	134314	0,0006	-0,0005	0,0648	-0,0341
61	214338	42,33	78674	135664	0,0000	-0,0005	-0,0168	0,0101
62	214352	42,32	77006	137346	0,0001	-0,0002	-0,0212	0,0124
63	214363	42,25	80562	133801	0,0001	-0,0017	0,0462	-0,0258
64	214400	42,19	78039	136361	0,0002	-0,0014	-0,0313	0,0191
65	214409	42,18	78039	136370	0,0000	-0,0002	0,0000	0,0001
66	214425	42,1	79927	134498	0,0001	-0,0019	0,0242	-0,0137
67	214448	42,05	76918	137530	0,0001	-0,0012	-0,0376	0,0225
68	214457	42,04	76918	137539	0,0000	-0,0002	0,0000	0,0001
69	214473	42	78806	135667	0,0001	-0,0010	0,0245	-0,0136
70	214585	41,93	81790	132795	0,0005	-0,0017	0,0379	-0,0212
71	214608	41,92	78781	135827	0,0001	-0,0002	-0,0368	0,0228
72	214633	41,91	80669	133964	0,0001	-0,0002	0,0240	-0,0137
73	214804	41,82	82022	132782	0,0008	-0,0021	0,0168	-0,0088
74	214928	41,75	80449	134479	0,0006	-0,0017	-0,0192	0,0128
75	214953	41,67	82337	132616	0,0001	-0,0019	0,0235	-0,0139
76	215024	41,63	81216	133808	0,0003	-0,0010	-0,0136	0,0090
77	215195	41,62	82569	132626	0,0008	-0,0002	0,0167	-0,0088
78	215237	41,59	80449	134788	0,0002	-0,0007	-0,0257	0,0163
79	215262	41,5	82337	132925	0,0001	-0,0022	0,0235	-0,0138
80	215333	41,49	81216	134117	0,0003	-0,0002	-0,0136	0,0090
81	215504	41,45	82569	132935	0,0008	-0,0010	0,0167	-0,0088
82	215909	41,39	81872	134037	0,0019	-0,0014	-0,0084	0,0083
83	215914	41,38	83581	132333	0,0000	-0,0002	0,0209	-0,0127
84	216085	41,37	84934	131151	0,0008	-0,0002	0,0162	-0,0089
85	216439	41,3	85325	131114	0,0016	-0,0017	0,0046	-0,0003
86	217537	41,29	87672	129865	0,0051	-0,0002	0,0275	-0,0095
87	217542	41,22	89381	128161	0,0000	-0,0017	0,0195	-0,0131
88	217896	41,15	89772	128124	0,0016	-0,0017	0,0044	-0,0003
89	219263	41,14	89512	129751	0,0063	-0,0002	-0,0029	0,0127
90	219617	41,07	89903	129714	0,0016	-0,0017	0,0044	-0,0003
91	220426	41,05	93406	127020	0,0037	-0,0005	0,0390	-0,0208
92	221503	40,98	92268	129235	0,0049	-0,0017	-0,0122	0,0174
93	222312	40,97	95771	126541	0,0037	-0,0002	0,0380	-0,0208
average	213538,18	43,19	76844,98	136693,2043	0,0006	-0,0014	0,0042	-0,0012

Table A2.18. Instance 3 with 80 candidate facilities and 400 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	199882	38,65	62354	137528	-	-	-	-
2	199954	38,01	63503	136451	0,0004	-0,0166	0,0184	-0,0078
3	199949	37,94	63503	136446	0,0000	-0,0018	0,0000	0,0000
4	200079	37,86	62102	137977	0,0007	-0,0021	-0,0221	0,0112
5	200147	37,64	62737	137410	0,0003	-0,0058	0,0102	-0,0041
6	200201	37,55	62557	137644	0,0003	-0,0024	-0,0029	0,0017
7	200214	37,54	63886	136328	0,0001	-0,0003	0,0212	-0,0096
8	200268	36,76	63706	136562	0,0003	-0,0208	-0,0028	0,0017
9	200334	36,63	65287	135047	0,0003	-0,0035	0,0248	-0,0111
10	200388	36,54	65107	135281	0,0003	-0,0025	-0,0028	0,0017
11	200534	36,46	63401	137133	0,0007	-0,0022	-0,0262	0,0137
12	200542	36,45	63401	137141	0,0000	-0,0003	0,0000	0,0001
13	200588	36,36	63221	137367	0,0002	-0,0025	-0,0028	0,0016
14	200654	36,23	64802	135852	0,0003	-0,0036	0,0250	-0,0110
15	200708	36,14	64622	136086	0,0003	-0,0025	-0,0028	0,0017
16	200998	36,09	67276	133722	0,0014	-0,0014	0,0411	-0,0174
17	202806	36,08	69977	132829	0,0090	-0,0003	0,0401	-0,0067
average	200485,06	37,00	64202,47	136282,59	0,0009	-0,0043	0,0074	-0,0021

Table A2.19. Instance 4 with 80 candidate facilities and 400 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	204746	41,9	70710	134040	-	-	-	-
2	204801	41,76	70710	134090	0,0003	-0,0033	0,0000	0,0004
3	204838	41,68	69007	135830	0,0002	-0,0019	-0,0241	0,0130
4	204857	41,62	70710	134150	0,0001	-0,0014	0,0247	-0,0124
5	204868	40,61	72833	132040	0,0001	-0,0243	0,0300	-0,0157
6	204934	40,48	72833	132090	0,0003	-0,0032	0,0000	0,0004
7	204960	40,38	71130	133840	0,0001	-0,0025	-0,0234	0,0132
8	204980	40,3	72833	132150	0,0001	-0,0020	0,0239	-0,0126
9	204993	40,29	72833	132160	0,0001	-0,0002	0,0000	0,0001

10	205028	40,28	72833	132200	0,0002	-0,0002	0,0000	0,0003
11	205035	40,13	72833	132200	0,0000	-0,0037	0,0000	0,0000
12	205038	40,11	72833	132210	0,0000	-0,0005	0,0000	0,0001
13	205072	40,1	71130	133940	0,0002	-0,0002	-0,0234	0,0131
14	205081	40,03	71130	133960	0,0000	-0,0017	0,0000	0,0001
15	205118	39,99	72833	132280	0,0002	-0,0010	0,0239	-0,0125
16	205152	39,91	71130	134030	0,0002	-0,0020	-0,0234	0,0132
17	205594	39,83	75423	130160	0,0022	-0,0020	0,0604	-0,0289
18	205622	39,76	73720	131910	0,0001	-0,0018	-0,0226	0,0134
19	206323	39,66	75859	130460	0,0034	-0,0025	0,0290	-0,0110
20	206360	39,65	74156	132200	0,0002	-0,0003	-0,0224	0,0133
21	206391	39,45	75958	130430	0,0002	-0,0050	0,0243	-0,0134
22	206428	39,37	74255	132170	0,0002	-0,0020	-0,0224	0,0133
23	206471	39,36	75958	130510	0,0002	-0,0003	0,0229	-0,0126
24	206941	39,09	78548	128390	0,0023	-0,0069	0,0341	-0,0162
25	206978	39,01	76845	130130	0,0002	-0,0020	-0,0217	0,0136
26	207679	38,88	78984	128700	0,0034	-0,0033	0,0278	-0,0110
27	207716	38,8	77281	130440	0,0002	-0,0021	-0,0216	0,0135
28	208570	38,79	78889	129680	0,0041	-0,0003	0,0208	-0,0058
29	208718	38,74	78193	130510	0,0007	-0,0013	-0,0088	0,0064
30	209063	38,73	80423	128640	0,0017	-0,0003	0,0285	-0,0143
31	209271	38,66	81028	128240	0,0010	-0,0018	0,0075	-0,0031
32	209308	38,65	79325	129980	0,0002	-0,0003	-0,0210	0,0136
33	209317	38,58	79325	130000	0,0000	-0,0018	0,0000	0,0002
34	210298	38,52	80237	130060	0,0047	-0,0016	0,0115	0,0005
35	210656	38,51	82467	128180	0,0017	-0,0003	0,0278	-0,0145
36	211637	38,45	83379	128260	0,0047	-0,0016	0,0111	0,0006
37	212590	38,43	83062	129530	0,0045	-0,0005	-0,0038	0,0099
38	213542	38,41	83974	129570	0,0045	-0,0005	0,0110	0,0003
average	206973,00	39,66	75673,95	131298,9474	0,0011	-0,0023	0,0049	-0,0009

Table A2.20. Instance 5 with 80 candidate facilities and 400 demand points

Efficient Point #	Total Costs	Conditional β-mean values	TF	TV	Total Cost Change	Conditional β-mean Change	TF change	TV change
1	190311	41,42	65017	125294	-	-	-	-
2	190333	41,2	65017	125316	0,0001	-0,0053	0,0000	0,0002
3	190379	40,86	65842	124537	0,0002	-0,0083	0,0127	-0,0062
4	190403	40,85	65720	124683	0,0001	-0,0002	-0,0019	0,0012
5	190574	40,12	65017	125557	0,0009	-0,0179	-0,0107	0,0070
6	190634	40,11	65842	124792	0,0003	-0,0002	0,0127	-0,0061
7	190643	39,59	65842	124801	0,0000	-0,0130	0,0000	0,0001
8	190707	39,39	65842	124865	0,0003	-0,0051	0,0000	0,0005
9	190859	39,32	66644	124215	0,0008	-0,0018	0,0122	-0,0052
10	190904	39,13	66644	124260	0,0002	-0,0048	0,0000	0,0004
11	191119	39,02	66644	124475	0,0011	-0,0028	0,0000	0,0017
12	191241	38,94	66939	124302	0,0006	-0,0021	0,0044	-0,0014
13	191395	38,86	67741	123654	0,0008	-0,0021	0,0120	-0,0052
14	191439	38,68	67741	123698	0,0002	-0,0046	0,0000	0,0004
15	191654	38,55	67741	123913	0,0011	-0,0034	0,0000	0,0017
16	191780	38,51	68537	123243	0,0007	-0,0010	0,0118	-0,0054
17	192141	38,39	69269	122872	0,0019	-0,0031	0,0107	-0,0030
18	192315	38,35	67741	124574	0,0009	-0,0010	-0,0221	0,0139
19	192338	38,28	67741	124597	0,0001	-0,0018	0,0000	0,0002
20	192339	38,25	67619	124720	0,0000	-0,0008	-0,0018	0,0010
21	192398	38,21	67741	124657	0,0003	-0,0010	0,0018	-0,0005
22	192508	38,13	68345	124163	0,0006	-0,0021	0,0089	-0,0040
23	192525	38,1	68415	124110	0,0001	-0,0008	0,0010	-0,0004
24	192676	37,85	70366	122310	0,0008	-0,0066	0,0285	-0,0145
25	192717	37,84	69269	123448	0,0002	-0,0003	-0,0156	0,0093
26	193028	37,59	69564	123464	0,0016	-0,0066	0,0043	0,0001
27	193252	37,24	70366	122886	0,0012	-0,0093	0,0115	-0,0047
28	193850	37,22	71971	121879	0,0031	-0,0005	0,0228	-0,0082
29	194428	37,21	71971	122457	0,0030	-0,0003	0,0000	0,0047
average	191754,83	38,87	67694,76	124060,0690	0,0008	-0,0038	0,0037	-0,0008

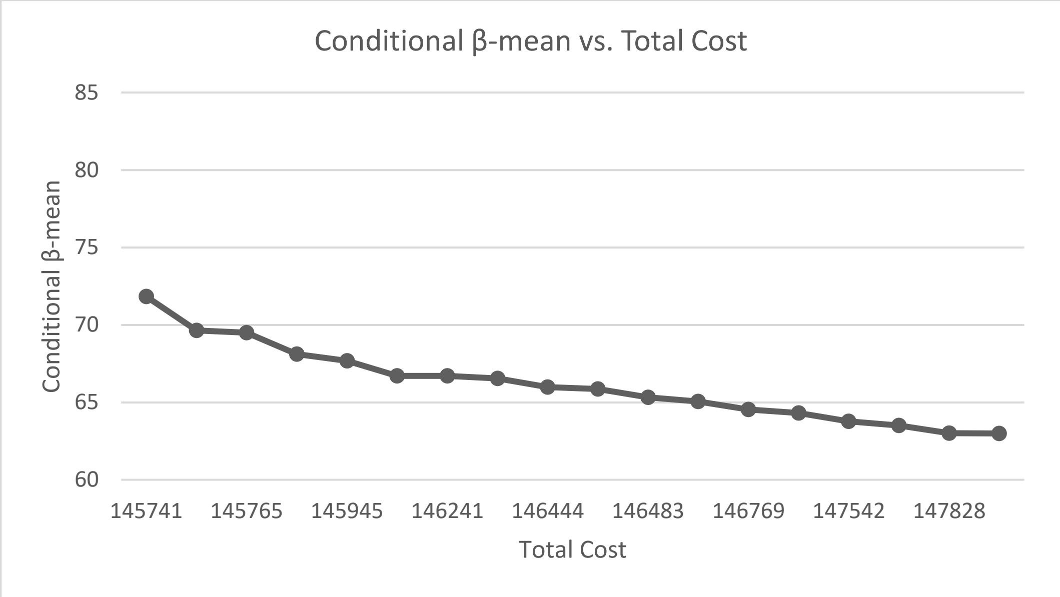

Figure A2.1. A trade-off between the conditional β-mean and the total costs (30-200-1)

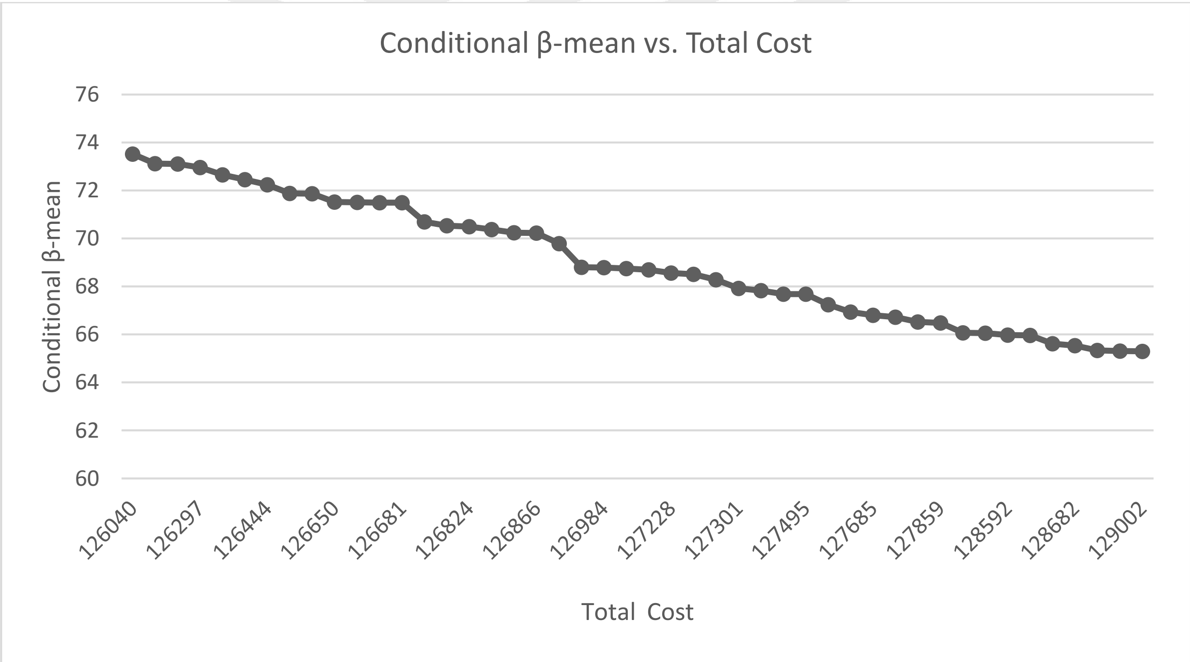

Figure A2.2. A trade-off between the conditional β-mean and the total costs (30-200-2).

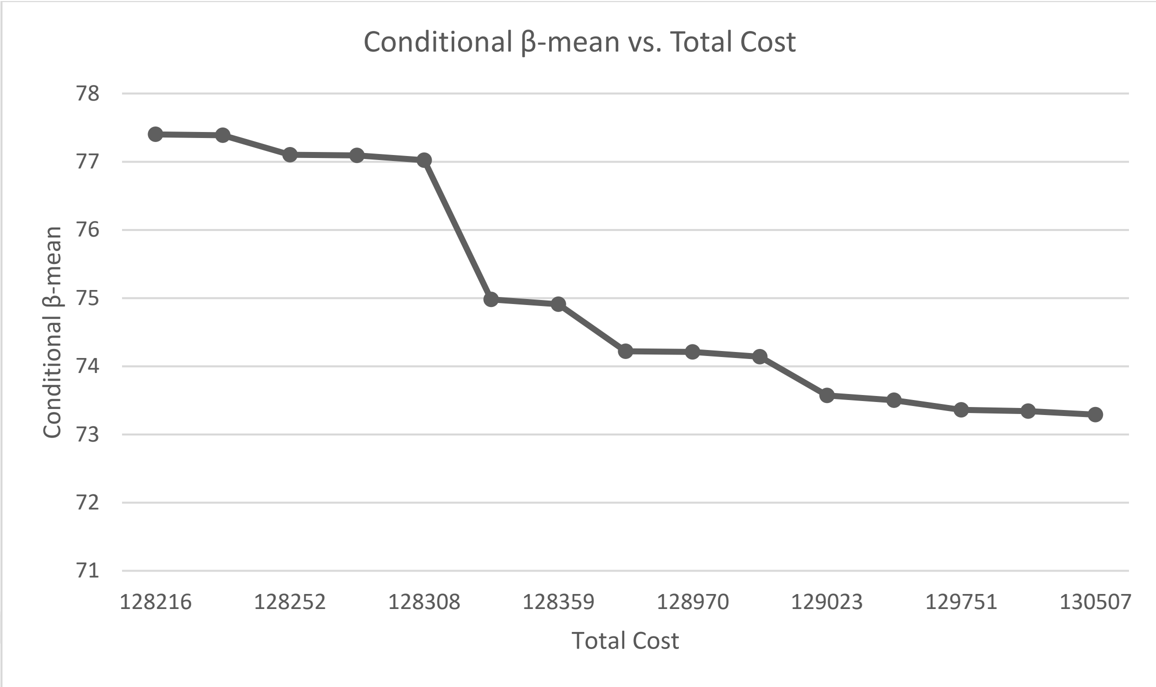

Figure A2.3. A trade-off between the conditional β-mean and the total costs (30-200-3).

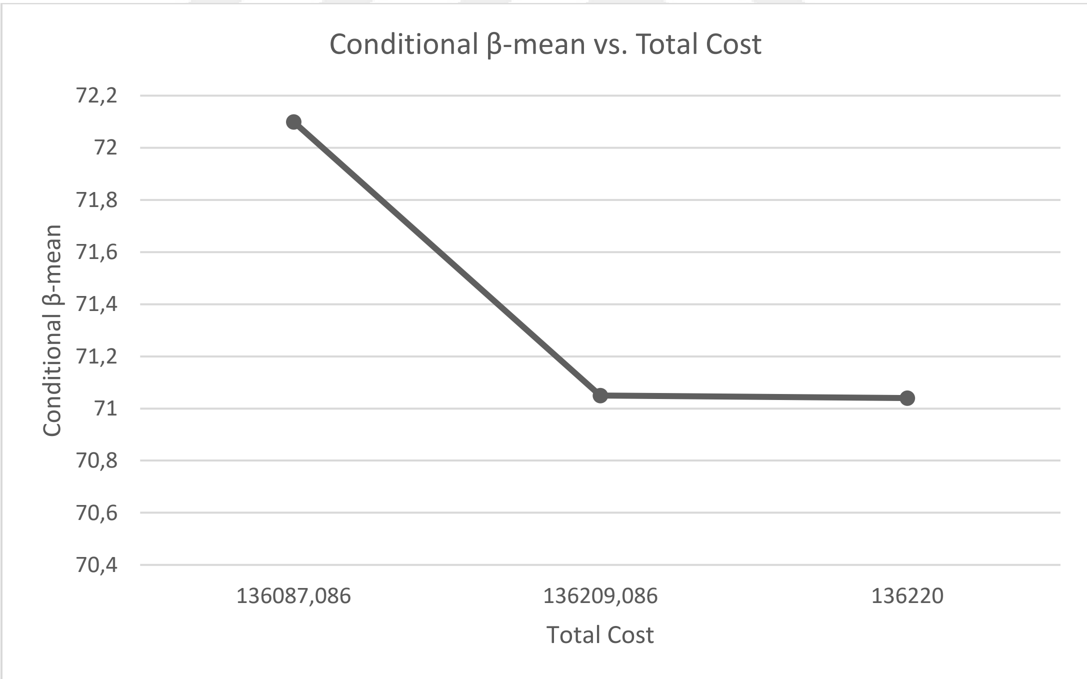

Figure A2.4. A trade-off between the conditional β-mean and the total costs (30-200-4).

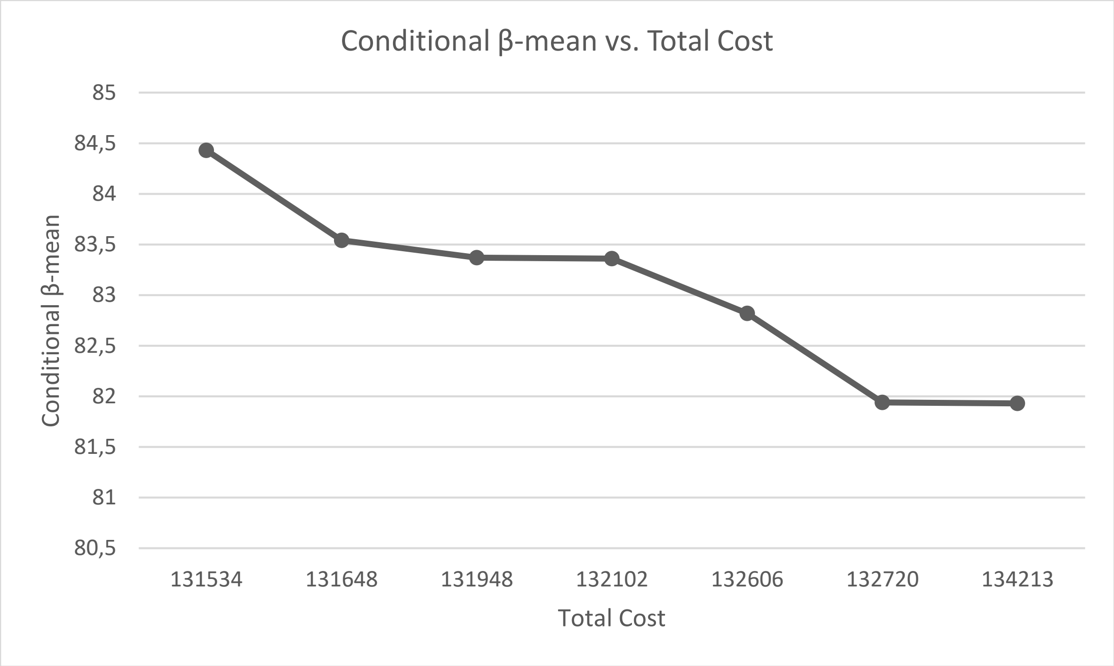

Figure A2.5. A trade-off between the conditional β-mean and the total costs (30-200-5).

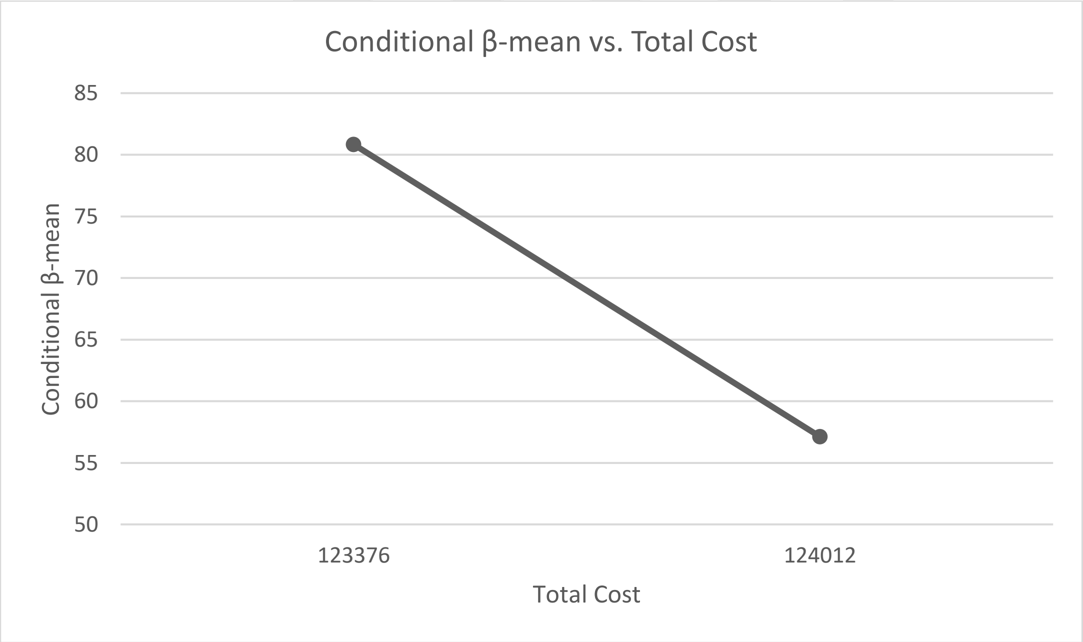

Figure A2.6. A trade-off between the conditional β-mean and the total costs (60-200-1).

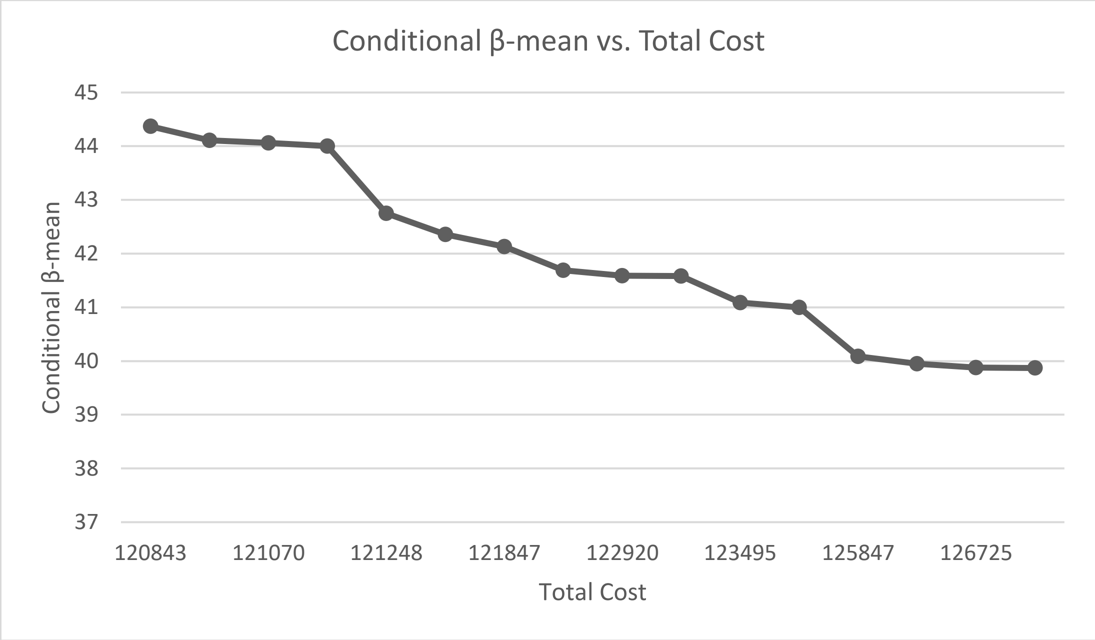

Figure A2.7. A trade-off between the conditional β-mean and the total costs (60-200-2).

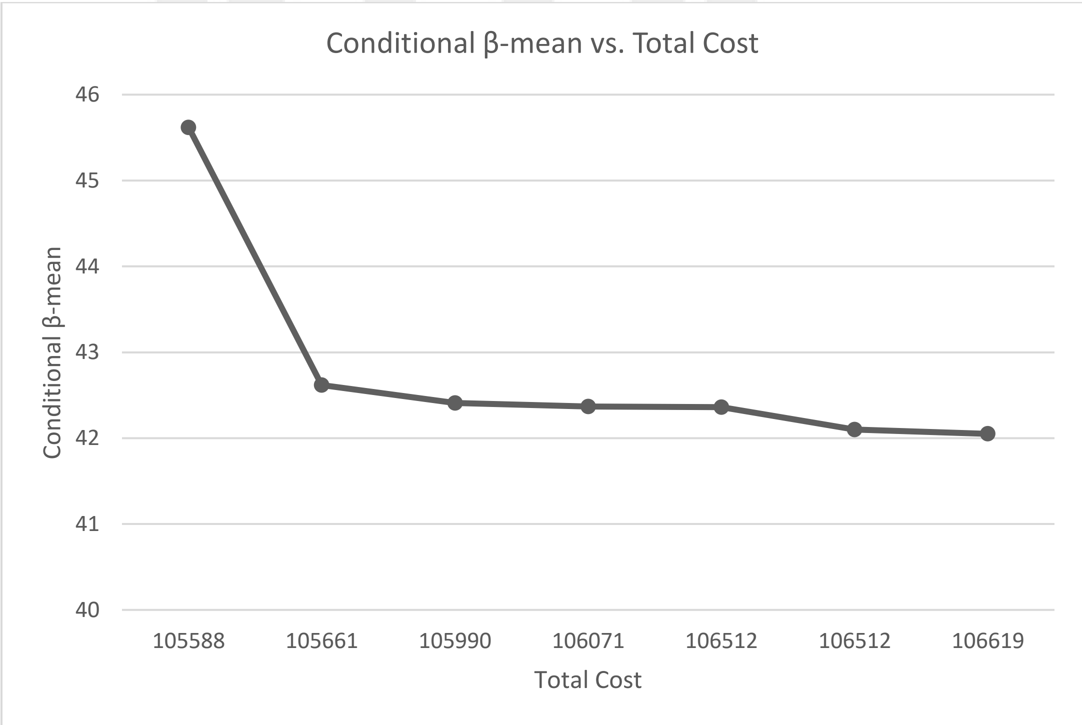

Figure A2.8. A trade-off between the conditional β-mean and the total costs (60-200-3).

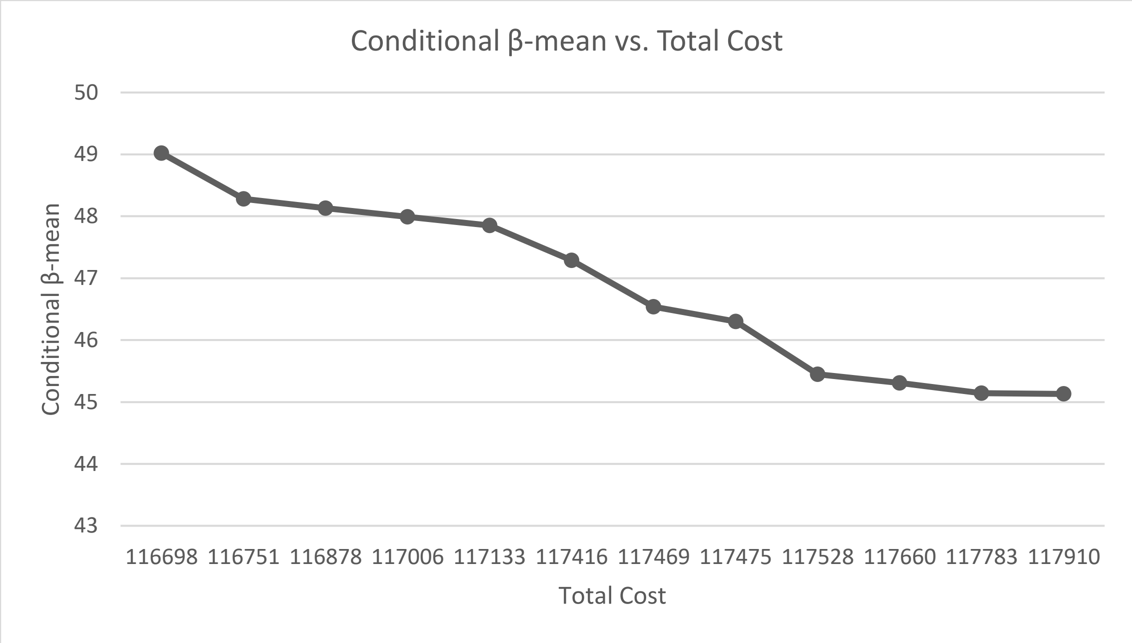

Figure A2.9. A trade-off between the conditional β-mean and the total cost values (60-200-4)

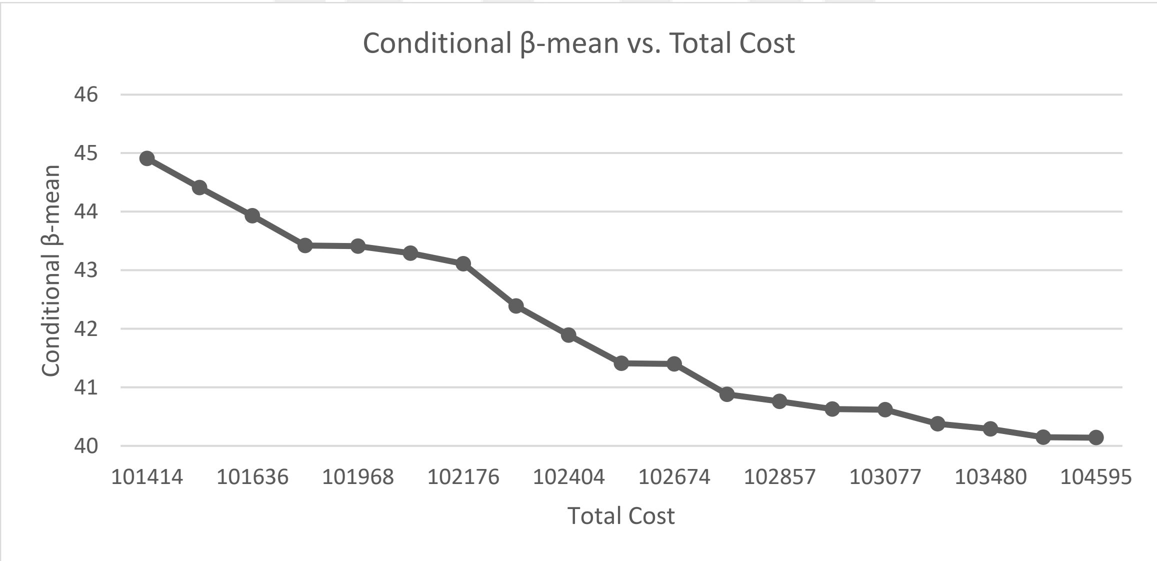

Figure A2.10. A trade-off between the conditional β-mean and the total cost values (60-200-5)

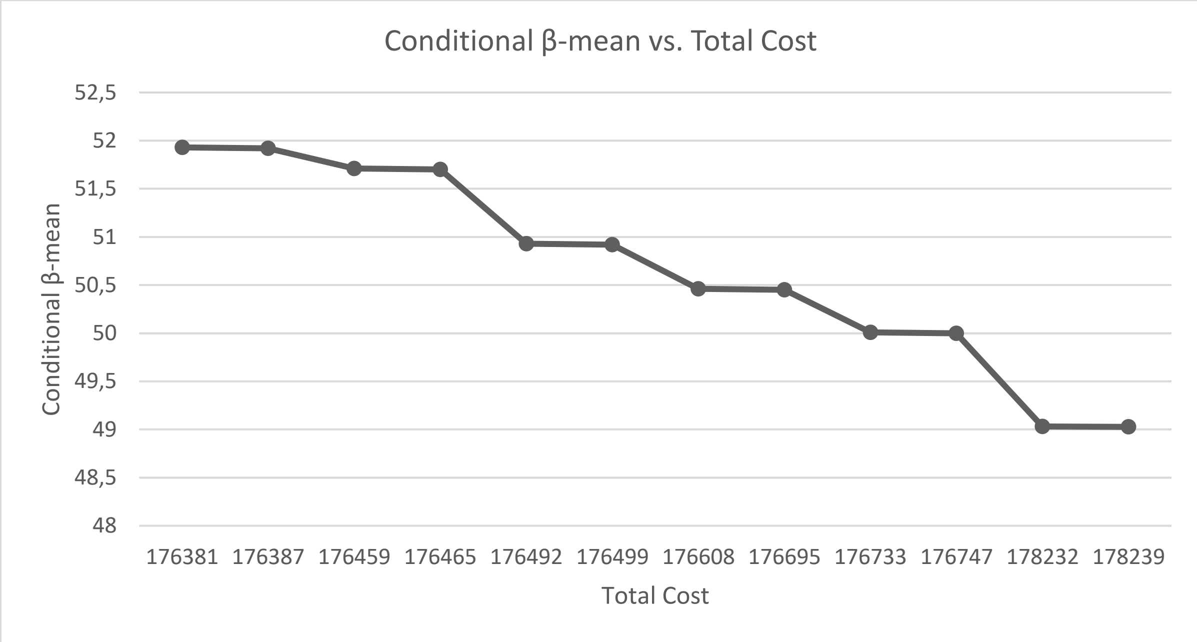

Figure A2.11. A trade-off between the conditional β-mean and the total cost values (60-300-1)

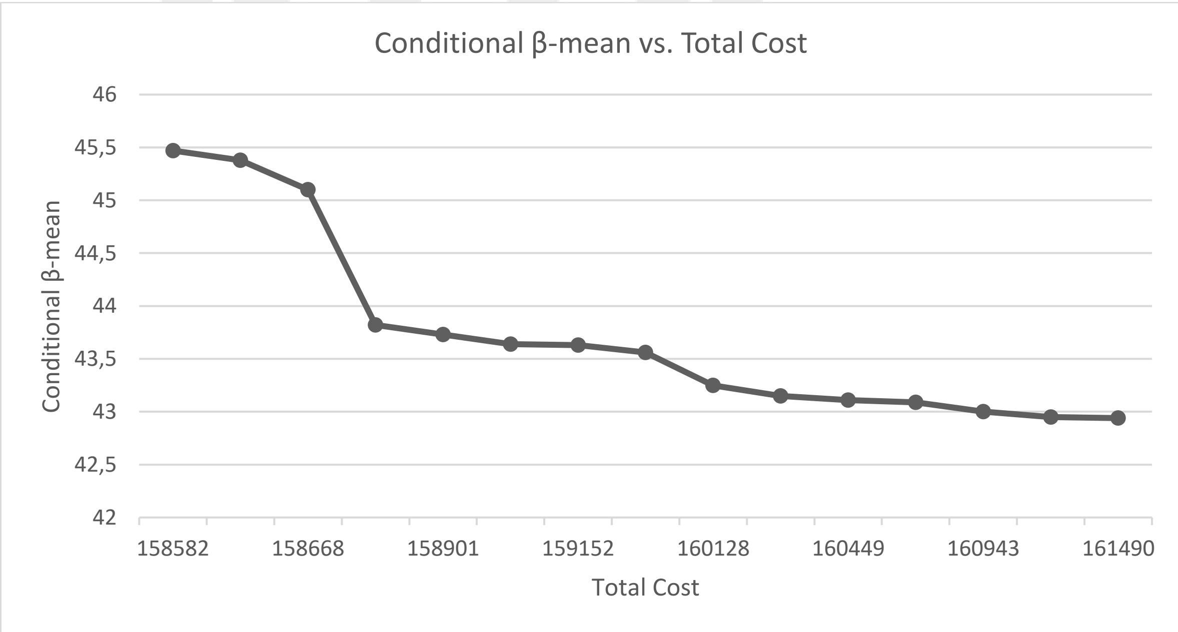

Figure A2.12. A trade-off between the conditional β-mean and the total cost values (60-300-2)

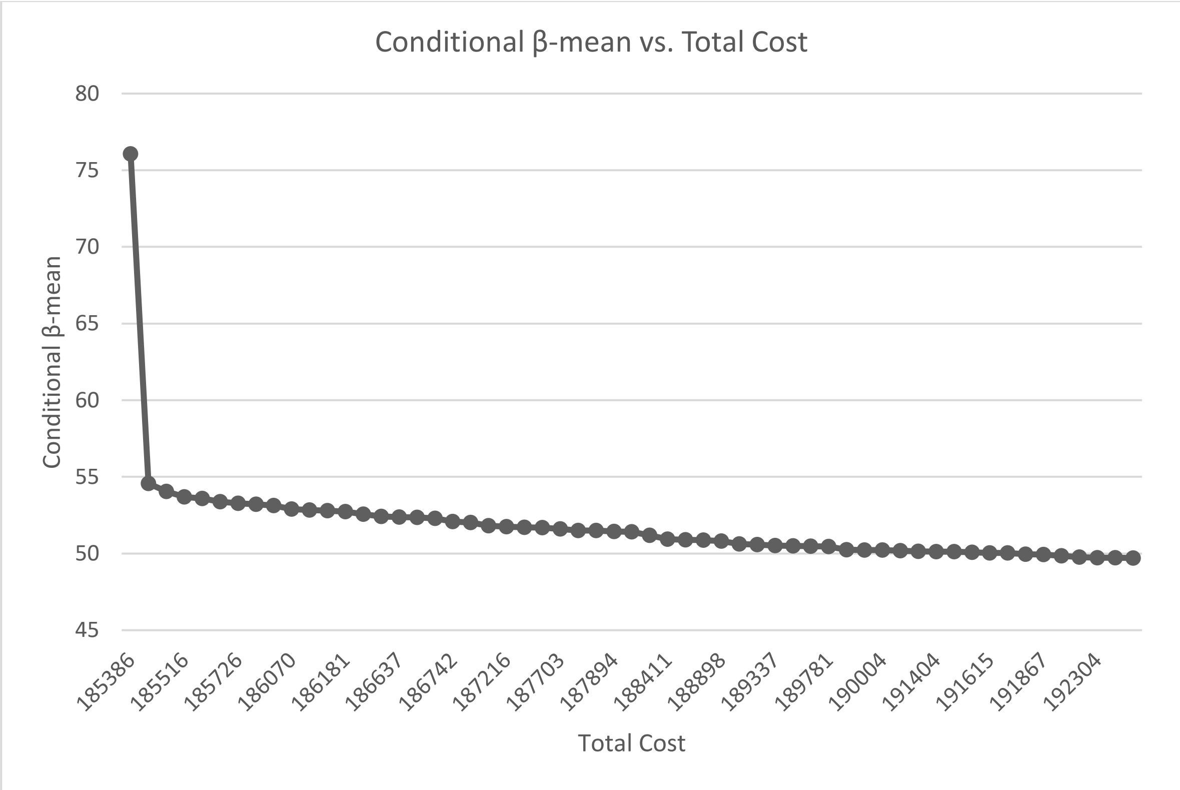

Figure A2.12. A trade-off between the conditional β-mean and the total cost values (60-300-3)

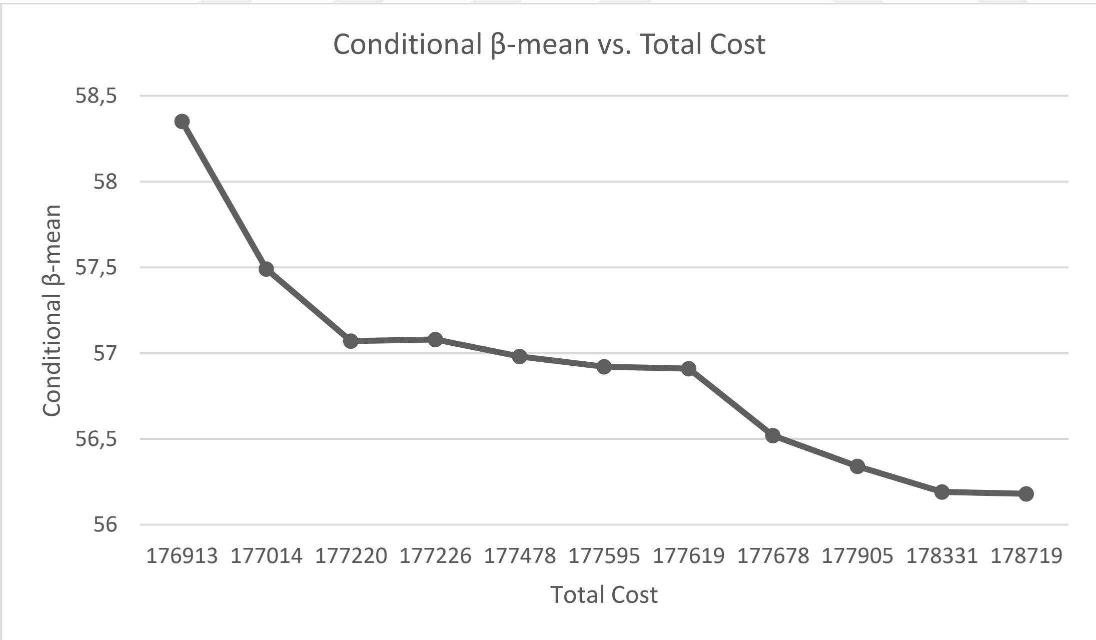

Figure A2.13. A trade-off between the conditional β-mean and the total cost values (60-300-4)

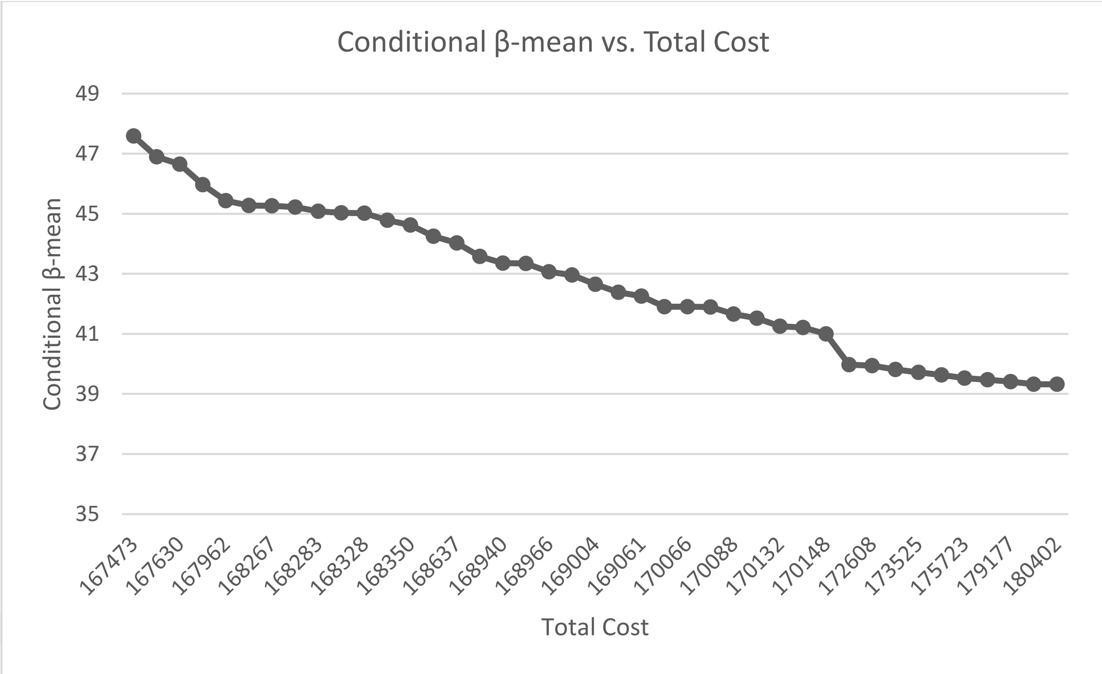

Figure A2.14. A trade-off between the conditional β-mean and the total cost values (60-300-5)

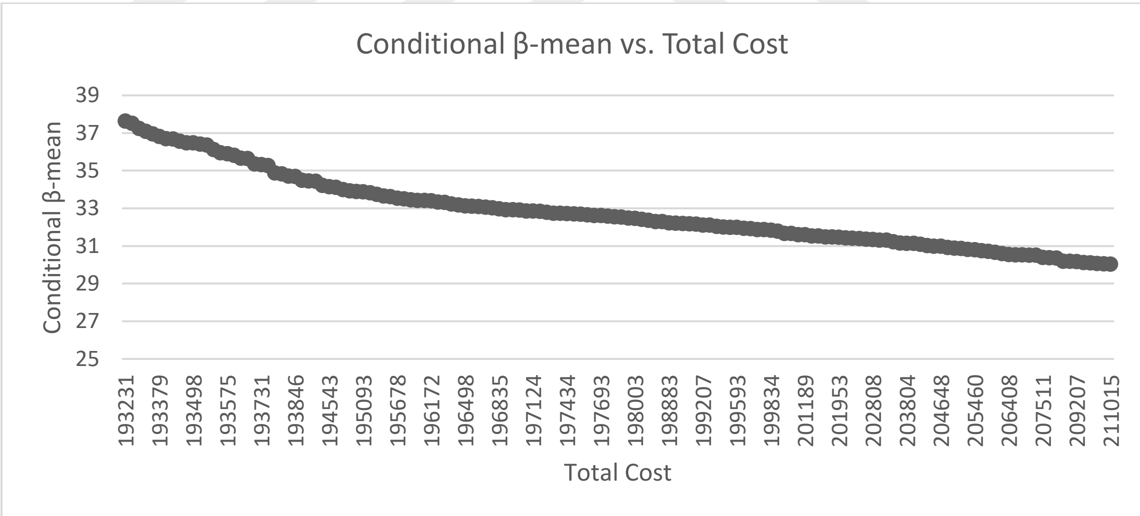

Figure A2.15. A trade-off between the conditional β-mean and the total cost values (80-400-1)

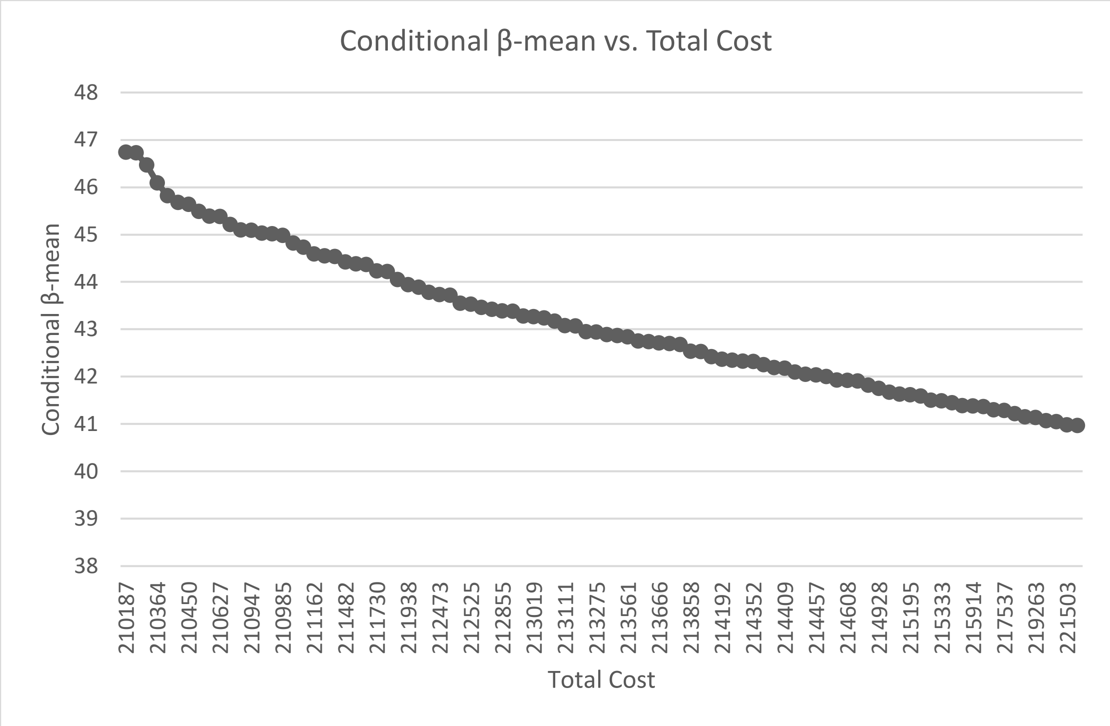

Figure A2.16. A trade-off between the conditional β-mean and the total cost values (80-400-2)

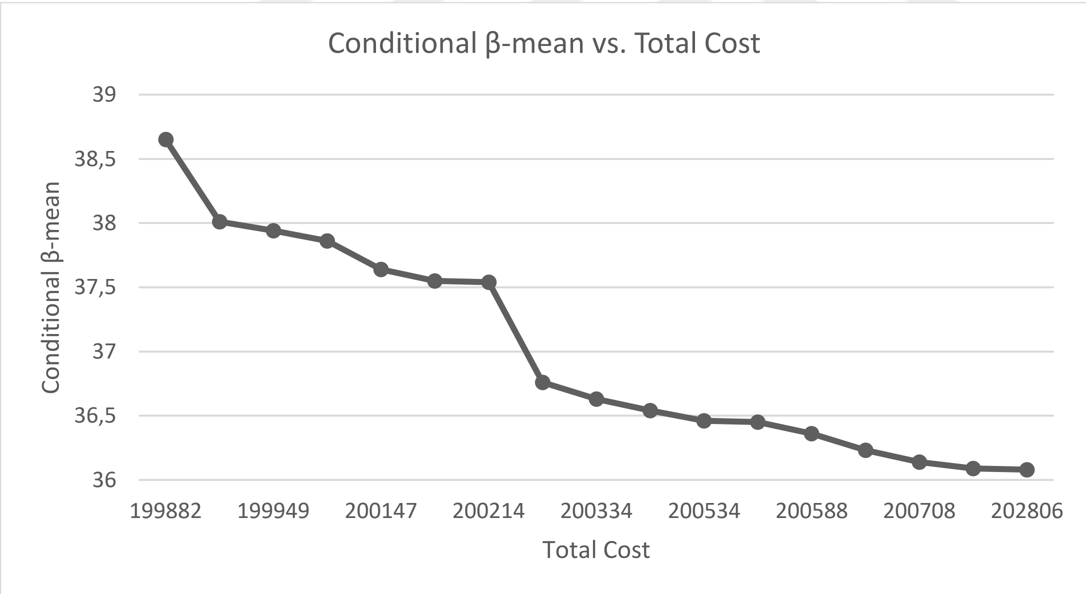

Figure A2.17. A trade-off between the conditional β-mean and the total cost values (80-400-3)

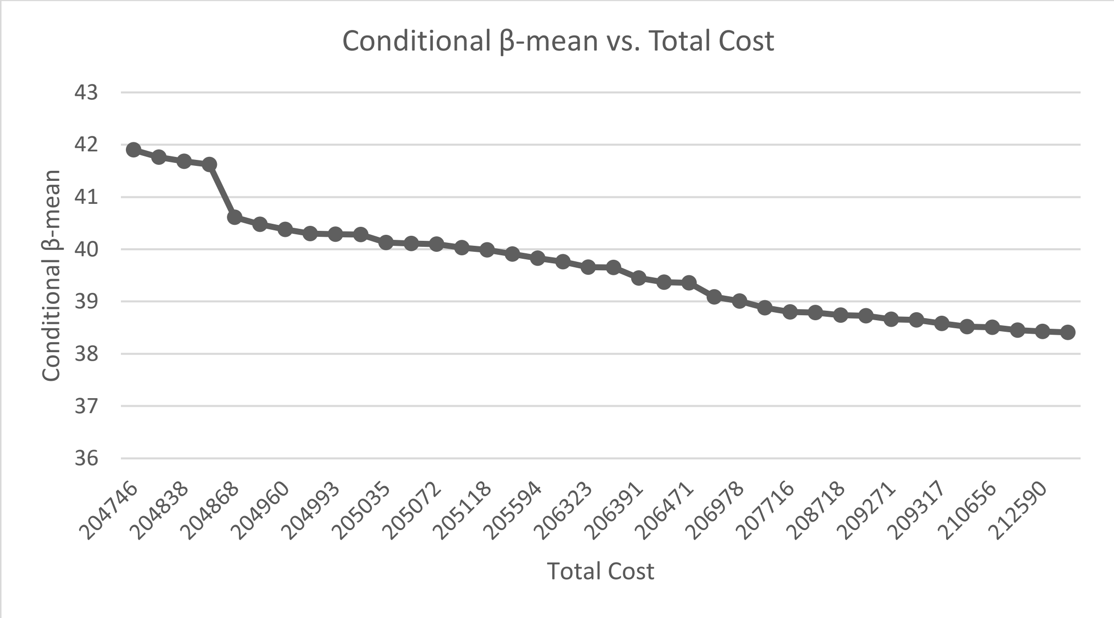

Figure A2.19. A trade-off between the conditional β-mean and the total cost values (80-400-4)

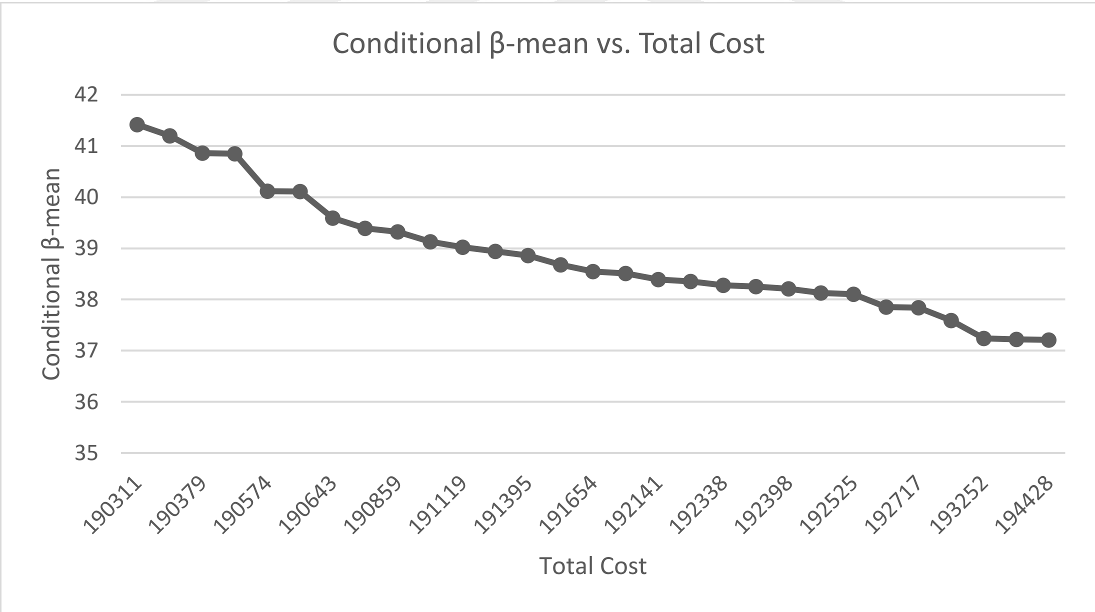

Figure A2.20. A trade-off between the conditional β-mean and the total cost values (80-400-5)